CHAPTER 45

Pseudospectral Domain Decomposition Techniques for the Navier-Stokes Equations

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Abstract. Pseudospectral approximations to the solution of fourth-order differential equations are constructed. These approximations enable boundary conditions of both Dirichlet and Neumann type to be satisfied by the representation. The equivalence between a variational and a collocation formulation of the problem is demonstrated. The interface conditions in a collocation scheme for a multi-domain problem are derived from the variational formulation. The method is applied to the laminar flow of an incompressible fluid in an L-shaped domain. Numerical results describing the main features of the flow are presented.

1. Introduction. In this paper the solution of the Navier-Stokes equations for the steady, two-dimensional laminar flow of an incompressible fluid through an L-shaped channel is considered. In this approach we use the generalized Legendre Gauss pseudospectral collocation method. The method developed here is an extension of the one developed by Malek and Phillips (1991) for fourth order linear problems in one and two dimensions.

The Navier-Stokes equations may be solved using either primitive variable or stream function formulations. Here we use the stream function formulation since it is simpler and preferable to work with a single equation than with a coupled system. The introduction of a stream function into the governing equations results in a nonlinear fourth order differential equation. This equation is solved iteratively using a Newton linearization technique. A disadvantage of the stream function formulation is that the resulting matrix systems are more badly conditioned than their primitive variable counterparts.

The flow domain is divided into a number of rectangular subdomains. The stream function within each subdomain is approximated by a pseudospectral representation which interpolates values of the stream function at interior collocation points and values of the stream function and its normal derivative on the boundaries and subdomain interfaces. The

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representations are then automatically C^1 continuous in the flow domain. A collocation scheme which was derived by Malek and Phillips (1991) for linear fourth order differential equations is then used to determine the unknowns in the pseudospectral representations at each Newton step. This scheme results in C^3 continuous approximations asymptotically. The Newton process converges extremely rapidly.

There have been a number of papers dedicated to the solution of the stream function formulation of the Navier-Stokes equations using spectral and pseudospectral approximations. Maday and Metivet (1986) study Chebyshev spectral and pseudospectral approximations to the Navier-Stokes equations. They prove convergence of these schemes and derive error estimates in weighted Sobolev spaces. Bernardi and Maday (1988) give a survey of different strategies which may be employed for linear fourth order problems. A Chebyshev spectral element method is described by Karageorghis and Phillips (1989) for the solution of the Navier-Stokes equations in a channel contraction using a stream function formulation.

2. Governing Equations. In terms of dimensionless variables the two-dimensional steady incompressible Navier-Stokes equations are

$$(1) \qquad (\mathbf{v}.\nabla)\mathbf{v} = -\nabla p + \frac{1}{Re}\nabla^2\mathbf{v} \,,$$

$$(2) \qquad \nabla.\mathbf{v} = 0 \,,$$

where $\mathbf{v} = (u, v)$ is the velocity vector, p is the pressure and Re is the Reynolds number. The Navier-Stokes equations (1)-(2) are to be solved in some domain Ω with no-slip boundary conditions $\mathbf{v} = 0$ on rigid walls and with $\mathbf{v}$ specified in the entry and exit sections.

The introduction of a stream function, $\psi(x, y)$, defined by

$$u = -\frac{\partial \psi}{\partial y} \,, \quad v = \frac{\partial \psi}{\partial x} \,,$$

means that the continuity equation (2) is satisfied identically. The pressure may then be eliminated from (1) to give

$$(3) \qquad \nabla^4\psi - Re[\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi)] = 0 \,.$$

The nonlinear equation (3) is solved in the L-shaped domain subject to the boundary conditions shown in Fig. 1. We assume no-slip boundary conditions on the channel walls and Poiseuille flow at entry ($y = a$, $-c \le x \le c$) and exit ($x = b$, $-1 \le y \le 1$).

