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Domain decomposition and parallel processing in microwave applicator design.

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Introduction

In this paper, spatial domain decomposition with a direct solver is presented. It is used with the frequency domain finite element method to compute the electric field in microwave heating applicators. Parallel implementation of the scheme is discussed, and the analysis of a loaded microwave heating cavity is presented. It is demonstrated how this method can be used to very rapidly determine the change in reflection coefficient as a function of localised changes in geometry.

The finite element method is a powerful technique for the electromagnetic analysis of microwave heating cavities, and a number of authors have reported on its use, notably Dibben and Metaxas [DM94] and Chassecourte et al. [CLM93].

It was found by Dibben and Metaxas [DM94] that for multimode microwave cavities loaded with a food-like material, the matrices produced by the frequency domain finite element method become extremely ill-conditioned. If an iterative method is to be used with the highly ill-conditioned system that is produced by the frequency domain formulation, very sophisticated and complicated preconditiong would be required. An alternative approach is to use a direct solver. Although direct solvers require a great deal more storage than iterative schemes, with the continual decrease in the cost of computer memory, they are becoming an attractive alternative, even for three dimensional structures of medium size.

The direct solution spatial domain decomposition technique described here, has been

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Eleventh International Conference on Domain Decomposition Methods
Editors Choi-Hong Lai, Petter E. Bjørstad, Mark Cross and Olof B. Widlund

extensively used in structural analysis in past decades [Prz63], but comparatively few applications have thus far been reported in electromagnetics [WRH92, SC95].

The domain decomposition method

The problem domain is subdivided into arbitrarily shaped subregions and a mesh is created for each subregion, with the meshes of adjacent subregions conforming on the shared boundaries. Using elemental matrices found from applying the FEM in the normal way [Jin93]pp. 244–250, a finite element matrix and vector is built up for each subregion independently. Each such system is then reordered so that the unknowns lying on the boundary $\{x_{BB}\}$ of the subregion are numbered last, and the unknowns lying in the interior $\{x_{II}\}$ are numbered first:

$$\begin{bmatrix} A_{II} & A_{IB} \\ A_{BI} & A_{BB} \end{bmatrix} \begin{Bmatrix} x_{II} \\ x_{BB} \end{Bmatrix} = \begin{Bmatrix} f_{II} \\ f_{BB} \end{Bmatrix} \tag{1}$$

The interior unknowns can now be written in terms of the boundary unknowns, by using the first row of the above equation:

$$\{x_{II}\} = [A_{II}]^{-1}\{f_{II}\} - [A_{II}]^{-1}[A_{IB}]\{x_{BB}\} \tag{2}$$

Substituting this into the second row of equation (1) allows the internal unknowns of the subregion to be eliminated, resulting in the reduced system in terms of only the boundary unknowns:

$$[A_{SC}]\{x_{BB}\} = \{f_{SC}\} \tag{3}$$

where

$$\begin{aligned} [A_{SC}] &= [A_{BB}] - [A_{BI}][A_{II}]^{-1}[A_{IB}] \\ \{f_{SC}\} &= \{f_{BB}\} - [A_{BI}][A_{II}]^{-1}\{f_{II}\} \end{aligned} \tag{4}$$

$[A_{SC}]$ is the Schur complement matrix of the system [SBG96]pp. 105–106. In the same way, the internal unknowns are eliminated for every subregion. The reduced systems of the subregions can then be combined into a reduced system for the whole region. After this is solved, the internal unknowns for each subregion are recovered using equation (2).

It is possible to execute this process in several steps. The internal unknowns for each subregion are eliminated, after which adjacent subregions are grouped together, and their reduced linear systems combined. The internal unknowns of these larger subregions, corresponding to unknowns on shared boundaries, are then eliminated, and the resulting larger subregions combined with adjacent ones into even larger subregions, until the whole domain is encompassed. In this way a pyramidal or hierarchical structure is created which is traversed upwards in order to eliminate unknowns, and then downwards in order to find their values.

