CHAPTER 44

# Coupling Particles and Finite Differences for the Nonlinear Radiative Transfer Equations

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**Abstract.** The radiative transfer equations have a transport behaviour in "transparent" materials and a diffusion one in opaque materials. So our domain decomposition philosophy consists of solving the equations adapted to each material : the transport equation for the transport zones, and the diffusion equation for opaque zones. The main problem is then to correctly couple the two models. We have developped a new particle method for the transport model which is very similar in its form to the usual finite differences scheme used for the diffusion model. Thanks to that similarity (which does not exist in the continous models), we were able to build an implicit conservative scheme between the two methods.

**The underlying physics and the basic equations.** The radiative transfer equations modelize the emission and absorption of light by hot matter (stars, plasmas...). The first equation (the transfer equation) represents the transport, the emission-absorption phenomena and the scattering of photons (interactions light-matter), while the second one (the energy balance equation) establishes the balance

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between the radiative energy and the intern energy of matter :

$$\begin{cases} \dfrac{1}{c}\dfrac{\partial I_k}{\partial t} + \Omega.\nabla I_k + Q(I_k) + \sigma_k I_k = \dfrac{\sigma_k}{4\pi} b_k(\phi) \\ \dfrac{1}{c}\dfrac{\partial E(\phi)}{\partial t} + \sigma\phi - \displaystyle\int \sum_k \sigma_k I_k \, d\Omega = 0 \end{cases}$$

$I_k(x, t, \Omega)$ is the radiative intensity in group of frequency $k$, ($x$ position, $t$ time, $\Omega$ direction of propagation); $I_k$ is an energy by unit of time, frequency, solid angle and area orthogonal to $\Omega$.

$c$ is the speed of light.

$\sigma_k$ is the opacity of matter.

$\phi = aT^4$ ($T$ temperature, $a$ radiation constant).

$b_k$ is the reduced Planck function ($\sum_k b_k = 1$).

$Q$ is the integral operator (on $\Omega$) representing the elastic angular scattering of photons on free electrons.

$E$ is the specific density of intern energy.

For more details on these equations see [1] or [2].

As these equations are solved on a lagrangian mesh, we prefer to use particles methods, because the construction of a good angular discretization is a very difficult problem for this kind of transport equation.

When the opacity ($\sigma$) increases, the phenomena has no longer a transport behaviour but becomes a diffusion process which can be modelized by the so called Rosseland diffusion equation :

$$\frac{1}{c}\frac{\partial}{\partial t}(E(\phi) + \phi) - \nabla.(\frac{1}{3\sigma}\nabla\phi) = 0$$

As particles methods becomes both too expensive and useless detailed in opaque media (great $\sigma$), we prefer to use this non linear diffusion equation (which is much more easy and less expensive to solve then the full equations) in such media.

**The Domain Decomposition.** Our "philosophy" is to treat transparent materials (transport phenomena, small $\sigma$) by a particle method for the full equations, and the opaque ones by a standard finite differences scheme for the Rosseland diffusion equation. Our aim is then to couple these different approximations of the same physical phenomena.

We have developped a new time discretization for the full equations (transport model) which leads to a new kind of numerical method that we call Particle Matrix Method. In this method, the particles are used to build a "transfer matrix" which will be used to solve the energy balance equation [3] [4] [6]. This method is a great improvement to the classical time discretization [5].

The scheme is fully implicit and similar to the diffusion one, so that the coupling scheme between transport and diffusion zones becomes "natural".

This coupling scheme is based on the integral solution (by semi groups) of the transfer equation, and on Marshak relation (Robin boundary condition) at the interface between transport and diffusion zones.

After time and space discretizations, the energy equation can be written "everywhere" in the following nice form :

$$E(\phi) + M\phi = S$$

where $M$ contains the usual five points diffusion matrix for the diffusion zones, the particle matrix for the transport zones and the coupling terms due to Marshak relation. The coupling is fully implicit and conservative [3] [4].

After tracking particles to get the transport matrix, after building the diffusion matrix, the energy equation is solved by the Newton method, for which at each iteration a non symetric linear system has to be solved. After trying different methods (ORTHOMIN, CGR, CGS, CGSTAB), the best results were obtained with CGSTAB associated with a diagonal preconditionning [7].

**Conclusion.** This kind of domain decomposition is interesting each time that different modelizations of the same phenomena are necessary. Here, we have a transport phenomena which can be approximated in some regions by a diffusion process.

Thanks to a "new" natural form of the discretized transport equation, we were able to build a good implicit and conservative coupling scheme with the Rosseland equation.

In these proceedings, P. le Tallec shows that it is also possible to couple Boltzmann and Navier Stokes equations.

## References.

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