Let us consider for the moment the solution of the Navier-Stokes equations (1)-(2) subject to homogeneous velocity boundary conditions i.e. $\mathbf{v} = \mathbf{0}$ on $\partial\Omega$. We define the Sobolev spaces

$$\begin{aligned} H^2(\Omega) &= \{w : w \in L^2(\Omega),\ Dw \in L^2(\Omega),\ D^2w \in L^2(\Omega)\}, \\ H_0^2(\Omega) &= \{w \in H^2(\Omega) :\ w = \frac{\partial w}{\partial n} = 0 \text{ on } \partial\Omega\}, \end{aligned}$$

where $L^2(\Omega)$ is the space of square integrable functions on Ω, D represents differentiation with respect to x or y and $\partial/\partial n$ represents differentiation in a direction normal to $\partial\Omega$. Let us introduce the bilinear form $b(.,.)$ on $H^2(\Omega) \times H^2(\Omega)$ defined by

$$(4) \qquad b(\psi, \phi) = (\nabla^2\psi, \nabla^2\phi).$$

Girault and Raviart (1979) show that the solution of (3) satisfies the variational problem: Find $\psi \in H_0^2(\Omega)$ such that

$$b(\psi,\phi) - Re \iint_\Omega \nabla^2\psi\left(\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}\right)dx\,dy = 0\,, \tag{5}$$

for all $\phi \in H_0^2(\Omega)$.

3. Newton Linearization. The governing equation (3) is a nonlinear partial differential equation for the stream function and is solved iteratively using a Newton-type method to linearize it (see Phillips (1984)). Let us rewrite (3) as

$$L(\psi) = 0\,, \tag{6}$$

where L is a nonlinear operator. The iteration begins by finding the solution, $\psi^{(0)}$, of the Stokes problem with no-slip boundary conditions on the rigid walls and Poiseuille flow in the entry and exit sections (see Fig. 1). Suppose that ψ^* is some subsequent approximation to the solution of (3) satisfying the boundary conditions. We replace L by its linearization about ψ^* and then solve the linearized problem

$$L'(\psi^*).\phi = -L(\psi^*)\,, \tag{7}$$

for ϕ subject to homogeneous Dirichlet and Neumann boundary conditions where $L'(\psi)$ is the Frechet derivative of L at ψ defined by

$$L'(\psi).\phi = \nabla^4\phi - Re\,T(\psi,\phi), \tag{8}$$

where

$$T(\psi,\phi) = \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\phi) - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\phi) + \frac{\partial\phi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) - \frac{\partial\phi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi).$$

The new approximation to the solution is thus $\psi^* + \phi$. This completes a single Newton step and is repeated until convergence is reached. The variational form of (7) is thus: Given $\psi^* \in H^2(\Omega)$ find $\phi \in H_0^2(\Omega)$ such that

$$b(\phi,\chi) + Re \iint_\Omega \nabla^2\psi^*\left[\frac{\partial\phi}{\partial y}\frac{\partial\chi}{\partial x} - \frac{\partial\phi}{\partial x}\frac{\partial\chi}{\partial y}\right]dxdy + Re \iint_\Omega \nabla^2\phi\left[\frac{\partial\psi^*}{\partial y}\frac{\partial\chi}{\partial x} - \frac{\partial\psi^*}{\partial x}\frac{\partial\chi}{\partial y}\right]dxdy$$

$$= -\iint_\Omega L(\psi^*)\chi\,dxdy, \tag{9}$$

for all $\chi \in H_0^2(\Omega)$.

4. Pseudospectral Approximations. Malek and Phillips (1991) derive an optimal set of collocation points for the model fourth order problem in one and two dimensions. They construct the generalized Lagrange interpolating polynomial which interpolates the function at the interior nodes and the function and its derivative at the boundary nodes. The use of this polynomial facilitates the imposition of both Dirichlet and Neumann boundary conditions for fourth order differential equations. If the inner product in the discrete variational formulation of the fourth order model problem is defined by the generalized Gaussian quadrature rule associated with this interpolating polynomial, namely

$$\int_{-1}^{1} f(x)dx = \sum_{j=2}^{N-1} w_j f(\xi_j) + w_1(f(-1)+f(1)) + \overline{w}_1(f'(-1)-f'(1)), \tag{10}$$

