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47. Shape Optimization for an Acoustic Problem

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Introduction

In this paper, an optimal shape design for an interfacial boundary between different media, through which sound propagates, is discussed. For example, designs for sound-proof walls along high-speed train routes or highways, walls of concert halls, etc are included in the same category.

For the above-mentioned problems, an algorithm to search for an optimal shape was proposed and tested numerically in the three-dimensional problems in [KS99], in which Fuzzy Optimization Method (FOM)[KS97] was used effectively.

Originally, FOM was invented as a local minimizer search algorithm. In order to look for a global minimizer, Multi-start Fuzzy Optimization Method (MS-FOM), which is a hybrid algorithm with FOM and Genetic Algorithms (GAs), has been developed on the basis of FOM[KOPS98].

An application of MS-FOM to such optimization problems makes it possible not only to look for a global minimizer but also to clarify the structure of the manifold of the cost functional defined in the parameter space. This fact depends mainly upon the functions of MS-FOM, one of which is counting-up of all local minimizers.

Here, the algorithm to search for an optimal shape by use of MSFOM is briefly stated and numerical results using it are presented. An observation of such results indicates the rather precise structure of the cost manifold, i.e., the distribution of local maximizers and minimizers in the parameter space. Such observation may be impossible by an application of other global minimizer searching algorithms, for example, Genetic Algorithms.

Finally, physical meanings of a set of local maximizers will be discussed from the view point of resonance phenomena corresponding to the variations of eigenfrequencies of coupled media based on the shape change of the interfacial boundary. Through the discussion mentioned above, shape optimization for an acoustic problem arouses careful treatment to look for a global minimizer.

Shape optimization problem

Configuration

- Γ_{top} and Γ_{bottom} are rigid boundaries, i.e., the density of these walls is infinity. Hence, a sound wave is completely reflected at these boundaries.

- Γ_{in} is a vibrating plate which generates a sound wave.

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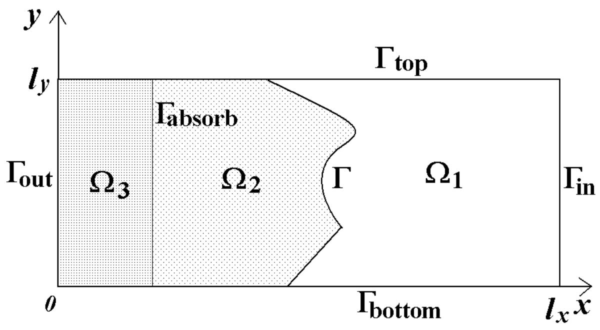

Figure 1: Geometry

- Ω_1 is occupied by water.
- Ω_2 is assumed to be made of pine timber, the role of which is to absorb the sound wave coming through Ω_1.
- Γ is the boundary between Ω_1 and Ω_2. We will try to optimize its shape to transmit the sound wave into Ω_2 as much as possible.
- Ω_3 is a so-called Fictitious Domain, i.e., artificial domain to approximate the boundary condition at infinity. In this domain, Helmholtz eq. with complex wave number is assumed, which is derived from Navier-Stokes eq. including the viscosity term. A sound wave transmitted from Ω_2 is almost completely damped in this domain and is not reflected into Ω_2.
- Γ_{absorb}, on which the amplitude of an absorbed sound wave in Ω_2 is computed.
- Γ_{out}, on which no sound waves exist because of the damping effect in the domain Ω_3.
- $\Omega = \Omega_1 \cup \Gamma \cup \Omega_2 \cup \Omega_3 = (0, l_x) \times (0, l_y)$
- $u^{(i)}(x, y)$ $(i = 1, 2, 3)$: Complex sound pressure.
- k_i $(i = 1, 2, 3)$: Wave number.
- ω : Angular velocity of the incident wave.
- ρ_i $(i = 1, 2, 3)$: Density of medium.
- $\mathbf{n}$: Outward normal vector on the boundaries.
- Γ : Interfacial boundary between Ω_1 and Ω_2.
- α : An incident angle of plane wave.

where $i = 1$ means water, $i = 2$ means pine and $i = 3$ means the fictitious domain.

Parameterization of the interfacial boundary

In order to parameterize the shape of the interfacial boundary, a scaling function for wavelet is introduced as follows;
Let

$$\eta_0(x) = \begin{cases} 1 & x \in [0,1], \\ 0 & else, \end{cases} \tag{1}$$

$$f_0(x) = (-0.585x^2 + 1.867x)\eta_0(x), \tag{2}$$
$$f_1(x) = (1.170x^2 - 2.734x + 1.282)\eta_0(x), \tag{3}$$
$$f_2(x) = (0.585x^2 + 0.867x - 0.282)\eta_0(x). \tag{4}$$

and

$$\phi(x) = f_0(x) + f_1(x-1) + f_2(x-2). \tag{5}$$

We define

$$\phi_{L,m}(x) = \sqrt{N_L} \cdot \phi(N_L x - m) \ \ (m \in Z) \tag{6}$$

where $N_L = 2^L$. Then $\{\phi_{L,m}\}$ constitutes an orthonormal set, i.e.,

$$\int_R \phi_{L,m}\phi_{L,m'}dx = \delta_{m,m'}. \tag{7}$$

By using these scaling functions, we parameterize the interfacial boundary by means of a superposition of $\phi_{L,m}(y)$, i.e.,

$$\Gamma(y) = \sum_m \gamma_m \cdot \phi_{L,m}(y). \tag{8}$$

Admissible set for the deformation of the interfacial boundary is defined by

$$\mathcal{A}_1 = \{\gamma_m \in R | \, |\gamma_m| \le K \ (m = 1,2,3,\cdots,M_1)\}. \tag{9}$$

