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43. Domain decomposition methods for welding problems

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Introduction

The welding of metals and alloys is a widely used industrial process. Many types of analysis have been carried out on such problems [MUB67]. The numerical thermal analysis of welding is required to take into account such features as temperature dependent material properties, phase change, non-uniform distribution of energy from heat source etc. In this paper, a 2-D non-linear electric arc-welding problem is considered. It is assumed that the moving arc generates an unknown quantity of energy which makes the problem an inverse problem with an unknown source. Robust algorithms to solve such problems efficiently, and in certain circumstances in real-time, are of great technological and industrial interest.

There are other types of inverse problems which involve inverse determination of heat conductivity or material properties [CDJ63][TE98], inverse problems in material cutting [ILPP98], and retrieval of parameters containing discontinuities [IK90]. As in the metal cutting problem, the temperature of a very hot surface is required and it relies on the use of thermocouples. Here, the solution scheme requires temperature measurements lied in the neighbourhood of the weld line in order to retrieve the unknown heat source. The size of this neighbourhood is not considered in this paper, but rather a domain decomposition concept is presented and an examination of the accuracy of the retrieved source are presented.

This paper is organised as follows. The inverse problem is formulated and a method for the source retrieval is presented in the second section. The source retrieval method is based on an extension of the 1-D source retrieval method as proposed in [ILP$^+$99] for metal cutting problems. A parallel algorithm based on the concept of coupling heterogeneous numerical models in different subdomains is given in the third section. Accuracy of the numerical simulation is compared with results that are generated by a known heat source [ASW85][DM93] and with temperature measurements that are obtained by using experimental thermocouples as shown in [ASW85].

The inverse welding problem

Three assumptions are needed in this problem. These assumptions are (1) the material properties are homogeneous across the domain of interest, (2) application of a welding tool along a weld path is equivalent to the application of a heat source along

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the path and (3) the rate of change of temperature on either side of the weld is directly proportional to the strength of the heat source [ILP+99]. The welding problem considered in this paper is the welding of two thin metal plates using the technology of arc-welding. For simplicity, the electric arc is moving along the weld path, $y = y_w$ with a speed of U_w. A straight weld line is depicted in 1 as a dotted line. Without loss of generality, the welding line can be a straight line or a general path. If the welding path was a straight line and that the welding tool travelled along $y = y_w = 0$, then due to the symmetry of the problem only the upper half of the domain needs to be considered. This simplifies the model description and programming effort. Since the thickness of the plate, d, is small compared to the other dimensions, only 2-D heat conduction needs to be considered. Hence, using the first two assumptions, the mathematical model which governs the heat conduction of the plate can be written as the following 2-D nonlinear, unsteady, parabolic, heat conduction equation,

$$c_e \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}(k(T)\frac{\partial T}{\partial x}) + \frac{\partial}{\partial y}(k(T)\frac{\partial T}{\partial y}) - 2h_{eff}A(T - T_a) + \delta(y - y_w)Q_w \quad (1)$$

subject to the initial condition $T(x, y, 0) = T_i(x, y)$ and boundary conditions defined by the functionals $B_0[T(0, y, t), 0, y, t] = 0$, $B_1[T(l, y, t), l, y, t] = 0$, $C_0[T(x, -h, t), x, -h, t] = 0$ and $C_1[T(x, h, t), x, h, t] = 0$. Here $T(x, y, t)$ is the temperature distribution, $k(T)$ is the conductivity of the metal plates, t is the time, h_{eff} is the effective heat transfer, A is the surface area, T_a is the ambient temperature, $c_e = \rho c - L\frac{\partial f_l}{\partial T}$ is the effective specific heat, ρ is the density, c is the specific heat capacity, L is the latent heat, $\frac{\partial f_l}{\partial T}$ is the variation of liquid fraction, $\delta(y - y_w)$ is the Dirac Delta function, $Q_w = Q_w(x, t)$ is the heat transfer rate generated from the moving arc. T_i, B_0, B_1, C_0 and C_1 are known functions. The source term, Q_w, in (1) is

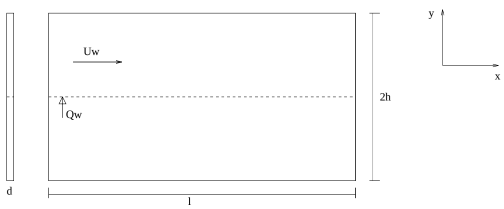

Figure 1: A simple welded work-piece.

an unknown, and the inverse problem here is to retrieve this unknown heat source.

