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Non-overlapping Domain Decomposition Applied to Incompressible Flow Problems

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1. Introduction

A non-overlapping domain decomposition method with Robin-type transmission conditions which is known for scalar advection-diffusion-reaction problems [**2**],[**5**] is generalized to cover the Oseen equations. The presented method, which is later referred to as DDM, is an additive iteration-by-subdomains algorithm. Hence parallelism is given in a very natural way. The formulation is based on the continuous level to study the DDM without dealing with a special discretization. A convergence result for the "continuous" algorithm is presented. To treat incompressible Navier-Stokes problems, the

A parallel implementation based on a finite element discretization has been done. Numerical results indicating linear convergence with a rate independent of the mesh size are presented for both the (linear) Oseen equations and the (nonlinear) Navier-Stokes equations.

We denote by $L^2(\Omega)$ the space of square integrable functions with norm $\|\cdot\|_{0,\Omega}$ and inner product $(\cdot,\cdot)_\Omega$. $H^s(\Omega)$ denotes the usual Sobolev space with norm $\|\cdot\|_{s,\Omega}$. For $\Gamma \subset \partial\Omega$ we write $\langle\cdot,\cdot\rangle_\Gamma$ for the inner product in $L^2(\Gamma)$ (or, if needed, for the duality product between $H^{\frac{1}{2}}_{00}(\Gamma)$ and $H^{-\frac{1}{2}}_{00}(\Gamma)$). The space $H^{\frac{1}{2}}_{00}(\Gamma)$ consists of functions $u \in H^{\frac{1}{2}}(\Gamma)$ with $d^{-\frac{1}{2}}u \in L^2(\Gamma)$ where $d(x) = \operatorname{dist}(x, \partial\Gamma)$ [**3**, Chap 1., Sec. 11.4]. We explain the DDM for the Oseen equations in Section 2 and look into its analysis in Section 3. Then we explain how to discretize the method (Section 4) and apply it to the Navier-Stokes equations (Section 5). Numerical results are presented in Section 6.

2. Definition of the DDM for the Oseen equations

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with Lipschitz, piecewise C^2-boundary $\partial\Omega$. We consider the following boundary value problem for the Oseen equations

$$(1)\qquad \left\{\begin{array}{rcll} -\nu\triangle \boldsymbol{u} + \boldsymbol{\nabla} p + (\boldsymbol{b}\cdot\boldsymbol{\nabla})\boldsymbol{u} + c\boldsymbol{u} &=& \boldsymbol{f} & \in (L^2(\Omega))^2 \\ \boldsymbol{\nabla}\cdot\boldsymbol{u} &=& 0 & \in L^2(\Omega) \\ \boldsymbol{u} &=& \boldsymbol{g} & \in (H^{\frac{1}{2}}(\partial\Omega_D))^2 \\ \nu\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}} - p\boldsymbol{n} + \eta\boldsymbol{u} &=& \boldsymbol{h} & \in (L^2(\partial\Omega_R))^2 \end{array}\right. ,$$

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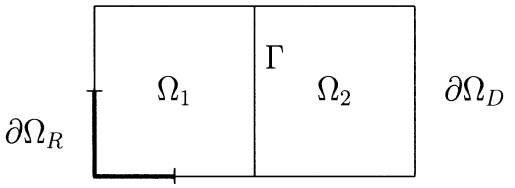
FIGURE 1.