Definition of optimization problem

Define the state equation;

$$\begin{cases} (\triangle + k_i^2)\, u^{(i)}(\Gamma,a) = 0 & \text{in } \Omega_i, \ \ (i=1,2,3), \\ u^{(1)}(\Gamma,a) = u^{(2)}(\Gamma,a) = a & \text{on } \Gamma, \\ \dfrac{\partial u^{(i)}(\Gamma,a)}{\partial n} = 0 & \text{on } \Gamma_{top} \cup \Gamma_{bottom} \ (i=1,2,3), \\ u^{(1)}(\Gamma,a) = e^{ik_1 \cos\alpha l_x} e^{ik_1 \sin\alpha y} & \text{on } \Gamma_{in}, \\ u^{(2)}(\Gamma,a) = 0 & \text{on } \Gamma_{out}, \end{cases} \tag{10}$$

and the cost function;

$$J_c(\Gamma, a) = -\int\limits_{\Gamma_{absorb}} \left|u^{(2)}(\Gamma, a)\right|^2 d\Gamma$$
$$+\frac{1}{\varepsilon}\int\limits_{\Gamma}\left|\frac{1}{\rho_1}\frac{\partial u^{(1)}(\Gamma, a)}{\partial n} - \frac{1}{\rho_2}\frac{\partial u^{(2)}(\Gamma, a)}{\partial n}\right|^2 d\Gamma. \quad (11)$$

In the definition of the cost function, the constraint caused by the transmission condition is included as a penalty term with a small positive parameter ε.

The Dirichlet datum a is defined on Γ by

$$a = \sum_m a_m \cos(\frac{\pi m}{l_y} y). \quad (12)$$

Admissible set for a is represented by

$$\mathcal{A}_2 = \{a_{mm'} \in \mathbf{C} \ \ (m, m' = 0, 1, 2, \cdots, M_2) | |a_{mm'}| \leq L\}. \quad (13)$$

Therefore, our minimization problem is;

$[P_r]$: Minimize $J_c(\Gamma, a)$ for $(\Gamma, a) \in \mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2$.

Numerical solution of Helmholtz equation

In order to compute the sound field in the domain bounded by a complicated interfacial boundary, the coordinate transformation is used as follows;

1. Generate mesh system in the deformed domain, which is the transformation from physical domain to computational one;

$$x = x(\xi, \eta), \ y = y(\xi, \eta). \quad (14)$$

2. Transform differential operators by use of (14).

3. Transform Helmholtz eq. by use of (14).

Transformed Helmholtz equation is discretized by use of finite difference method. Discretized Helmholtz eq. constitutes a large-scale system of equations. In order to solve this system of equations, GPBi-CG method[Zha97] is adopted.

Hybridized algorithm by FOM and GAs

In this section, Multi-start Fuzzy Optimization Method, which is a hybridized algorithm by Fuzzy Optimization Method (FOM) and Genetic Algorithms (GAs), is briefly summarized. Let us define operators F, M and R as follows.

- F : Algorithm due to Fuzzy Optimization Method. This procedure is a down-hill process on the cost manifold. (Refer to [KS97] for the detailed implementations.)

- M : Mountain crossing algorithm. This procedure is a up-hill process on the cost manifold. (Refer to [KOPS98] for the detailed implementations.)
- R : Rearrangement algorithm by GAs. In this procedure, starting points for the next down-hill process are rearranged by use of GAs.

Solution algorithm of (P_r)

The algorithm of Multi-start FOM is stated in the following way;

Step 1 Give an initial population W^0 (the set of searchers).

Step 2 Compute $U^n := FW^n$ (the set of local minimizers obtained).

Step 3 Compute $V^n := MU^n$ (the set of quasi-local maximizers obtained).

Step 4 Compute $W^n := RV^n$ (the set of rearranged searchers).

Step 5 Increase generation number $n := n + 1$ and repeat steps from **2** to **4** until the generation number n is beyond the preset one.

It should be noted that the operation R is applied in order to obtain a good viewing point, which is taken by the surviving searchers through the fitness selection rule. It is observed through our numerical experiments that the viewpoints for restarting initial points are rather effective.