then an equivalent collocation scheme may be derived. Additionally, the corresponding interface conditions may be determined in the case of domain decomposition. The above quadrature rule is exact for all polynomials of degree $2N-1$ or less when ξ_i, $2 \le i \le N-1$, are the $N-2$ zeros of $P_N''(x)$, where $P_N(x)$ is the Legendre polynomial of degree N. The boundary weights in the above quadrature rule are given by

$$w_1 = \frac{8(2N^2+2N-3)}{3(N-1)N(N+1)(N+2)},\ \overline{w}_1 = \frac{8}{(N-1)N(N+1)(N+2)},$$

and the interior weights by

$$w_j = \frac{32N(N+1)}{(N-1)(N+2)(N+3)^2}\frac{1}{(1-\xi_j^2)[P_{N-1}(\xi_j)]^2},\ \text{for } 2 \le j \le N-1.$$

The L-shaped domain is divided into three rectangular subdomains Ω_1, Ω_2 and Ω_3 (see Fig. 1). In each subdomain the collocation points (ξ_i^k, η_j^k) and associated weights (w_i^k, z_j^k) are found by transforming the subdomain onto the unit square. Let ψ_{ij}^k denote the approximation to ψ in subdomain k at the point (ξ_i^k, η_j^k). The approximations to the normal derivatives of ψ at the boundaries and interfaces are defined similarly. Then we expand ψ in each subdomain using pseudospectral approximations, for example in subdomain I we have

$$\psi^1(x,y) = \sum_{i=1}^{N}\sum_{l=1}^{N} \psi_{il}^1 h_i(\frac{x}{c}) h_l[2(\frac{y-a}{-1-a})-1]$$

$$+\sum_{i=1}^{N}(\frac{-1-a}{2})(\psi_y^1)_{iN} h_i(\frac{x}{c})\overline{h}_N[2(\frac{y-a}{-1-a})-1]\,, \tag{11}$$

where $h_i(x)$, $1 \le i \le N$ and $\overline{h}_i(x)$, $i = 1, N$ are the generalized Lagrange interpolation polynomials defined by

$$h_i(\xi_j) = \delta_{i,j},\ h_i'(\pm 1) = 0,\ 1 \le i,j \le N,$$

$$\overline{h}_i(\xi_j) = 0,\ i = 1, N,\ 1 \le j \le N,\ \overline{h}_i'(\xi_j) = \delta_{i,j},\ i,j = 1, N,$$

with $\xi_1 = -1$ and $\xi_N = 1$. The approximations in subdomains II and III are defined similarly. If we set

$$\psi_{iN}^1 = \psi_{i1}^2\,,\quad (\psi_y^1)_{iN} = (\psi_y^2)_{i1}\,,\quad 1 \le i \le N\,,$$

$$\psi_{Nl}^2 = \psi_{1l}^3\,,\quad (\psi_x^2)_{Nl} = (\psi_x^3)_{1l}\,,\quad 1 \le l \le N\,,$$

we can easily verify that these approximations are C^1 continuous in Ω. The representations automatically satisfy all the boundary conditions.

5. Derivation of the Collocation Scheme. Let γ_{12} and γ_{23} denote the interfaces between subregions I and II, and II and III, respectively. Define the subspace U of $H^2(\Omega_1) \times H^2(\Omega_2) \times H^2(\Omega_3)$ by

$$U = \{\Psi = (\psi^1, \psi^2, \psi^3) :\ \psi^1 = \psi^2, \frac{\partial \psi^1}{\partial y} = \frac{\partial \psi^2}{\partial y} \text{ on } \gamma_{12}, \psi^2 = \psi^3, \frac{\partial \psi^2}{\partial x} = \frac{\partial \psi^3}{\partial x} \text{ on } \gamma_{23}\},$$

and the subspace U_0 of U by

$$U_0 = \{\Psi = (\psi^1, \psi^2, \psi^3) \in U :\ \psi^m = \frac{\partial \psi^m}{\partial n} = 0 \text{ on } \partial\Omega \text{ for } m = 1,2,3\}.$$