Implementation

A domain decomposition FE program was created in C++ and Fortran 90 using the above concept, to solve the vector wave equation [Met96]pp. 41–43:

$$\nabla \times \frac{1}{\mu}\nabla \times \mathbf{E} + \sigma_e \frac{\partial \mathbf{E}}{\partial t} + \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = -\frac{\partial \mathbf{J}_s}{\partial t} \tag{5}$$

Using this program, solutions were computed in closed cavities fed by a waveguide, with the boundary condition on the metal walls being zero tangential electric field, and excitation by an imposed field distribution across the width of the waveguide.

The code was written specifically with parallel execution in mind, and to this end, the elimination of the internal unknowns for each subregion runs as a separate program. Programs communicate using the PVM (Parallel Virtual Machine) library of message passing routines, allowing the code to be used on non-homogenous clusters of workstations.

Because the subregions are combined in a hierarchical manner, the parallelism decreases as one moves upwards in the pyramid. To achieve load balancing, a *pool of tasks* paradigm was implemented, with a single controller processor continually assigning new subregions to the processors as they become available. In this way, reasonable parallel efficiency is achieved, as long as the domain is partitioned into many more subregions than there are processors.

A feature of the direct solution scheme that was deliberately exploited, is that it if the model geometry is changed, it is necessary to recompute the Schur complement matrices for only those subregions affected by the change. This means that if a geometry change is fairly localised, the bulk of the computation does not have to be repeated, allowing a new solution to be very rapidly computed.

Although the intention is eventually to use an unstructured tetrahedral mesh, initially a regular mesh of brick edge-elements [Jin93]pp. 244–250 was used to facilitate geometric mesh decomposition.

Results

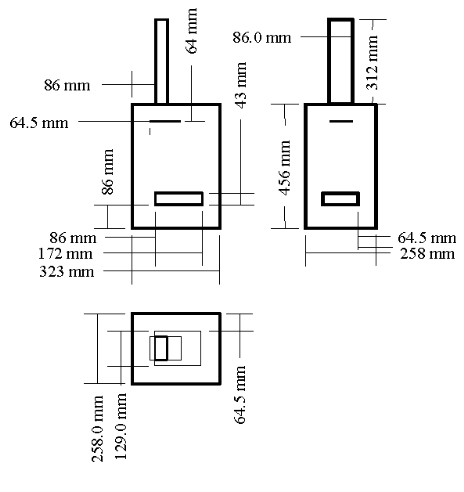

Figure 1 Dimensions of geometry modelled.

For a test problem, a multimode rectangular microwave heating cavity fed by a waveguide was chosen, as shown in Figure 1.
A block of lossy material with permittivity 2-1j was placed some way above the bottom of the cavity. From the computed electric field in the waveguide, it was possible to determine the reflection coefficient of the structure. It was decided to attempt to reduce this by suspending

a thin metal plate below the waveguide aperture, and varying its length in order to improve the matching. Meshing the problem resulted in 78,000 elements, and 138,000 free unknowns.

Using spatial domain decomposition, the geometry was divided into twenty-one subregions at the lowest level, with six levels in the hierarchy. To do the initial elimination of unknowns and field solution for the entire structure took more than two hours, using 16 processors on a Hitachi SR2201 parallel machine.

After this, the length of the thin plate was varied from 7.16mm to 200.6 mm in 28 steps, requiring 28 new field solutions to be computed. However, since the changing plate was confined to only one subregion, a field calculation for each new plate size could be very rapidly computed - in around 45 minutes on a *single* processor. Moreover, the calculations for each new plate size could be done completely in parallel, so that is possible to compute the field patterns, and hence reflection coefficients, for all twenty eight plate sizes simultaneously using twenty eight processors, in 45 minutes.

