CHAPTER 48

A Multiblock Multigrid Solution Procedure for Multielement Airfoils

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Abstract. A block-structured grid formulation is presented and discussed. The compressible Euler equations are solved on the decomposed domain with a multigrid method based on Runge-Kutta time stepping and centered spatial differencing. The flexibility of the multiblock approach is demonstrated by computing low speed inviscid flow over two different multielement airfoil configurations.

Introduction. In the past few years substantial progress has been made in the development of effective algorithms for solving the Euler and Navier-Stokes equations. Yet the generation of an appropriate grid on which to compute the flow over complex aerodynamic configurations continues to be a major obstacle. One approach for resolving this problem is the use of block-structured grids. With this method the domain is partitioned into a number of subdomains or blocks. The blocks are generally determined so as to allow the construction of a body-fitted grid, having as much regularity as possible, around each component of the configuration being considered. Such grids facilitate the implementation of boundary conditions and the resolution of the flow field near the body. The flow solver is applied to each block, and the blocks communicate at the interface boundaries. If the block interface boundaries are treated appropriately, then they are transparent to the numerical scheme, and the scheme behaves the same as it does for a single-block formulation. This domain decomposition approach permits a single computer program to be used for computing the flow over a wide variety of complex geometries, without requiring modifications in the program for each new configuration.

In this paper we consider a block-structured grid for calculating the inviscid flow over multielement airfoil configurations. A combination of algebraic and hyperbolic grid generation techniques is employed to construct the grid in the domain. Although we enforce C_0 continuity of the mesh lines at the interface boundaries, the present computational technique allows the number of mesh intervals at the intersection of adjacent blocks to be different. This provides greater flexibility in generating regular grids as well as a capability to introduce local mesh refinement. Moreover, the framework exists so that one can easily remove the requirement of C_0 continuity. The compressible Euler equations are solved in each subdomain with a multigrid method. An explicit Runge-Kutta scheme is the driver of the multigrid method. Spatial derivatives are approximated with a cell-centered, finite-volume method.

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Numerical Method. The spatial derivatives in the time-dependent Euler equations are approximated with central differences. A cell-centered, finite-volume technique, which is described in (Swanson, 1991), is used to obtain the spatial discretization. For sufficiently smooth meshes the discretization is second-order accurate. Adaptive numerical dissipation terms are appended to the resulting semidiscrete formulation. These terms, which are a blending of second and fourth differences, are included to provide shock capturing capability and to give the necessary background dissipation for convergence. In smooth regions of a flow field the dissipation terms are third order. Additional discussion of the numerical dissipation is given in (Swanson, 1987). The semidiscrete equations are integrated in time with a modified five stage explicit Runge-Kutta scheme presented in (Jameson, 1985). On the first, third, and fifth stages there is a weighted evaluation of the dissipation terms, which results in a good parabolic stability limit. The decoupling of the temporal and spatial discretizations makes the scheme amenable to convergence acceleration techniques, which are very beneficial in the computation of steady flows.

Three techniques are employed to accelerate convergence to steady state. The first one is local time stepping, where the solution at any point in the domain is advanced at the maximum time step allowed by stability. This results in faster signal propagation, and thus, faster convergence. The second technique is variable coefficient implicit residual smoothing. It can be regarded as simply a mathematical step applied after each Runge-Kutta stage to extend the local stability range. The third technique is multigrid. A multigrid method involves the application of a sequence of meshes to a discrete problem to accelerate convergence of the time-stepping scheme. Successively coarser meshes can be generated by starting with the desired fine mesh and eliminating every other mesh line in each coordinate direction. An equivalent fine grid problem is defined on each coarse grid. Appropriate operators are introduced to transfer information between the meshes. In the method applied here a fixed W-type cycle is used to execute the multigrid strategy. The efficiency of the multigrid process depends strongly upon effective high frequency damping characteristics of the driving scheme. Such damping behavior is provided by the five stage Runge-Kutta scheme. The good smoothing of the highest frequencies on the coarser meshes allows rapid removal of the low frequency errors in the fine grid solution. There are two additional advantages of the multigrid method. First, less computational effort is required on the coarser meshes. Second, information is propagated faster on the coarser meshes due to larger allowable time steps. To provide a well conditioned initial solution, a Full Multigrid (FMG) method is used.

Grid Generation and Multiblock Strategy. For a multielement airfoil configuration, we construct a C-type boundary curve around each airfoil element, which defines the blocks of the domain (see Figure 1). We also prescribe grid point distributions along these boundary curves and the surfaces of the airfoils. An algebraic grid generation procedure, which is based on transfinite interpolation, is used to determine the interior grid lines. To generate the remaining portion of the grid, the outer boundary curve of the entire configuration is defined as an initial data surface for a hyperbolic grid generator. The C-type topology for the airfoil elements facilitates good resolution in the leading edge region, which is needed to properly capture the leading edge suction peak. Embedding C-type grids in a global mesh does produce singular points in the flow field. In general, one must ensure that these singular points do not occur too close to the leading edge of an airfoil element, since this can cause significant deterioration in the accuracy of the stagnation flow prediction. Such a loss in accuracy would give an incorrect prediction of the lift of the element.