where $\partial\Omega = \overline{\partial\Omega}_D \cup \overline{\partial\Omega}_R$, $\partial\Omega_D \cap \partial\Omega_R = \emptyset$, and $\nu > 0$, $\boldsymbol{b} \in (H^1(\Omega))^2$ with $\boldsymbol{\nabla} \cdot \boldsymbol{b} \in L^\infty(\Omega)$, $c \in L^\infty(\Omega)$, $\eta \in L^\infty(\partial\Omega_R)$. $\boldsymbol{n}$ is the outer normal on $\partial\Omega$. If $\partial\Omega_R = \emptyset$ we also require $\int_{\partial\Omega} \boldsymbol{g} \cdot \boldsymbol{n} = 0$. Then if $c - \frac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{b} \geq 0$ and $\eta + \frac{1}{2}\boldsymbol{b} \cdot \boldsymbol{n} \geq \eta_0 = \text{const} > 0$ problem (1) has a unique solution which belongs to $(H^1(\Omega))^2 \times L^2(\Omega)$ if $\mu(\partial\Omega_R) > 0$, and to $(H^1(\Omega))^2 \times L_0^2(\Omega)$ with $L_0^2(\Omega) = \{f \in L^2(\Omega) \mid \int_D f \mathrm{d}x = 0\}$ otherwise.

The reason why we consider a mixed boundary value problem will become clear in the next section. Here we allow the additional term $c\boldsymbol{u}$ within the momentum equation which occurs if a simultaneous linearization and semi-discretization in time of the non-stationary Navier-Stokes equations is performed.

A heuristical approach to non-overlapping domain decomposition methods for this type of problems is as follows. We divide Ω into two subdomains Ω_k, $k = 1, 2$ also having a Lipschitz, piecewise C^2-boundary. The artificial boundary $\partial\Omega_1 \cap \partial\Omega_2$ is denoted by Γ (Figure 1). For simplicity we assume $\overline{\partial\Omega_R} \subsetneq \partial\Omega_1 \cap \partial\Omega$.

Then the original boundary value problem is equivalent to the following split formulation ($\boldsymbol{n}_i$ always denotes the outer normal of Ω_i)

$$(2)\quad \begin{cases} -\nu\triangle\boldsymbol{u}_1 + \boldsymbol{\nabla}p_1 + (\boldsymbol{b}\cdot\boldsymbol{\nabla})\boldsymbol{u}_1 + c\boldsymbol{u}_1 &= \boldsymbol{f} &\in (L^2(\Omega_1))^2 \\ \boldsymbol{\nabla}\cdot\boldsymbol{u}_1 &= 0 &\in L^2(\Omega_1) \\ \boldsymbol{u}_1 &= \boldsymbol{g} &\in (H^{\frac{1}{2}}(\partial\Omega_D)|_{\partial\Omega_D\cap\partial\Omega_1})^2 \\ \nu\frac{\partial\boldsymbol{u}_1}{\partial\boldsymbol{n}_1} - p_1\boldsymbol{n}_1 + \eta\boldsymbol{u}_1 &= \boldsymbol{h} &\in (L^2(\partial\Omega_R))^2 \end{cases}$$

$$(3)\quad \begin{cases} -\nu\triangle\boldsymbol{u}_2 + \boldsymbol{\nabla}p_2 + (\boldsymbol{b}\cdot\boldsymbol{\nabla})\boldsymbol{u}_2 + c\boldsymbol{u}_2 &= \boldsymbol{f} &\in (L^2(\Omega_2))^2 \\ \boldsymbol{\nabla}\cdot\boldsymbol{u}_2 &= 0 &\in L^2(\Omega_2) \\ \boldsymbol{u}_2 &= \boldsymbol{g} &\in (H^{\frac{1}{2}}(\partial\Omega_D)|_{\partial\Omega_D\cap\partial\Omega_2})^2 \end{cases}$$

together with the continuity requirements on Γ

$$(4)\qquad \boldsymbol{u}_1 = \boldsymbol{u}_2 \qquad \text{in } (H^{\frac{1}{2}}(\Gamma))^2,$$

$$(5)\qquad \nu\frac{\partial\boldsymbol{u}_1}{\partial\boldsymbol{n}_1} - p_1\boldsymbol{n}_1 = -\nu\frac{\partial\boldsymbol{u}_2}{\partial\boldsymbol{n}_2} + p_2\boldsymbol{n}_2 \qquad \text{in } (H_{00}^{-\frac{1}{2}}(\Gamma))^2.$$

These two continuity conditions can be used to construct a non-overlapping domain decomposition method for this problem, which can be considered as an iterative decoupling of the split formulation. For example using (4) iteratively to calculate solutions on Ω_1, i.e. $\boldsymbol{u}_1^k = \boldsymbol{u}_2^{k-1}$, and (5) for Ω_2 we would get a Dirichlet-Neumann-algorithm [7].