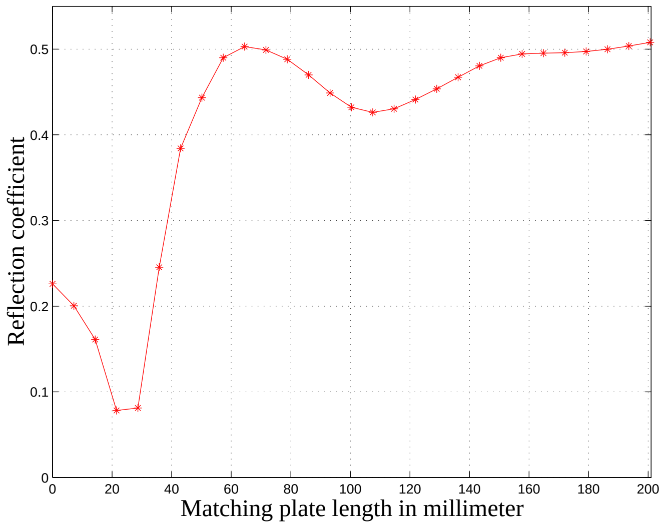

Figure 2 Reflection coefficient against matching plate length.

The results are displayed in Figure 2. From this graph it can be seen that by introducing a small piece of plate 21.5 mm long, the matching of the cavity can be significantly improved - from 0.22 to 0.08.

The field patterns for three points on the graph are displayed in Figure 3. The first has no plate, while in the second the field concentration around the short little plate can be clearly seen. In the third, with a slightly longer plate, the large standing wave ratio in the waveguide, indicating poor matching, can be observed.

Conclusions

Domain Decomposition allows the Finite Element Method to be used in the frequency domain with a direct solver to analyse resonant structures several wavelengths in size. Once an initial

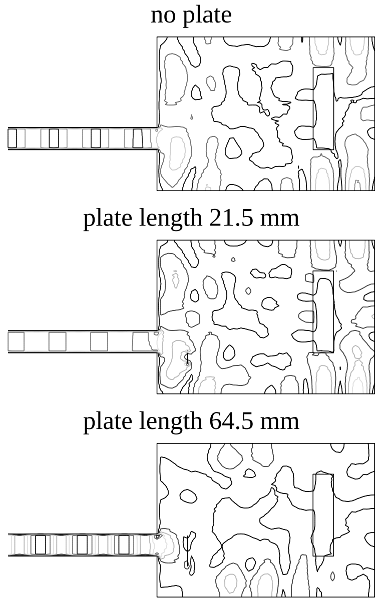

Figure 3 Electric field pattern with three different plate lengths.

solution for the complete structure has been computed, localised changes in the geometry can be isolated in a subregion, and a field solution for the entire structure recomputed very rapidly. This might allow FE-based optimisation of electromagnetic cavities to be done in the future.

Significant parallelism is introduced into the solution process which can be exploited given suitable computer resources.

REFERENCES

[CLM93] Chassecourte P., Lamaudiere J., and Maestrali B. (1993) *Electromagnetic Field Modelling of a Loaded Microwave Cavity.* Presented at 28th IMPI Microwave Power Symposium, Montreal, Canada, July 11 – 14.

[DM94] Dibben D. and Metaxas A. (1994) Finite-Element Time-Domain Analysis of Multimode Applicators Using Edge Elements. *International Journal of Microwave Power and Electromagnetic Energy* 29(4): 242–251.

[Jin93] Jin J. (1993) *The Finite Element Method in Electromagnetics.* John Wiley & Sons, Inc., New York NY.

[Met96] Metaxas A. (1996) *Foundations of Electroheat.* John Wiley & Sons, Inc., Chichester, UK.

[Prz63] Przemieniecki J. S. (1963) Matrix structural analysis of substructures. *Am. Inst. Aero. Astro. Journal* 1: 138–147.

[SBG96] Smith B., Bjørstad P., and Gropp W. (1996) *Domain Decomposition: Parallel Multilevel Methods for Elliptic Partial Differential Equations.* Cambridge University Press, Cambridge.

[SC95] Spring C. and Cangellaris A. (1995) Parallel implementation of domain decomposition methods for the electromagnetic analysis of guided wave systems. *Journal of Electromagnetic Waves and Applications* 9(1/2): 175–192.

[WRH92] Weeber K., Ratnajeevan S., and Hoole H. (March 1992) The subregion method in magnetic field analysis and design optimisation. *IEEE Transactions on Magnetics* 28(2): 1561–1564.