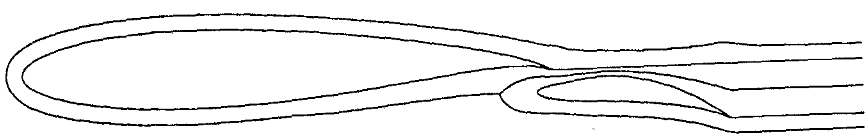

Figure 1 Block Structure for Karman-Trefftz Airfoil with a Flap.

Although we have obtained fairly reasonable grids with the process just described, we have imposed constraints in the present work that do limit the quality of the grid. These constraints are as follows: 1) use as few blocks as possible, 2) apply simple interface condition (i.e., C_0 continuity). It should be emphasized that these conditions were introduced to facilitate development and keep computational time as small as possible, and they are not inherent to the multiblock method.

At this point we briefly outline the solution strategy with the block-structured grids. Each block includes two additional or auxiliary cells normal to its boundaries. These auxiliary cells are required to treat the fourth difference numerical dissipation in the same manner as it is in the interior of the domain. The solution procedure is as follows. First, we calculate auxiliary cell flow quantities for a given block from values existing in adjoining blocks. Then the solution is updated in the given block, and the convective flux across each cell face at an interface boundary is stored. The stored fluxes are used in the boundary conditions for the next block to be considered. This strategy is equivalent to computing auxiliary cell information directly from the adjacent block if C_0 continuity exist at the interface. Also, it guarantees conservation across interface boundaries when grid refinement is applied. The solution in all blocks is updated at a given stage of the Runge-Kutta scheme before moving on to the next stage; and thus, there is no time lag between the blocks. This also means that all blocks are updated at a given multigrid level before going to the next multigrid level, which minimizes the effect of the multiblock strategy on the multigrid performance.

Results. The first case considered is that of flow over a Karman-Trefftz airfoil with a flap. An incompressible analytical solution exists for this case (Williams, 1973) and can be used for comparison purposes. The computational domain was broken up into three blocks, as shown in Figure 1. The first block is a C-grid around the flap, the second is a C-grid around the main airfoil, and the third is a C-grid around both elements. Figures 2 and 3 show a portion of the computational grid. The computed surface pressures for this case, which was run at a

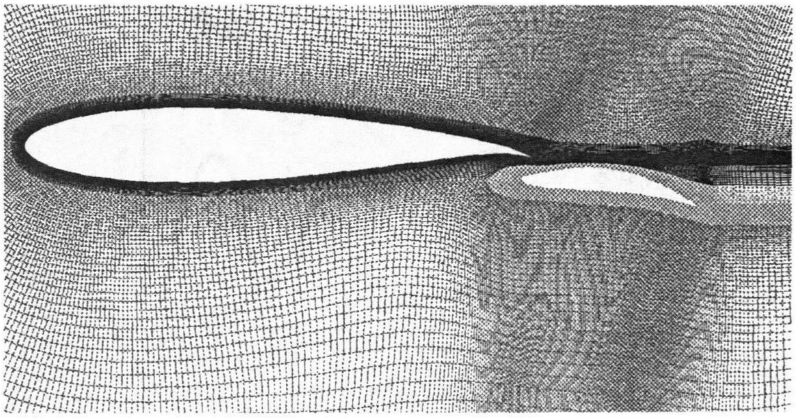

Figure 2 Grid for Karman-Trefftz Airfoil with a Flap.

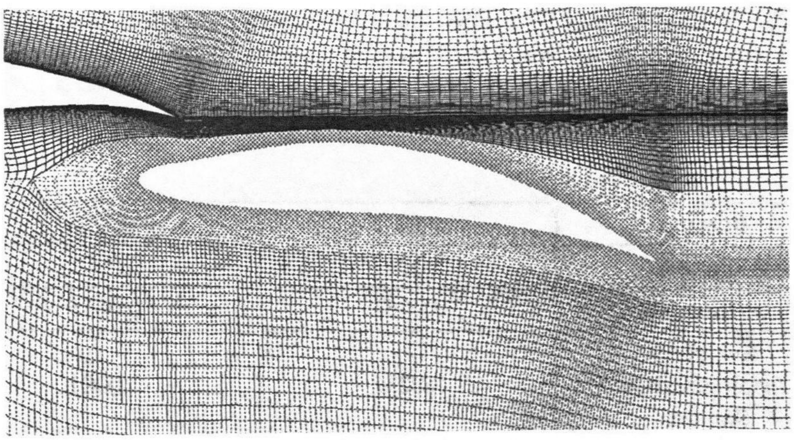

Figure 3 Detail of Flap Area.