We can show, using Green's theorem, that the variational problem: Given $\Psi \in U$, find $\Phi \in U_0$ such that

$$b(\Phi, \Theta) + ReF(\Psi, \Phi, \Theta) = -\sum_{k=1}^{3} \iint_{\Omega_k} L(\psi^k)\theta^k \, dxdy, \tag{12}$$

for all $\Theta \in U_0$, where the bilinear form $b(.,.)$ and the functional $F(\Psi, \Phi, \Theta)$ are defined by

$$b(\Phi, \Theta) \ = \ \sum_{k=1}^{3} \iint_{\Omega_k} (\nabla^2 \phi^k)(\nabla^2 \theta^k) \, dxdy,$$

and

$$F(\Psi, \Phi, \Theta) \ = \ Re \sum_{k=1}^{3} \iint_{\Omega_k} \{\nabla^2 \psi^k [\frac{\partial \phi^k}{\partial y}\frac{\partial \theta^k}{\partial x} - \frac{\partial \phi^k}{\partial x}\frac{\partial \theta^k}{\partial y}] + \nabla^2 \phi^k [\frac{\partial \psi^k}{\partial y}\frac{\partial \theta^k}{\partial x} - \frac{\partial \psi^k}{\partial x}\frac{\partial \theta^k}{\partial y}]\} dxdy,$$

is equivalent to the interface problem: Given $\Psi \in U$, find $\Phi \in U_0$ such that

$$\begin{aligned} & L'(\psi^k)\phi^k = -L(\psi^k) \ \in \ \Omega_k, \ k = 1,2,3, \\ & \phi^k = \frac{\partial \phi^k}{\partial n} = 0 \text{ on } \partial\Omega_k \cap \partial\Omega, \ k = 1,2,3, \\ & \frac{\partial^2 \phi^1}{\partial y^2} = \frac{\partial^2 \phi^2}{\partial y^2}, \frac{\partial^3 \phi^1}{\partial y^3} = \frac{\partial^3 \phi^2}{\partial y^3} \text{ on } \gamma_{12}, \\ & \frac{\partial^2 \phi^2}{\partial x^2} = \frac{\partial^2 \phi^3}{\partial x^2}, \frac{\partial^3 \phi^2}{\partial x^3} = \frac{\partial^3 \phi^3}{\partial x^3} \text{ on } \gamma_{23}. \end{aligned} \tag{13}$$

Define the finite dimensional spaces W_i, $i = 1,2,3$, by

$$W_i = P_{N+1}(\Omega_i) \cap H^2(\Omega_i),$$

where $P_{N+1}(\Omega_i)$ is the space of algebraic polynomials of degree at most $N+1$ in each coordinate direction in Ω_i. We also define the finite dimensional spaces X_{N+1} and Y_{N+1} associated with U and U_0, respectively, by

$$X_{N+1} = \{\Phi = (\phi^1, \phi^2, \phi^3) \in W_1 \times W_2 \times W_3 : \Phi \in U\},$$

and

$$Y_{N+1} = \{\Phi = (\phi^1, \phi^2, \phi^3) \in W_1 \times W_2 \times W_3 : \Phi \in U_0\}.$$

The dimension of the space Y_{N+1} is $(3N-2)(N-2)$.

We define a discrete inner product on X_{N+1} by

$$(\Phi,\Theta)_{N+1}=\sum_{k=1}^{3}(\phi^k,\theta^k)_{N+1}^k,$$

where $(\phi^k,\theta^k)_{N+1}^k$ is obtained by applying the quadrature rule (10) to the function $\phi^k\theta^k$ in each coordinate direction, for $k=1,2,3$. The discrete bilinear form $b_{N+1}(.,.)$ is defined on $X_{N+1}\times X_{N+1}$ by

$$b_{N+1}(\Phi,\Theta)=\sum_{k=1}^{3}(\nabla^4\phi^k,\theta^k)_{N+1}^k.$$

Further the discrete functional G on $X_{N+1}\times X_{N+1}\times X_{N+1}$ is defined by