We use however a linear combination of both conditions for all subdomains, i.e. to get a new solution on Ω_i within the iteration, we impose

$$(6)\qquad \nu\frac{\partial\boldsymbol{u}_i^k}{\partial\boldsymbol{n}_i} - p_i^k\boldsymbol{n}_i + \lambda_i\boldsymbol{u}_i^k = \nu\frac{\partial\boldsymbol{u}_j^{k-1}}{\partial\boldsymbol{n}_i} - p_j^{k-1}\boldsymbol{n}_i + \lambda_i\boldsymbol{u}_j^{k-1} \quad \text{on } \partial\Omega_i \cap \partial\Omega_j$$

Such an approach does not require a red-black partition which is often needed for DN-algorithms. Furthermore it is known for the related method for advection-diffusion-reaction problems that the behaviour of the solution can be modelled "adaptively" if λ_i is chosen as an appropriate function [**2**],[**5**],[**1**],[**6**].

More precisely we use a restricted class of weighting factors λ_i as given below, but we allow some kind of relaxation of the interface condition (6).

The algorithm. We now consider a non-overlapping partition into N subdomains $\bar{\Omega} = \bigcup_{i=1}^{N} \bar{\Omega}_i$, $\Omega_i \cap \Omega_j = \emptyset \ \forall i \neq j$ with each Ω_i having the same boundary regularity as Ω. We denote $\Gamma_i = \partial\Omega_i \setminus \partial\Omega$ and $\bar{\Gamma}_{ij} = \partial\Omega_i \cap \partial\Omega_j$.

Furthermore we use the following notations:

"interface operator"	:	$\Phi_i(\boldsymbol{u}, p) = \nu \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}_i} - p\boldsymbol{n}_i + (-\frac{1}{2}\boldsymbol{b} \cdot \boldsymbol{n}_i + \rho_i)\boldsymbol{u}$
initial interface condition	:	$\Phi_{i,0}$
"relaxation parameter"	:	$\theta \in (0, 1]$

Instead of λ_i we use $-\frac{1}{2}\boldsymbol{b} \cdot \boldsymbol{n}_i + \rho_i$ with ρ_i to be chosen, because the necessary restrictions are easier formulated for ρ_i.

Now the domain decomposition algorithm for the Oseen problem (1) reads:
For $k \in \mathbb{N}$ solve for all subdomains Ω_i $(i = 1, \ldots, N)$ in parallel:

$$\left\{ \begin{array}{rcl} -\nu\triangle \boldsymbol{u}_i + \boldsymbol{\nabla} p_i + (\boldsymbol{b} \cdot \boldsymbol{\nabla})\boldsymbol{u}_i + c\boldsymbol{u}_i &=& \boldsymbol{f} \\ \boldsymbol{\nabla} \cdot \boldsymbol{u}_i &=& 0 \end{array} \right. \tag{7}$$

with the given boundary conditions on $\partial\Omega_i \cap \partial\Omega$ together with the interface condition

$$\Phi_i(\boldsymbol{u}_i^k, p_i^k) = \left\{ \begin{array}{ll} \theta\Phi_i(\boldsymbol{u}_j^{k-1}, p_j^{k-1}) + (1-\theta)\Phi_i(\boldsymbol{u}_i^{k-1}, p_i^{k-1}) & k > 1 \\ \Phi_{i,0} & k = 1 \end{array} \right. \tag{8}$$

on Γ_{ij}.

3. Convergence Analysis

Before formulating the convergence result for the algorithm above we start with its well-posedness.

