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Quasi-Simultaneous Coupling for Wing and Aerofoil Flow

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INTRODUCTION

For the design and analysis of aerofoils and wings, a fast robust and accurate computer code is an essential tool. These days the prediction of aerodynamic characteristics is generally accomplished using Navier-Stokes simulation. Whilst this approach potentially offers generality, the computational cost involved currently limits their use for practical application. A possible alternative, is to use viscous-inviscid interaction (VII), which is an application of Schwarz non-overlapping domain-decomposition with Neumann-Dirichlet boundary conditions. VII methods have been shown to be very computationally economical and for many cases of aerodynamic design VII methods match the experimental data as well as Navier-Stokes simulation. However, despite the likelihood of there being similar advantages for three-dimensional problems, only a limited amount of research has been done in this area. The development of fully three-dimensional VII methods proves difficult [LW87].

The research described in this paper is preliminary work on two-dimensional quasi-simultaneous VII, carried out as a first stage in the development of a general three-dimensional viscous-inviscid interaction code. A Dirichlet interaction law has been derived for the coupling of the boundary layer to a panel method for two-dimensional incompressible flow that is more efficient than the well-known thin-aerofoil theory interaction law.

Quasi-simultaneous coupling [Vel81] has also been implemented in the DERA

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Viscous Full Potential (VFP) program for the calculation of transonic flow over swept tapered wings [AS85]. This is a case where the three-dimensional boundary layer equations reduce to a system of ODE's and the calculations are therefore performed section-wise, similar to the 2D problem.

QUASI-SIMULTANEOUS COUPLING

Various coupling techniques have been proposed over the years and a comprehensive review of the various viscous-inviscid interaction schemes can be found in Lock and Williams [LW87]. The technique used here to couple the boundary layer to the inviscid region is the quasi-simultaneous (QS) method [Vel81]. QS has a very fast convergence rate and is able to calculate in separated regions. The QS method is described in more detail and put into a domain-decomposition context in the paper by Veldman and Lai in these proceedings [VL99]. The QS coupling resembles the direct method, where the velocity (or pressure) distribution is calculated in the inviscid region and then used to determine the displacement effect from the boundary layer equations. There is however a significant difference between direct and QS. The QS method performs the boundary layer calculations simultaneously with a local representation of the external flow, termed the interaction law.

This interaction law functions as a boundary condition to the boundary layer equations. In this way, the problems the direct method encounters at separation are avoided. The choice of interaction law is important since convergence speed depends on how well this law describes the interaction with the outer flow.

In most papers [CHR95, VLDBS90] in which the coupling is described between the boundary layer and the external flow, the interaction law used is based on thin-aerofoil theory. This gives a symmetric and diagonally dominant interaction law matrix. However, for most cases this interaction law is not close to the inviscid model, which results in a loss of convergence speed.

This dilemma is also described in Arnold and Thiede [AT93]. They instead derive a Laplace interaction law for the coupling of their Smith and Hess panel method [Kat91] and boundary layer. This gives them a more accurate representation. However, because of the panel method used, the calculations are performed in the panel midpoints causing the interaction law matrix to become less well behaved.

Here a new interaction law is derived from the influence matrix of the Dirichlet panel method [Kat91]. It allows the calculations to be performed in the panel endpoints, which leads again to a well behaved interaction law matrix.

The other application of quasi-simultaneous coupling that is to be discussed in this paper is of a swept tapered wing in transonic flow. For this case it is not easy to find an approximation to the external flow, as it is governed by the transonic small perturbation equation which is essentially non-linear. An interaction law based on thin-aerofoil theory has therefore initially been applied, to show the use of QS for this quasi-three-dimensional case. It is hoped to derive a more accurate representation in the near future.