$$G_{N+1}(\Psi,\Phi,\Theta)=\sum_{k=1}^{3}(T(\psi^k,\phi^k),\theta^k)_{N+1}^k+D_{N+1}^k(\phi^k,\theta^k),$$

with

$$D_{N+1}^1(\phi^1,\theta^1)=-\sum_{i=2}^{N-1}w_i^1[\frac{\partial^2\phi^1}{\partial y^2}\frac{\partial\theta^1}{\partial y}+\frac{\partial^3\phi^1}{\partial y^3}\theta^1](\xi_i^1,-1),$$

$$D_{N+1}^2(\phi^2,\theta^2)=\sum_{i=2}^{N-1}w_i^2[\frac{\partial^2\phi^2}{\partial y^2}\frac{\partial\theta^2}{\partial y}+\frac{\partial^3\phi^2}{\partial y^3}\theta^2](\xi_i^2,-1)+\sum_{j=2}^{N-1}z_j^2[\frac{\partial^2\phi^2}{\partial x^2}\frac{\partial\theta^2}{\partial x}-\frac{\partial^3\phi^2}{\partial x^3}\theta^2](c,\eta_j^2),$$

$$D_{N+1}^3(\phi^3,\theta^3)=-\sum_{j=2}^{N-1}z_j^3[\frac{\partial^2\phi^3}{\partial x^2}\frac{\partial\theta^3}{\partial x}-\frac{\partial^3\phi^3}{\partial x^3}\theta^3](c,\eta_j^3).$$

The discrete variational problem corresponding to (12) is then: Given $\Psi\in X_{N+1}$, find $\Phi\in Y_{N+1}$ such that

$$b_{N+1}(\Phi,\Theta)+G_{N+1}(\Psi,\Phi,\Theta)=-\sum_{k=1}^{3}(L(\phi^k),\theta^k)_{N+1}^k,\tag{14}$$

for all $\Theta\in Y_{N+1}$.

THEOREM 1. *The discrete variational problem (14) is equivalent to the following collocation problem: Given $\Psi\in X_{N+1}$, find $\Phi\in Y_{N+1}$ such that*

$$L'(\psi^k)\phi^k = -L(\psi^k)\ at\ (\xi_i^k,\eta_j^k),\ 2\le i,j\le N-1,\ k=1,2,3,\tag{15}$$

$$\frac{\partial^3}{\partial y^3}(\phi^2-\phi^1) = -(z_N^1r^1+z_1^2r^2+\overline{z}_N^1\frac{\partial r^1}{\partial y}+\overline{z}_1^2\frac{\partial r^2}{\partial y})\ at\ (\xi_i^1,-1),\ 2\le i\le N-1,\tag{16}$$

$$\frac{\partial^2}{\partial y^2}(\phi^2-\phi^1) = -(z_N^1r^1+z_1^2r^2)\ at\ (\xi_i^1,-1),\ 2\le i\le N-1,\tag{17}$$

$$\frac{\partial^3}{\partial x^3}(\phi^3-\phi^2) = -(w_N^2r^2+w_1^3r^3+\overline{w}_N^2\frac{\partial r^2}{\partial x}+\overline{w}_1^3\frac{\partial r^3}{\partial x})\ at\ (c,\eta_j^2),\ 2\le j\le N-1,\tag{18}$$

$$\frac{\partial^2}{\partial x^2}(\phi^3-\phi^2) = (w_N^2r^2+w_1^3r^3)\ at\ (c,\eta_j^2),\ 2\le j\le N-1,\tag{19}$$

where $r^k = L'(\psi^k)\phi^k + L(\psi^k)$ *is the residual in* Ω_k.