free-stream Mach number of 0.125 and zero degrees angle of attack, are compared with the exact solution in Figure 4. Good agreement between the solutions can be seen in this figure, as

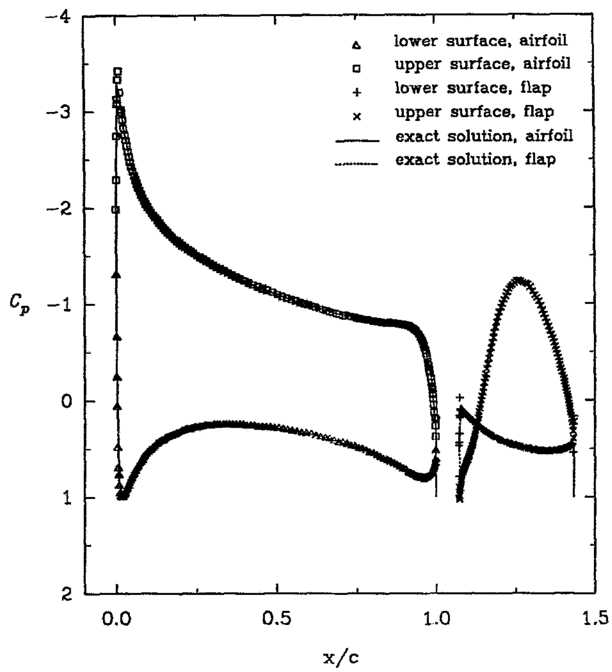

Figure 4 Comparison of Computed Surface Pressures with the Incompressible Analytic Solution.

well as in a comparison of total lift and drag for the configuration, which is presented in Table 1. Table 1 also includes results from a highly adapted unstructured grid calculation (Mavriplis, 1990). Finally, the convergence history for this calculation is shown in Figures 5.

Scheme	Cl	Cd
Multiblock	2.0297	-0.0001
Unstrucured	2.0362	-0.0016
Analytical Incompressible Solution	2.0281	-0.0001

Table 1. Comparison of lift and drag for Karman-Trefftz configuration.

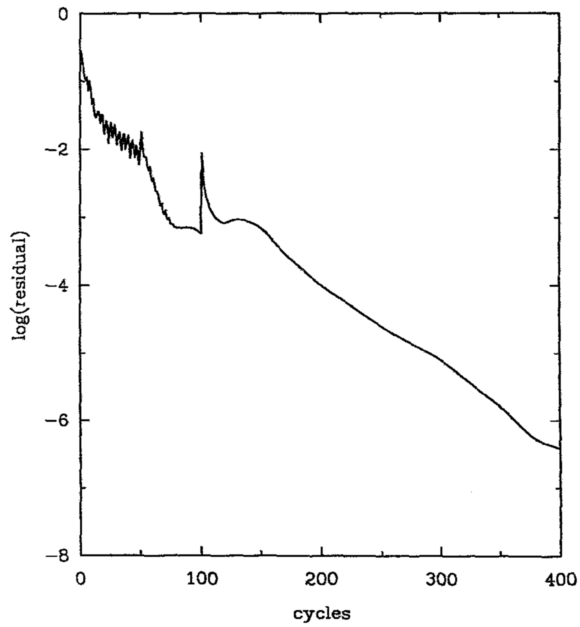

Figure 5 Convergence History for Karman-Trefftz Configuration.

The block structure for the second test case, flow over a three element airfoil, is shown in Figure 6. As before, a C-grid is constructed around each element, and a final C-grid is

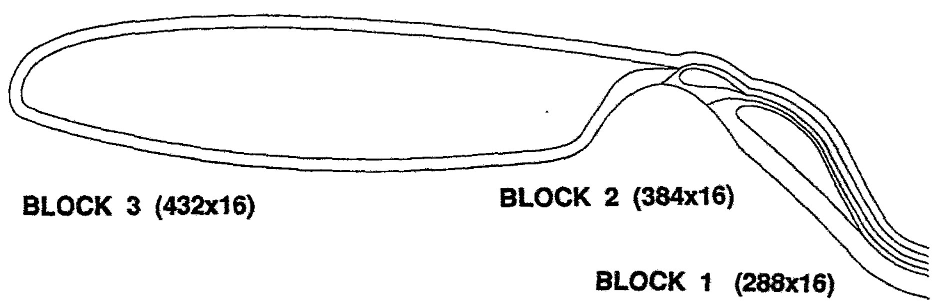

Figure 6 Block Structure for 3–Element Configuration.