LEMMA 1. *In addition to the regularity of the data prescribed in Section 2 we assume $c - \frac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{b} \geq 0$, $\eta + \frac{1}{2}\boldsymbol{b} \cdot \boldsymbol{n} \geq \eta_0 = \text{const} > 0$, and for all subdomains Ω_i*

1. *$\rho_i \in L^\infty(\Gamma_i)$ with $\rho_i \geq \rho_i^0 = \text{const} > 0$*
2. *$\Phi_{i,0} \in L^2(\Gamma_i)$*
3. *$c - \frac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{b} \geq c_i = \text{const} > 0$ or $\mu(\partial\Omega_i \cap \partial\Omega) > 0$.*

Then the domain decomposition algorithm is well-defined, i.e. all local boundary value problems have for all k a unique solution in $(H^1(\Omega_i))^2 \times L^2(\Omega_i)$. Furthermore we have

$$\Phi_i(\boldsymbol{u}_i^k, p_i^k) \in L^2(\Gamma_{ij}) \ \forall k. \tag{9}$$

We emphasize that the local pressure solution p_i^k is unique in $L^2(\Gamma_i)$ for all k.

Now we denote by Π_i the L^2-projection onto the space $L_0^2(\Omega_i)$, more precisely

$$\Pi_i : L^2(\Omega_i) \to L_0^2(\Omega_i) \qquad q \mapsto q - \frac{1}{\mu(\Omega_i)} \int_{\Omega_i} q \mathrm{d}x. \tag{10}$$

THEOREM 2. *Let the solution $(\boldsymbol{u}, p)$ of (1) be regular enough to have $\nu \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{n}_i} - p\boldsymbol{n}_i \in (L^2(\Gamma_{ij}))^2$ for all i, j. If $\rho_i = \rho_j$ a.e. then we have under the assumptions of Lemma 1 for all $\theta \in (0, 1]$*

$$\begin{aligned} \|\boldsymbol{u}_i^k - \boldsymbol{u}_i\|_{1,\Omega_i} &\longrightarrow 0, \\ \|\Pi_i \left(p_i^k - p_i\right)\|_{0,\Omega_i} &\longrightarrow 0 \end{aligned}$$

for $k \to \infty$, where $(\boldsymbol{u}_i, p_i)$ is the restriction of $(\boldsymbol{u}, p)$ to Ω_i.
Furthermore, if $\mu(\partial\Omega_R) > 0$, i.e. a mixed boundary value problem is considered, we have for all $\theta \in (0, 1]$

$$\|p^k - p\|_{0,\Omega_i} \longrightarrow 0$$

for $k \to \infty$.

Remarks

- If $c(x) - \frac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{b}(\boldsymbol{x}) \geq C > 0$ then arbitrary subdomain partitions satisfying the regularity requirements are allowed. Especially internal cross-points can be treated. For $C = 0$ such partitions are not covered by the theorem, but nevertheless they work in numerical computations.
- The Stokes problem is covered.
- All results remain valid if different ρ_i for every velocity component are used.
- For a Dirichlet problem the pressure convergence is local: Only the locally normalized pressure will converge. I.e. we have pressure convergence up to a constant which can differ for different subdomains and iteration steps.
- Since the theorem yields no information about the convergence speed, we have no theoretical indication how to construct a "good" ρ_i. A heuristic approach to a "good" ρ_i for advection-diffusion-reaction equations is contained in [**6**]. For the Oseen equations this question is still open.

To prove the convergence for the velocity variable a key step is the relation

$$|||\boldsymbol{u}_i^k|||_i^2 + \int_{\Gamma_i} \frac{1}{4\rho_i} \left(\Phi_i(\boldsymbol{u}_i^k, p_i^k) - 2\rho_i \boldsymbol{u}_i^k\right)^2 = \int_{\Gamma_i} \frac{1}{4\rho_i} \left(\Phi_i(\boldsymbol{u}_i^k, p_i^k)\right)^2$$

with

$$|||\boldsymbol{u}|||_i^2 := \nu |u|_{1,\Omega_i}^2 + \|(c - \tfrac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{b})^{\frac{1}{2}} \boldsymbol{u}\|_{0,\Omega_i}^2 + \|(\eta + \tfrac{1}{2}\boldsymbol{\nabla} \cdot \boldsymbol{b})^{\frac{1}{2}} \boldsymbol{u}\|_{0,\partial\Omega_R \cap \partial\Omega_i}^2$$

which uses the L^2-regularity of the interface data. This part is established similar to the convergence of the related algorithm for advection-diffusion-reaction problems. ([**1**] contains that proof for $\theta = 1$.)