Proof. See Phillips and Malek (1991) for details of the proof. □

REMARK 1. *Note that in view of the expressions for the weights given in (13) and (14),*

$$w_1 = w_N = O(N^{-2}) \,, \quad \overline{w}_1 = -\overline{w}_N = O(N^{-4}) \,, \quad as\ N \to \infty \,,$$

and therefore from (31)-(34) we can write

$$\frac{\partial^3}{\partial y^3}(\psi^2 - \psi^1) = O(N^{-2})\ at\ (\xi_i^1, -1),\ 2 \le i \le N-1, \tag{20}$$

$$\frac{\partial^2}{\partial y^2}(\psi^2 - \psi^1) = O(N^{-2})\ at\ (\xi_i^1, -1),\ 2 \le i \le N-1, \tag{21}$$

$$\frac{\partial^3}{\partial x^3}(\psi^3 - \psi^2) = O(N^{-2})\ at\ (c, \eta_j^2),\ 2 \le j \le N-1, \tag{22}$$

$$\frac{\partial^2}{\partial x^2}(\psi^3 - \psi^2) = O(N^{-2})\ at\ (c, \eta_j^2),\ 2 \le j \le N-1, \tag{23}$$

as $N \to \infty$. *Thus we have second and third order continuity at the interfaces asymptotically, as* $N \to \infty$. The following algorithm is used in our computations.

ALGORITHM 1.

Step 0 Set $\psi^{(0)}$ to be the initial approximation to the solution of (6) where $\psi^{(0)}$ satisfies the boundary conditions of the problem. The initial approximation is chosen to be the solution to the Stokes problem or the solution to the Navier-Stokes equations for a lower value of the Reynolds number. Set $k = 1$.

Step 1 Approximate ϕ in (7) using pseudospectral representations and solve (7) for ϕ.

Step 2 Find the new approximation $\psi^{(k)}$ by simply adding together the corresponding coefficients in the pseudospectral representations of ϕ and $\psi^{(k-1)}$.

Step 3 Find the maximum absolute difference, $|\ \psi^{(k)} - \psi^{(k-1)}\ |$, between two successive approximations to ψ at all the collocation points. If $\max |\ \psi^{(k)} - \psi^{(k-1)}\ | \le \epsilon$ then stop, otherwise set $k \leftarrow k + 1$ and go to Step 1.

6. Numerical Results. The collocation equations (15)-(19) yield a system of $(3N - 2)(N - 2)$ equations for the $(3N - 2)(N - 2)$ unknowns. The collocation equations give rise to a linear algebraic system $A\mathbf{x} = \mathbf{b}$. The vector $\mathbf{x}$ contains the nodal values of ϕ and the normal derivative of ϕ at the interface nodes. This system is solved using a Crout factorization subroutine from the NAG Library. A more efficient direct solution technique which takes account of the inherent matrix structure is the almost block diagonal solver of Brankin and Gladwell (1990) which has been used in spectral calculations by Karageorghis and Phillips (1990,1991). However, this subroutine has not yet been incorporated into the present algorithm.

For a tolerance $\epsilon = 10^{-10}$ only a few Newton steps are required for convergence. Our numerical calculations show that for Reynolds numbers $Re = 10$ and $Re = 50$, the Newton scheme converges after only six and eight iterations, respectively. In these cases the initial guess is chosen to be the solution to the corresponding Stokes problem. For larger Reynolds numbers the numerical solution is obtained by means of a continuation process in the Reynolds number. This means, for example, that the solution of the problem for $Re = 50$ is used as an initial guess in order to determine the solution when $Re = 75$. Table 1 shows the maximum absolute difference between successive iterates of ψ at the collocation points for each Newton iteration until convergence is reached when $Re = 50$ for $N = 16$ and $N = 18$. The usual convergence history associated with Newton-type methods is clearly evident.