DERIVATION OF THE DIRICHLET INTERACTION LAW

For the 2D steady incompressible case the interaction law will be derived from the inviscid formulation. This will be explained here in more detail. The inviscid flow solution is found by using a two-dimensional potential method [Kat91]. Along the aerofoils surface a source sheet of strength σ is distributed together with a doublet sheet of strength μ. The latter is also extended into the wake. With free stream potential ϕ_∞, the total potential ϕ^* can be written as:

$$\phi^* = \phi_\infty + \frac{1}{2\pi}\int_{S_B} \sigma \ln r \, ds - \frac{1}{2\pi}\int_{S_B+S_W} \mu \frac{\partial}{\partial n}(\ln r)\, ds, \quad (1)$$

in which n is the vector normal to the surface in the direction of μ, s a coordinate along the surface and r the distance between a point at s and a field point (x, y). Furthermore, S_B and S_W are the contours of the aerofoil and the wake respectively.

For an enclosed region, equation (1) has a zero normal velocity boundary condition. If there is however a boundary layer present, the outer flow does not see the real aerofoil but sees instead a thickened one. To take this displacement effect of the boundary layer into account the normal velocity boundary condition has to be changed. The normal velocity at the boundary should be taken to be equal to the transpiration velocity V_n:

$$\frac{\partial \phi^*}{\partial n} = V_n = -\frac{\partial u_e \delta^*}{\partial s}, \quad (2)$$

in which δ^* is the displacement thickness and u_e the velocity at the edge of the boundary layer. The minus sign is a result of n being the inward normal on the body and pointing down in the wake. Physically, the above transpiration velocity represents the stream wise rate of change of mass defect, $m = u_e\delta^*$.

In the wake, the displacement effect is a jump in normal velocity, ΔV, across a convenient line that divides the flows coming from the upper and lower surfaces of the aerofoil:

$$\Delta V = \frac{\partial}{\partial s}(u_{e_{upper}}\delta^*_{upper}) + \frac{\partial}{\partial s}(u_{e_{lower}}\delta^*_{lower}). \quad (3)$$

However, for simplicity the wake is modelled as just one layer, with thickness $\delta^* = \delta^*_{upper} + \delta^*_{lower}$ and tangential velocity $u_e = u_{e_{upper}} = u_{e_{lower}}$.

Instead of using the Neumann boundary condition (2), the Dirichlet boundary condition is used, by specifying the potential inside the body surface ϕ^*_{int} on the boundary. For collocation points inside the body, the internal potential is:

$$\phi^*_{int} = \phi_\infty + \frac{1}{2\pi}\int_{S_B+S_W} \sigma \ln r \, ds - \frac{1}{2\pi}\int_{S_B+S_W} \mu \frac{\partial}{\partial n}(\ln r)\, ds. \quad (4)$$

With the Dirichlet boundary condition $\phi^*_{int} = \phi_{int} + \phi_\infty = \phi_\infty$, where ϕ_{int}, the perturbation potential is set to zero, equation (4) simply becomes:

$$\frac{1}{2\pi}\int_{S_B+S_W} \mu \frac{\partial}{\partial n}(\ln r)\, ds = \frac{1}{2\pi}\int_{S_B+S_W} \sigma \ln r \, ds. \quad (5)$$

Consequently the source distribution σ is found which is, along the aerofoil's surface:

$$\sigma \quad = \quad \frac{\partial \phi^*_{int}}{\partial n} - \frac{\partial \phi^*}{\partial n} = U_\infty \cdot n + \frac{\partial u_e \delta^*}{\partial s}. \tag{6}$$

The term $\partial u_e \delta^* / \partial s$ on the right hand side is an effect of the presence of the boundary layer. It's value is to be determined from the boundary layer equations. In the wake (6) holds with $U_\infty \cdot n$ omitted.

To determine the problem uniquely, an implicit Kutta condition is used to define the doublet strength in the wake. The final result is a system of algebraic equations for the unknown doublet strengths on the surface of the aerofoil.

Discretisation

To determine the unknown doublet strength, the aerofoil's surface and wake are discretised into a number of straight line panels, with N nodes on the aerofoil and N_w nodes in the wake. Each panel will have a constant doublet strength, μ, and a constant source strength, σ, as defined previously. The collocation points are taken in the midpoints of the panels since defining them in the endpoints would lead to singularities.