In order to deal with this additional unknown, temperature measurements near the weld line is required (see Figure 2). Thermocouples are attached at $y = y_s$, such that $y_w < y_s < h$. Let the temperature measured by means of the thermocouples be $T(x, y_s, t) = T^*(x, t)$. The measured temperatures are used as interior boundary conditions, as described in next Section, along subdomain interfaces and to retrieve the temperature distribution at the welding points. The heat source retrieval is based on the third assumption, i.e. in the neighbourhood of the weld,

$$\frac{\partial T}{\partial t} = \alpha(x, t)\delta(y - y_w)Q_w(x, t) \quad (2)$$

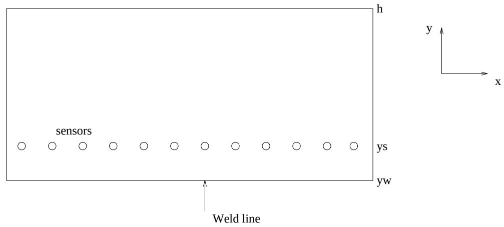

Figure 2: Thermocouples are located near the weld line.

where $\alpha > 1$ is a time dependent function that also depends on x. The condition $\alpha > 1$ is to ensure an increase in temperature at the weld due to an increase in Q_w. Integrating (2) across the weld at a given value of x gives

$$\int_{y_w^-}^{y_w^+} \frac{\partial T}{\partial t} dy = \alpha(x,t) Q_w(x,t) \tag{3}$$

where y_w^+ to y_w^- is the width of the weld along y-axis at a given instance of time under immediate influence of the electric arc. Integrating (1) across the weld and equating the result to (3) lead to

$$\frac{1}{c_e}\{k(T)\frac{\partial T}{\partial y}|_{y_w^+} - k(T)\frac{\partial T}{\partial y}|_{y_w^-} + \frac{\partial}{\partial x}(k(T)\frac{\partial T}{\partial x})(y_w^+ - y_w^-) - 2h_{eff}A(T-T_a)(y_w^+ - y_w^-)\}$$

$$= (\alpha(x,t) - \frac{1}{c_e})Q_w(x,t) \tag{4}$$

Let $\beta(x,t) = c_e\alpha(x,t) - 1$ and define the predicted heat source as $Q_p = \beta(x,t)Q_w(x,t)$ which may be computed as

$$Q_p(x,t) = k(T)\frac{\partial T}{\partial y}|_{y_w^+} - k(T)\frac{\partial T}{\partial y}|_{y_w^-} + \frac{\partial}{\partial x}(k(T)\frac{\partial T}{\partial x})(y_w^+ - y_w^-)$$

$$- 2h_{eff}A(T - T_a)(y_w^+ - y_w^-) \tag{5}$$

Then Q_p can be substituted into (1) to replace Q_w and the non-linear heat conduction problem may then be solved as a direct problem with $T_p(x,t)$ being the corresponding temperature distribution. Hence it is possible to evaluate $\beta(x,t)$ from the knowledge of $T_p(x,t)$ and $T(x,y_w,t)$ as

$$\beta(x,t) = \frac{T_p(x,t)}{T(x,y_w,t)} = \frac{Q_p(x,t)}{Q_w(x,t)} \tag{6}$$

where $T(x,y_w,t)$ is the temperature at the weld line corresponds to $Q_w(x,t)$. Hence $Q_w(x,t)$ may then be determined once $\beta(x,t)$ is known. Note that it is not necessary to compute $c_e\alpha(x,t) - 1$.