Dependence of $\| \psi^{(k)} - \psi^{(k-1)} \|_\infty$ *on the number of iterations* (k) *and discretization parameter* (N).

k	$N = 16$	$N = 18$
1	1.115 +1	1.308 +1
2	4.672 +0	5.142 +0
3	8.439 -1	1.116 +0
4	3.521 -1	4.694 -1
5	2.342 -2	2.062 -2
6	6.333 -5	9.884 -5
7	1.023 -8	1.474 -9
8	4.081 -13	2.014 -11

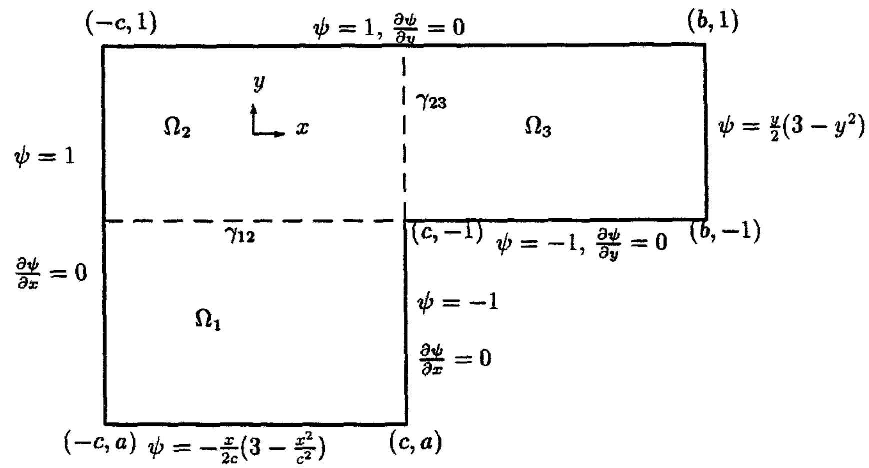

Figure 1. *The flow domain and boundary conditions*

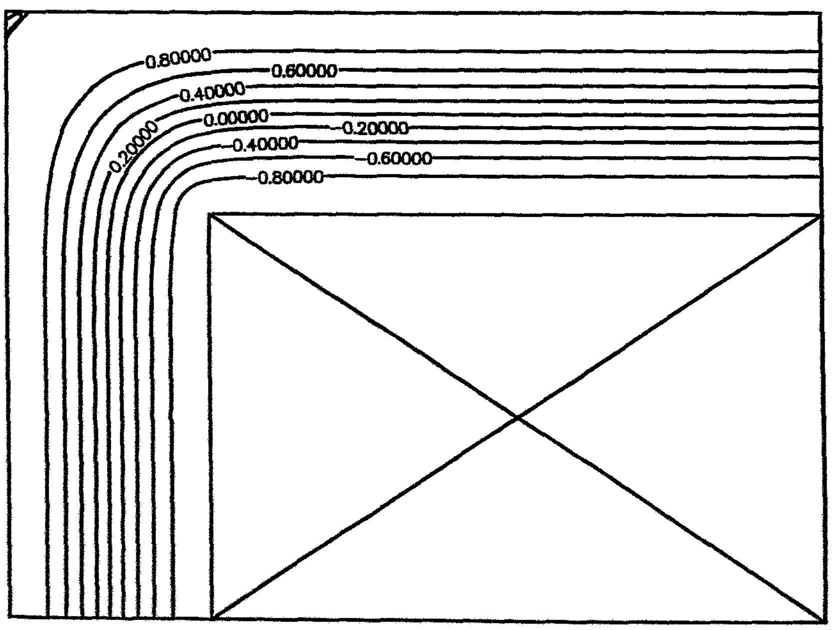

Figure 2. *Contours of* $\psi(x, y)$ *for* $Re = 0$ *and* $N = 14$

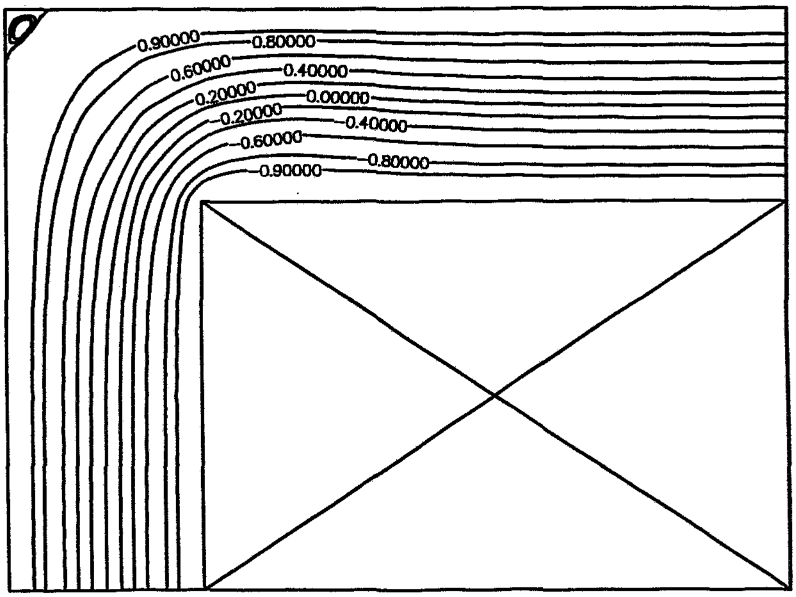

Figure 3. *Contours of $\psi(x, y)$ for $Re = 10$ and $N = 18$*

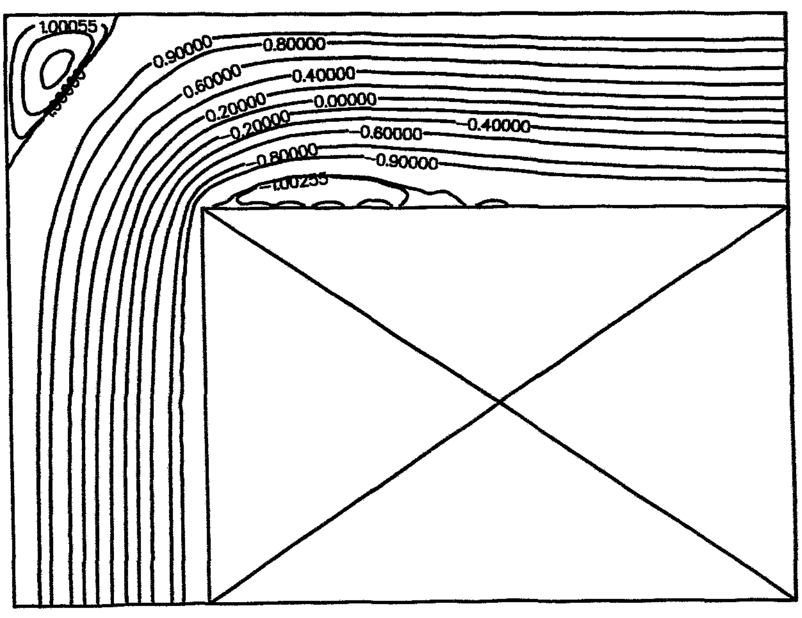

Figure 4. *Contours of $\psi(x, y)$ for $Re = 50$ and $N = 18$*

The contours of the stream function inside the L-shaped domain are shown in Figs. 2-4 for various values of the Reynolds number Re and discretization parameter N. The stream function contours are plotted using the option in the UNIRAS graphical library which bilinearly interpolates data given on a non-uniform grid. We are unable to use the piecewise cubic option in this package since this requires data given on a uniform grid. In all of the calculations presented we have chosen $a = -5$, $b = 7$ and $c = 1$. The values of a and b are chosen so that we do indeed obtain fully developed flow in the entry and exit sections, respectively. The numerical results indicate that the solution converges as the degree of the approximation, N, is increased in each subdomain. In Fig. 2 we give the contour plot of the stream function for the Stokes problem. The contour plots of the stream function for the Navier-Stokes problem, given in Figs. 3 and 4, show the development of vortices in the salient corner and downstream of the reentrant corner. The salient corner vortex grows steadily as the Reynolds number is increased. The vortex downstream of the reentrant corner grows and extends further downstream as the Reynolds number is increased. For a fixed value of the Reynolds number the entry and exit lengths are extended to ensure that a fully developed velocity profile is obtained. A converged solution to the discrete problem is obtained with mesh refinement i.e. as N is increased.

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