From equation (5), after discretisation and manipulation, a relation for the doublet strength can be determined in the midpoints of the panels. However, as the tangential velocity of the outer flow (OF) is the gradient of the doublet strength, a relation for u_{eOF} can be found in the endpoints:

$$u_{eOF_i} \quad = \quad u_{e0_i} + \sum_{j=1}^{N+N_w-1} D(i,j) u_{e_i} \delta^*_i. \tag{7}$$

In the above expression, u_{e0} is the real inviscid velocity and the second part on the right hand side represents the disturbance caused by the presence of the boundary layer. Matrix D also contains the central discretisation over a panel of the gradient $\partial u_e \delta^* / \partial s$. The values of u_{eOF} at the trailing edge points are obtained by extrapolation.

In the wake, where the dividing streamline is taken parallel to the free stream, the velocity is obtained with the total potential:

$$u_{eOF_i} \quad = \quad \nabla \phi^* \cdot t|_i,$$

with t the vector tangential to the surface. As ϕ^* is defined in the midpoints of the panels, u_{eOF} in the wake is calculated there as well. By simple averaging, the value of u_{eOF} can be found in the endpoints. After discretisation and manipulation, an equation similar to (7) is found for u_{eOF} in the wake region.

The now determined influence matrix D has a suitable structure, being diagonally dominant and with off-diagonals having equal and opposite sign.

A simple interaction law, $u_e \approx I[\delta^*]$, containing only the local influence, can now be derived from the above determined equations for u_{eOF} on the surface and wake.

$$\begin{aligned} u_{e_i} &\approx u_{e0_i} + D(i,i-1)u_{e_{i-1}}\delta^*_{i-1} + D(i,i)u_{e_i}\delta^*_i + D(i,i+1)u_{e_{i+1}}\delta^*_{i+1}, \\ &\approx u_{e0_i} + \sum_{j=i-1}^{i+1} I(i,j)u_{e_j}\delta^*_j, \end{aligned}$$

where $I(i,j)$ is the tridiagonal interaction law matrix, containing the main and off-diagonal values of matrix D. The above interaction law equation together with the integral boundary layer equations are to be solved with a Newton method to give a new displacement thickness.

Results NACA4412 aerofoil

Calculations with the Dirichlet QS program Viscous-Inviscid Boundary Layer Analysis (VIBLA) have been performed for several aerofoils. Here results will be presented for the NACA 4412 section at $Re = 4.17 \times 10^6$. The results shown in figure 1 are compared with experiment and calculations from Hastings and Williams [HW87].

In the first figure the lift coefficient is shown. Up to an incidence of 11^o the results correspond very well with experiment and with the results of Hastings and Williams. However, near stall the prediction is slightly less accurate as Cl_{max} is obtained too early. After an incidence of 12.6^o no calculations could be performed.

In the figure top right the pressure distribution is compared with experimental results at an incidence of 12.5^o. The results are in good agreement. Only in the trailing edge region, where there is separation, do the results differ. In the figure bottom left the displacement thickness is shown and compares very well with experiment. Near the leading edge the displacement thickness is slightly under predicted, which is possibly a result of not having modelled the presence of the transition strip. The shape factor is shown in the figure bottom right. In the region of separation, i.e. $H > 4$, the shape factor is over predicted. The poor shape factor predictions are due to the empirical closure relation for H_1. Separation took place at $x/c = 0.8$, which was also predicted by experiment.

DERA-VFP CODE FOR SWEPT, TAPERED WINGS IN TRANSONIC FLOW

The DERA Viscous Full Potential (VFP) code [Smi89] is based on a viscous-coupled method for computing the compressible flow over wing-body combinations. It combines a full-potential flow solver and an integral boundary layer method. The boundary layer equations are derived in a local co-ordinate system for each chord wise strip on the wing. This derivation assumes that each section of the wing behaves as if it were part of a larger simply-tapered wing with the same leading and trailing edge sweeps as the local section. Ashill and Smith [AS85] show that, for conventional wings, this gives results for the boundary layer development that are very similar to those obtained using a fully three-dimensional version of the boundary layer equations.

In the original code, the interaction between the external inviscid flow and the viscous regions was governed by the boundary-layer equations through the semi-inverse

(SI) method. The aim of the work described here was to replace the semi-inverse method with a quasi-simultaneous (QS) coupling method. The motivation for this study was the, already observed, improved performance (in terms of convergence rate and robustness) of the QS scheme when compared with the SI method (*e.g.*, King and Williams [KW88]) for two-dimensional flows.