The local pressure convergence comes from a modified a priori estimate which is based on the continuous version of the Babuška-Brezzi-condition. Global convergence of p in the case $\mu(\partial\Omega_R) > 0$ is based on the transmission of the local pressure mean values across interfaces. The full proof is given in [**6**].

4. The discrete algorithm

Since finite elements are favoured as discretization method, weak formulations of the subdomain problems should be considered. In the case of homogeneous Dirichlet boundary conditions on $\partial\Omega$ and $c = 0$ the local Oseen problems read in weak formulation:

Within step k find $(u_i^k, p_i^k) \in V_i \times Q_i = \{\boldsymbol{v} \in (H^1(\Omega_i))^2 \mid \boldsymbol{v} = 0 \text{ on } \partial\Omega \cap \partial\Omega_i\} \times L^2(\Omega_i)$ *with*

$$\begin{aligned} a_i(\boldsymbol{b}; \boldsymbol{u}_i^k, \boldsymbol{v}) - b_i(p_i^k, \boldsymbol{v}) + b_i(q, \boldsymbol{u}_i^k) &+ \langle(-\frac{1}{2}\boldsymbol{b}\cdot\boldsymbol{n}_i + \rho_i)\boldsymbol{u}_i^k, \boldsymbol{v}\rangle_{\Gamma_i} \\ &= (f, \boldsymbol{v})_{\Omega_i} + \sum_{j\neq i}\langle\Lambda_{ji}^{k-1}, \boldsymbol{v}\rangle_{\Gamma_{ij}} \end{aligned} \tag{11}$$

where

$$\begin{aligned} a_i(\boldsymbol{b}; \boldsymbol{u}, \boldsymbol{v}) &= \nu(\boldsymbol{\nabla u}, \boldsymbol{\nabla v})_{\Omega_i} + (\boldsymbol{b}\cdot\boldsymbol{\nabla u} + c\boldsymbol{u}, \boldsymbol{v})_{\Omega_i} \\ b_i(q, \boldsymbol{v}) &= (p, \boldsymbol{\nabla}\cdot\boldsymbol{v})_{\Omega_i} \\ \Lambda_{ji}^k &= \theta\Phi_i(\boldsymbol{u}_j^k, p_j^k) + (1-\theta)\Phi_i(\boldsymbol{u}_i^k, p_i^k). \end{aligned}$$

The discretization is performed by choosing finite dimensional subspaces V_i^h, Q_i^h of V_i, Q_i which consist of piecewise polynomial functions on the restriction of a global triangulation of Ω to Ω_i.

The evaluation of $\Phi_i(\boldsymbol{u}_j^{k-1}, p_j^{k-1})$ resp. $\Phi_i(\boldsymbol{u}_i^{k-1}, p_i^{k-1})$ can be avoided by means of the following formula

$$\Lambda_{ij}^k = \theta(\rho_i + \rho_j)\boldsymbol{u}_i^k - \theta\Lambda_{ji}^{k-1} + (1-\theta)\Lambda_{ij}^{k-1} \tag{12}$$

which does not use derivatives of the finite element solutions. Again the discrete algorithm starts with an initial guess $\Phi_{i,0}$ for the interface condition. Hence a good initial guess can reduce the number of iterations until convergence.