The domain decomposition method

Since the only unkown involved in the p.d.e. is the heat source, it makes sense to eliminate the unknown source term of the p.d.e. [ILPP98] for the governing equations on both sides of the welding path. The monitored thermocouple data provides an ideal interior partitioning. For the present study, y_w is chosen as zero. Hence the problem become symmetric and only half of the entire problem needs to be considered. The original domain is partitioned to two well defined, homogeneous, continuous and properly connected subdomains denoted by $D_1 = \{(x, y) : 0 < x < l \text{ and } 0 < y < y_s\}$ and $D_2 = \{(x, y) : 0 < x < l \text{ and } y_s < y < h\}$ and they are depicted as in Figure 3. The two subproblems can be written as follows:

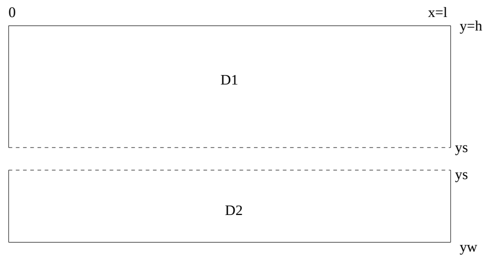

Figure 3: Visualization of subdomains.

$$
\begin{aligned}
SP_1:\quad & c_e \tfrac{\partial T_1}{\partial t} = \tfrac{\partial}{\partial x}(k(T_1)\tfrac{\partial T_1}{\partial x}) + \tfrac{\partial}{\partial y}(k(T_1)\tfrac{\partial T_1}{\partial y}) - 2h_{eff}A(T_1 - T_a)\ inD_1 \\
& \text{subject to } T_1(x, y, 0) = T_i(x, y), \\
& B_0[T_1(0, y, t)0, y, t] = 0,\ B_1[T_1(l, y, t), l, y, t] = 0, \\
& T_1(x, y_s, t) = T^*(x, t),\ C_1[T_1(x, h, t), x, h, t] = 0.
\end{aligned}
$$

$$
\begin{aligned}
SP_2:\quad & c^e \tfrac{\partial T_2}{\partial t} = \tfrac{\partial}{\partial x}(k(T_2)\tfrac{\partial T_2}{\partial x}) + \tfrac{\partial}{\partial y}(k(T_2)\tfrac{\partial T_2}{\partial y}) - 2h_{eff}A(T_2 - T_a)\ inD_2 \\
& \text{subject to } T_2(x, y, 0) = T_i(x, y), \\
& B_0[T_2(0, y, t), 0, y, t] = 0,\ B_1[T_1(l, y, t), l, y, t] = 0, \\
& T_2(x, y_s, t) = T^*(x, t),\ \tfrac{\partial T_2(x,0,t)}{\partial y} = 0.
\end{aligned}
$$

and are defined on two different subdomains of different sizes which are subjected to different set of boundary conditions. They are non-linear in nature and are completely decoupled from each other. Therefore they may be solved simultaneously or concurrently by using the Newton's method. Let $F(T)$ be defined as

$$F(T) = c_e \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}(k(T)\frac{\partial T}{\partial x}) - \frac{\partial}{\partial y}(k(T)\frac{\partial T}{\partial y}) + 2h_{eff}A(T - T_a) \equiv 0 \qquad (7)$$

which leads to the corresponding Jacobian $J(T)$ as follows,

$$J(T) = c_e \frac{\partial}{\partial t} - k' \frac{\partial^2 T}{\partial x^2} - k \frac{\partial^2}{\partial x^2} - k''(\frac{\partial T}{\partial x})^2 - 2k' \frac{\partial T}{\partial x}\frac{\partial}{\partial x} - k' \frac{\partial^2 T}{\partial y^2}$$

$$-k\frac{\partial^2}{\partial y^2} - k''(\frac{\partial T}{\partial y})^2 - 2k'\frac{\partial T}{\partial y}\frac{\partial}{\partial y} + 2h_{eff}A \tag{8}$$

The linearisation leads to an iterative scheme, to be performed in each of the subdomain, $T^{new} = T^{old} - J^{-1}(T^{old})F(T^{old})$ where superscript *new* denotes new iterates and *old* denotes old iterates. $F(T)$ and $J(T)$ are obtained by a second order finite volume method which leads to a set of large sparse linear system and it can be solved by means of a standard domain decomposition software package such as PETSc [BGMS97]. More processors may be used to achieve a secondary level of parallelism for the Newton's iterative scheme, which are separately controlled by different hosts assigned to each of the subproblems. Therefore the inverse welding problem has two levels of parallelism. One level being the differential equation level and the other being the discretised level [ILPP98]. The solution of SP_2 retrieves the temperature at the weld line. The temperature at and around the weld line is, in turn, used to retrieve the unknown source term as described above.