The interaction law VFP-QS

As previously explained, the QS method requires a suitable locally linearised representation of the external flow to be defined. For incompressible flow the most commonly used interaction law is derived from thin-aerofoil theory. For subsonic flow ($M < 0.7$, say) the thin-aerofoil representation is still valid if a compressibility correction term, such as the Prandtl-Glauert rule, is used [VLDBS90]. The linear relation between u_e and δ^* can be found as follows,

$$u_{e_i} = u_{e0_i} + u'_i,$$

where u_{e0} is the inviscid velocity and u' is the velocity perturbation induced by the boundary layer. Now, by the Prandtl-Glauert law:

$$u'_i = \frac{u'_{i\,incompr}}{\sqrt{1 - M_\infty^2}},$$

and $u'_{i\,incompr}$ may be found locally with the thin-aerofoil theory solution,

$$u'_{i\,incompr} \approx \frac{1}{\pi} \int_{x_{i-1}}^{x_{i+1}} \frac{V_n d\xi}{(x_i - \xi)},$$

in which V_n is the transpiration velocity.

It would be more computationally efficient to use an interaction law that takes account of the local flow conditions, including the local Mach number of the external flow. For transonic flow it is not easy to find a more accurate interaction law based on a linear representation for the external flow from the transonic small perturbation (TSP) equation. Not least because the TSP equation is essentially nonlinear. However, the function of the interaction law within a QS procedure is not to define the solution, but to 'point' the solution in the right direction. The more accurate the interaction law is in representing the outer flow, the better it can point the solution in the right direction and the faster the method will converge. As the interaction law described above for transonic flow is not representative, a penalty is probably introduced in terms of slower convergence.

Runge-Kutta-4 versus Newton in VFP

A four-stage Runge-Kutta (RK4) method is used in the original VFP program to solve the set of boundary layer equations which is not suitable for use with a QS scheme. As described above, an extra equation, the interaction law, has to be solved simultaneously with the boundary layer equations if QS coupling is applied. This would have to be applied at each stage of the RK4 scheme (i.e. at each of the four stages between grid points). To avoid the calculation of an interaction law at all the intermediate stages between grid points, a Newton iterative scheme has been used.

The use of a Newton scheme however has the disadvantage that the scheme is first-order accurate (using upwinded differences), whereas the RK4 scheme is fourth-order accurate. This consequently leads to a loss of formal accuracy.

Results W4 wing

Results are shown in figures 2, 3 and 4 for $M = 0.78$, $Re = 13.3 \times 10^6$ at an incidence $\alpha = 1^o$. Comparing solutions from the SI and QS versions of the VFP code, the agreement is generally good, particularly in view of the quite different integration procedures being used to compute the boundary layer in each case. In particular the shock is predicted by the QS method to be at the same location as predicted by the SI method. The solution of the QS scheme differs only slightly at the trailing edge and in the wake.

The calculations for both the SI scheme and the QS scheme took similar computer time, with the number of iterations being fixed. However, little experimentation has been carried out with the numerical parameters of the QS scheme, and there is scope for optimisation of these to reduce the run time.

CONCLUSIONS

Quasi-simultaneous coupling has been applied successfully for two-dimensional incompressible flow and quasi-three-dimensional transonic flow. For the 2D incompressible case a new Dirichlet interaction law has been derived and for the transonic case a thin-aerofoil theory interaction law with compressibility correction term has been used. For the latter case it is hoped to improve the interaction law by finding a more suitable approximation, to speed up convergence. The methods used are to be extended to the development of a fully three-dimensional interaction method.