positioned around the overall configuration. Notice, however, that the grid around the lower surface of the main element does not extend to the farfield boundary, but instead joins to the outer boundary of the grid around the center element. Figures 7 and 8 detail a portion of the computational grid. The flow conditions for this case are a free-stream Mach number of 0.20

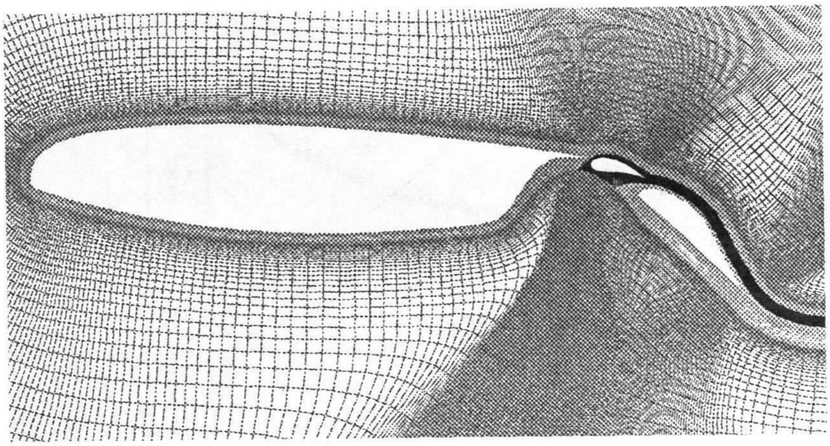

Figure 7 Grid for 3–Element Configuration.

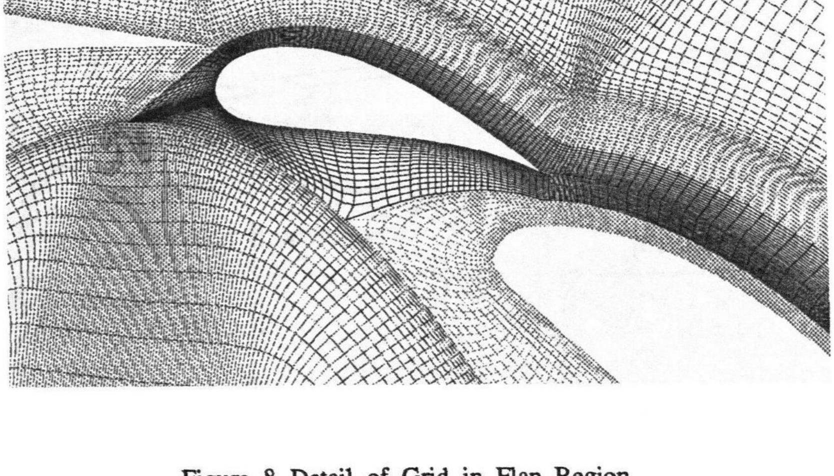

Figure 8 Detail of Grid in Flap Region.

and an angle of attack of 8 degrees. Figure 9 shows a comparison of the computed surface pressures with those obtained by solving the full potential equation (Mavriplis, 1990). Even

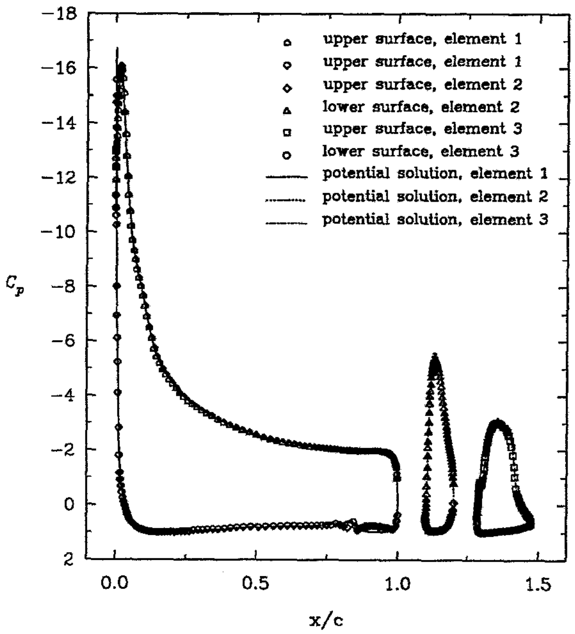

Figure 9 Comparison of Computed Surface Pressures with Full Potential Solution for 3–Element Configuration.

though there is some nonregularity in the grid, the convergence history for this case is similar to the one shown for the two-element airfoil.

Concluding Remarks. A multiblock formulation has been combined with a multigrid method to produce a versatile solver for the Euler equations. Low speed flow around two- and three-element airfoil configurations has been computed. The predicted pressure distributions in both cases have compared well with exact or highly resolved potential flow solutions. Good convergence behavior has been obtained for each problem.

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