Numerical examples

In this Section, a validation problem for comparison purposes is defined. The true source as given in [ASW85, DM93] is depicted in Figure 9 and the physical data for (1), as given below, are used to derive validation data for comparison. Geometry of the two plates is chosen as $l = 0.5m$, $2h = 0.33m$, $d = 0.008m$, $U_w = 0.00333m/s$. The model problem gives the temperature field of the steel plate. The physical data used in the numerical example are $Q_w = 1350\ W$, $T_a = 293\ K$, $h_{eff} = 60\ W/m^2K$, $\rho = 7850\,kg/m^3$, $c = 607\,J/kgK$, $L = 272\,kJ/kg$, $T_s = 1843\,K$ and $T_l = 1863\,K$. Here T_s is the solidus temperature and T_l is the liquidus temperature. For the present purpose, the liquid fraction f_l is evaluated as,

$$f_l = \begin{cases} 0 & \text{if } T < T_s \\ \frac{(T-T_s)}{(T_l-T_s)} & \text{if } T_s \le T \le T_l \\ 1 & \text{if } T > T_l \end{cases}$$

The nonlinear conductivity of steel is given by,

$$k(T) = \begin{cases} \frac{-27.2}{762}T \ + \ 64.9448 & \text{if } T \le 1035K \\ \frac{8}{881}T \ + \ 18.6016 & \text{if } T > 1035K \end{cases}$$

The initial condition is $T_i(x, y) = T_a$ and the boundary conditions are $B_0 = B_1 = k\frac{\partial T}{\partial x} + h_{eff}(T - T_a) = 0$ and $C_1 = k\frac{\partial T}{\partial y} + h_{eff}(T - T_a) = 0$. The source is applied only at cells which are at a given instant of time under immediate influence of the electric arc. A mesh size of 50×50 is used to obtain the following numerical results. Figure 4 shows the 2-D temperature distribution at $t = 75s$. At this time the arc passes the midsection of the plate ($x = 0.25m$,$y = y_w = 0m$). Therefore, the temperature is at its highest value at this section. Thermocouple temperature measurements are available for comparison from MPA, Stuttgart [ASW85]. Figures 5 to 7 show the comparison of numerical results with the thermocouple measurements. Figure 5 compares the numerical results with measured results when the arc passes the midsection of the plate. Figure 6 shows the comparison at a further 7.5s later, as expected cooling has begun (since the arc has moved away from the midsection). Figure 7 shows the temperature history at the midsection, it illustrates the rapid heating to the melting point when the arc approaches the midsection and the gradual cooling thereafter when the arc has passed the section. These results show that the

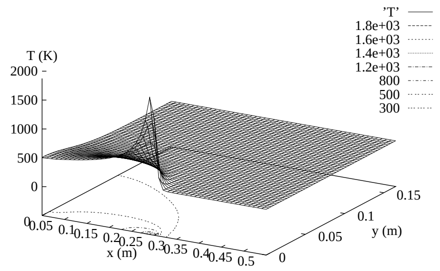

Figure 4: Temperature distribution at t = 75s.