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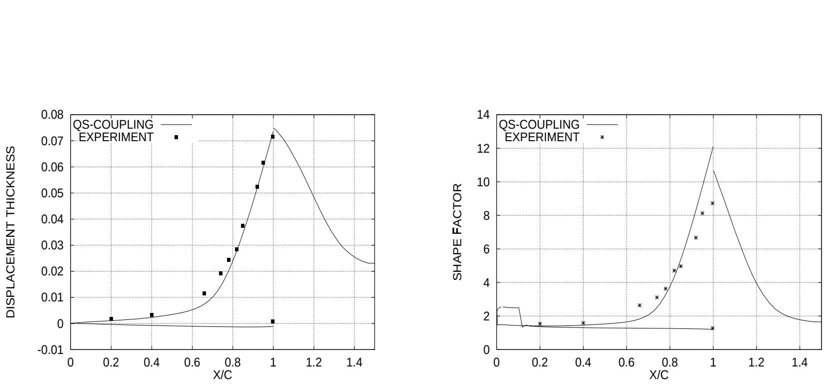

Figure 1 *Results for NACA 4412 at* $Re = 4.17 \times 10^6$

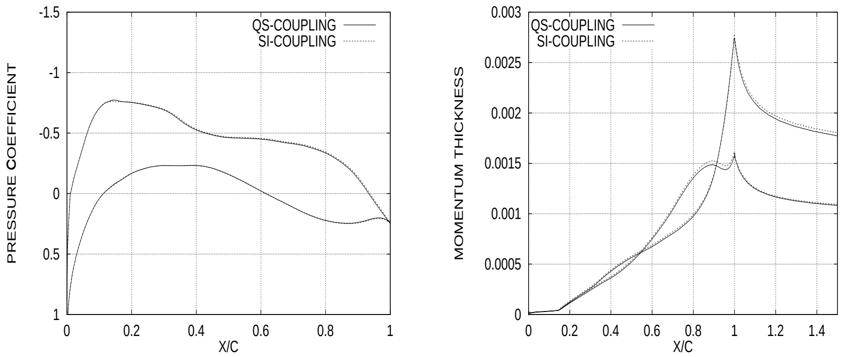

Figure 2 *Results W4 wing at* $\eta = 0.132$, $Re = 13.3 \times 10^6$, $M = 0.78$

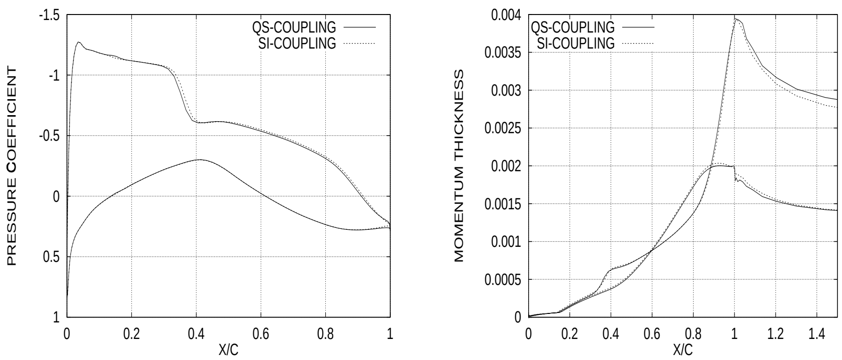

Figure 3 *Results W4 wing at* $\eta = 0.534$, $Re = 13.3 \times 10^6$, $M = 0.78$

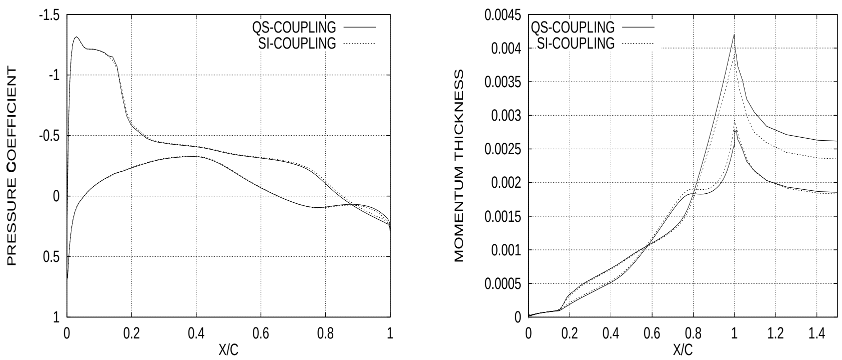

Figure 4 *Results W4 wing at* $\eta = 0.988$, $Re = 13.3 \times 10^6$, $M = 0.78$