5. Application to the Navier-Stokes equations

The stationary Navier-Stokes equations as a non-linear problem of the form

$$A[\hat{u}]\hat{u} = f$$

can be solved by a defect correction method

$$A[\hat{u}^{m-1}](\hat{u}^m - \hat{u}^{m-1}) = \omega_m\left\{f - A[\hat{u}^{m-1}](\hat{u}^{m-1})\right\} \tag{13}$$

or equivalently

$$A[\hat{u}^{m-1}](\hat{u}^m) = \omega_m f + (1-\omega_m)A[\hat{u}^{m-1}](\hat{u}^{m-1}) \tag{14}$$

with some damping factor $\omega_m > 0$. The idea is to solve the linear(ized) Oseen problem occuring within this iterative process using the domain decomposition algorithm as inner cycle. Then the local subproblems are as in (11) with $\boldsymbol{u}_i^k, p_i^k$ replaced by $\boldsymbol{u}_i^{m,k}, p_i^{m,k}$ and $\boldsymbol{b}$ by the velocity solution from the previous linearization step. If the formulation (14) is used, an appropriate initial interface condition for the domain decomposition within step m is the last calculated Λ_{ij}^k from step $m-1$. Hence results of step $m-1$ are re-used and it is not necessary to achieve convergence of the domain decomposition algorithm within every linearization step.

6. Numerical examples

As remarked in Section 2 an analogous method turned out to be very efficient for advection-diffusion-reaction problems [**1**],[**6**]. Hence for the numerical examples below we used the straightforward extension of the interface function proposed in [**1**]

$$\rho_i = \sqrt{(\boldsymbol{b}\cdot\boldsymbol{n}_i)^2 + \nu\lambda}. \tag{15}$$

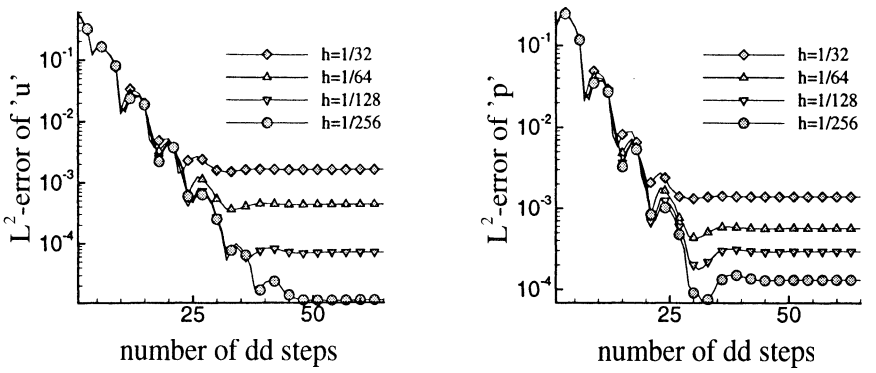


FIGURE 2. Convergence history for Example 1.

λ is a strictly positive function and could be chosen seperately for each velocity component.

Within our calculations we use continuous piecewise linear finite elements for the velocity as well as for the pressure. So we add to (11) residual terms in order to satisfy a modified Babuška-Brezzi-condition and to get a stable discretization (see [**4**] for details).

Numerical experiments showed that a relaxation parameter $\theta < 1$ gives global pressure convergence for the Dirichlet problem, too (cf. Theorem 2). But for the type of problems considered below there is no acceleration for smaller θ. So we chose $\theta = 1$ for all test cases.

Example 1. Linearized Navier-Stokes flow (Oseen flow):
We consider the Poiseuille flow in a 2d channel, where we use the quadratic profile as known velocity field. At the outflow part we impose a homogeneous Neumann boundary condition and prescribe the velocities elsewhere. The computational domain $[0,1] \times [0,1/4]$ is divided into 4 subdomains arranged in a row. The exact solution is given by $(u,v,p) = (64y(1/4-y), 0, -128\nu x)$ with $\nu = 10^{-3}$. We show in Figure 2 the convergence history of the discrete L^2-errors versus the iteration number of the DDM for different mesh sizes. (Due to their interface discontinuities the dd-solutions do not belong to the space of continuous finite element functions which is needed to calculate residuals directly. That is why we here only consider the error. An alternative is under development.)