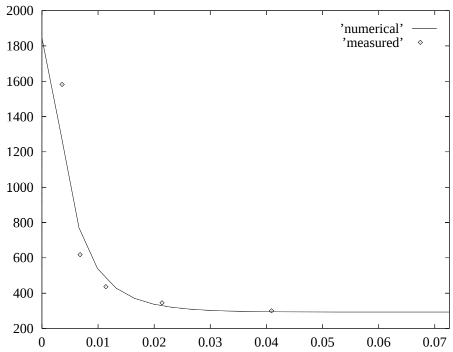

Figure 5: Temperature distribution at x=0.25m and t = 75s (Vertical axis - T in Kelvin and horizontal axis - y coordinates).

derived data matches with the experimental data. Thermocouples are now placed at $y_s = 0.0033m$ as suggested in[ASW85]. The inverse problem given by (1) is solved by using the mesh configuration of 200×200. Figure 8 shows the accuracy of the retrieved temperature field at $x = 0.25m$ and $t = 75s$. At this time step, the electric arc passes over the point $x = 0.25m$ and $y = y_w = 0m$ (midsection), and as expected it generates high temperature values (and gradients) around this point. Figure 9 shows the accuracy of the retrieved source term at $x = 0.25m$ and $y = y_w = 0m$ using the

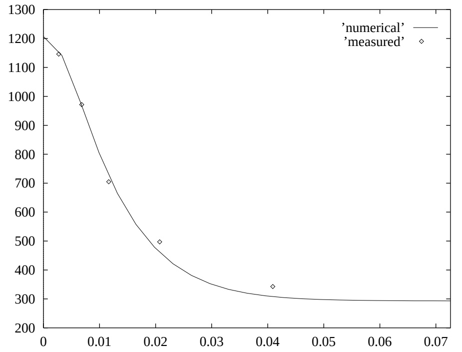

Figure 6: Temperature distribution at x=0.25m and t = 82.5s (Vertical axis - T in Kelvin and horizontal axis - y coordinates).

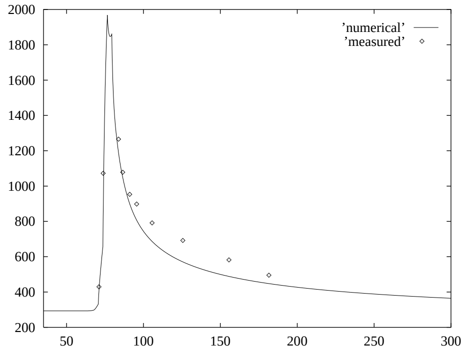

Figure 7: Temperature history at x=0.25m and y=0m (Vertical axis - T in Kelvin and horizontal axis - time in seconds).

proposed method as shown above. The source retrieval is only activated when the electric arc actually passes over this point.

Conclusion

A domain decomposition method is proposed for an inverse problem in arc-welding. The method is based on the partitioning of problems at the continuous problem level where the unknown heat source can be eliminated from the mathematical model and where the subproblems may be completely decoupled. The retrieved heat source compares well with the results generated by using a known heat source of a typical arc-welding and by using experiments.

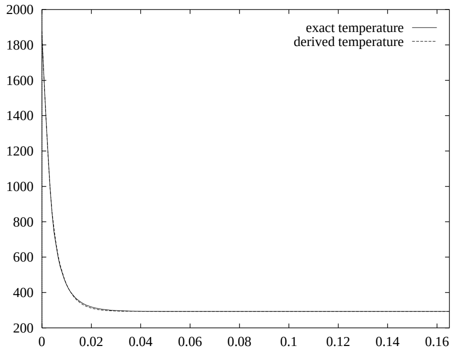

Figure 8: Accuracy of the temperature distribution at x=0.25m and t=75s (Vertical axis - T in Kelvin and horizontal axis - time in seconds).

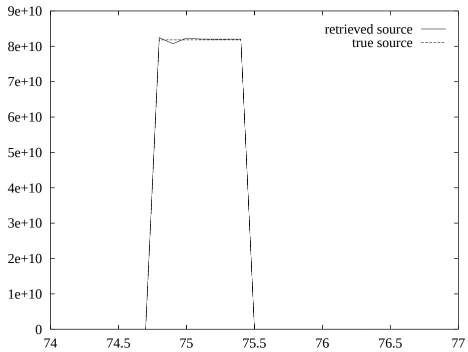

Figure 9: Accuracy of the source retrieval (Vertical axis - source strength and horizontal axis - y coordinates).

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