The results indicate that the DDM converges almost linearly until a certain error level is achieved which corresponds to the mesh size. The rate of convergence seems to be independent of the mesh size. In comparison to the related algorithm for scalar equations [**6**] the performance is worse and the choice of λ is more critical. Here it is chosen as $\lambda = 5/\nu$.

So far no mechanism of global data transport (like a coarse grid) is incorporated in the algorithm; hence it cannot be scalable. Table 1 shows the dependence of the number of subdomains for the finest grid used for this example. Neither load-balancing nor inexact subdomain solving has been used to obtain these results. As expected the number of iterations increases with the number of subdomains. Nevertheless, the computing time decreases and this suggests that this algorithm together

TABLE 1. Iteration numbers and computing time needed on the finest mesh ($h = 1/256$) to achieve for Example 1. an u-error smaller than $5 \cdot 10^{-5}$.

subdomain partition	number of dd-iterations	CPU-time [s] (fastest and slowest subdomain)
2×1	17	1260, 1630
4×1	54	1000, 1410
4×2	88	540, 1110

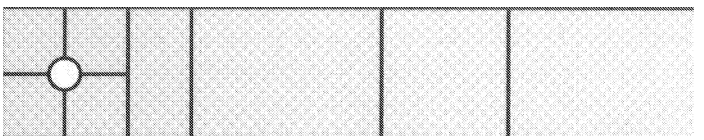


FIGURE 3. Subdomain partition for Example 2.

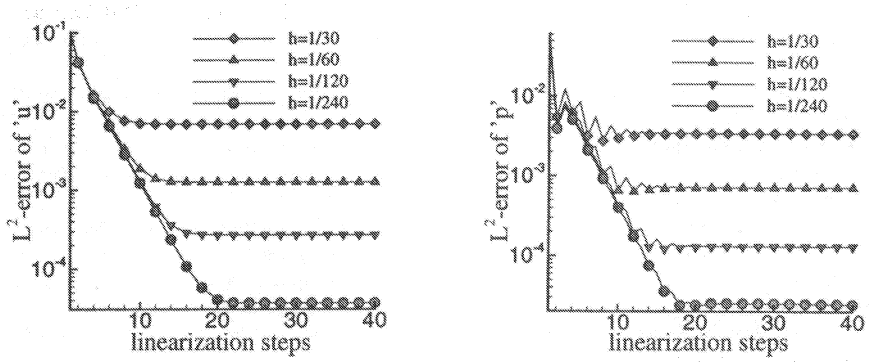


FIGURE 4. Convergence history for Example 2.

with a coarse grid solver can be very efficient for larger numbers of subdomains.

Example 2. Stationary Navier-Stokes flow around a cylinder:
We consider the stationary flow in a 2d-channel with an obstacle. We have a quadratic profile at the inflow, no-slip conditions at the walls and a homogeneous Neumann boundary condition at the outflow. The viscosity is $\nu = 10^{-3}$ and we choose $\lambda = 1/\nu$ in (15). The computational domain has been divided into 8 subdomains as shown in Figure 3.

In Figure 4 we show the convergence history of the discrete L^2-errors versus the number of linearization steps for different mesh sizes. Within each step we performed 10 steps of the domain decomposition algorithm. To calculate the errors we used a reference solution obtained by solving the global boundary value problem on the same mesh up to the level of the truncation error.

The graphs show the linear convergence of the outer iteration (linearization) with a rate independent of the mesh size. A direct computation without the DDM needs between 13 and 16 linearization steps. Hence with 10 dd steps within every linearization step we get nearly the same accuracy with roughly the same number of steps. In fact, the parallel calculation is cheaper with respect to computing time. So the DDM works quite well as kernel of a Navier-Stokes solver.

7. Summary

We described a non-overlapping domain decomposition algorithm for the Oseen (linearized Navier-Stokes) equations and proved its convergence on the continuous level. A discretized variant was proposed and applied to the Navier-Stokes problem. A finite element implementation which has not been fully optimized yielded reasonable results for the linear and non-linear problem. The method has also been applied to non-isothermal flow problems with promising results. Further investigations of both theory and implementation are under development.

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