Results and discussions

Local minima, local maxima and a global minimum

As mentioned in the previous section, MS-FOM makes it possible to discover a set of local minimizers. In fact, MS-FOM found at least four local minimizers A, B, C and D. However, MS-FOM does not guarantee non-existence of local minimizers apart from them. The values of the cost function of these local minimizers are 0.01039, 0.02800, 0.01758 and 0.02042, respectively. The local minimum A is the smallest among them and is concluded to be the global minimum. Figures from 2 to 5 show the sound fields corresponding to these local minimizers, respectively. Obviously, they correspond to the different shapes of the interfacial boundary.

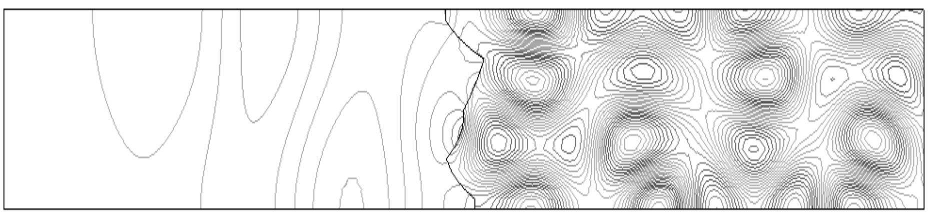

Figure 2: Real part of sound pressure corresponding to local minimizer A

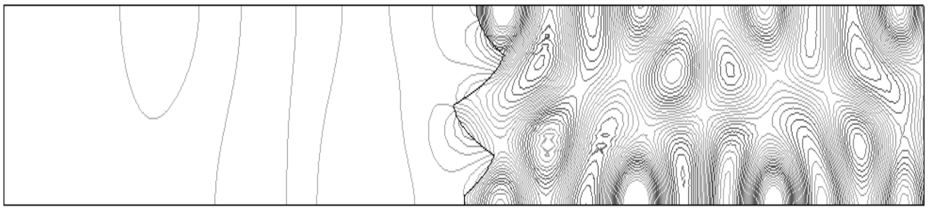

Figure 3: Real part of sound pressure corresponding to local minimizer B

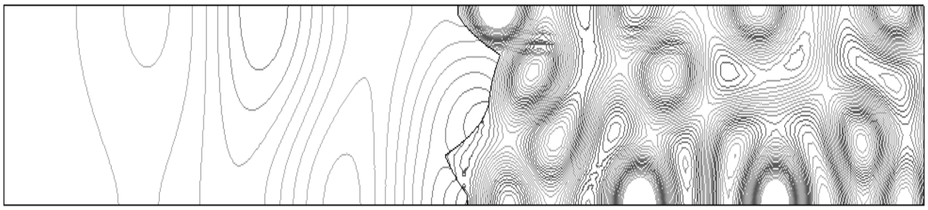

Figure 4: Real part of sound pressure corresponding to local minimizer C

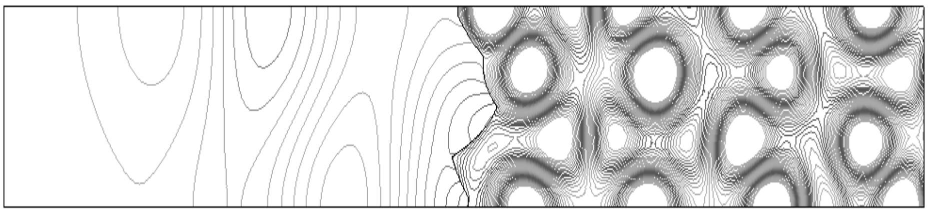

Figure 5: Real part of sound pressure corresponding to local minimizer D

Perspective of the cost manifold

In order to characterize the cost manifold, we draw some one-dimensional cross-sections of the cost manifold. Each one-dimensional cross section is a straight line in the 18-dimensional parameter space connecting two local minimizers. Concretely, figure 6 shows the values of the cost function on a straight line connecting local minimizers A and B, where A is the global minimizer. In this figure, 0 and 1 on the horizontal axis correspond to the local minimizers A and B, respectively. Figure 7 and 8 show the values of the cost function on straight lines connecting B and C, and B and D, respectively. We can see from these figures that the cost manifold has a lot of local minimizers and maximizers. Furthermore, we conjecture from these figures that the cost manifold originally forms convex envelopes and expect that the global minimizer concluded in our computations seems to be a reliable global minimizer. Then, what physical meanings do local maximizers have?

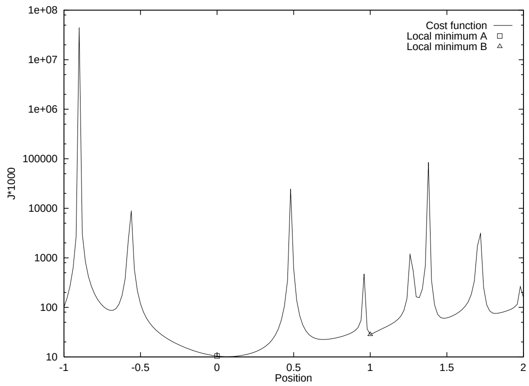

Figure 6: A cross section of the cost manifold connecting local minimizers A and B

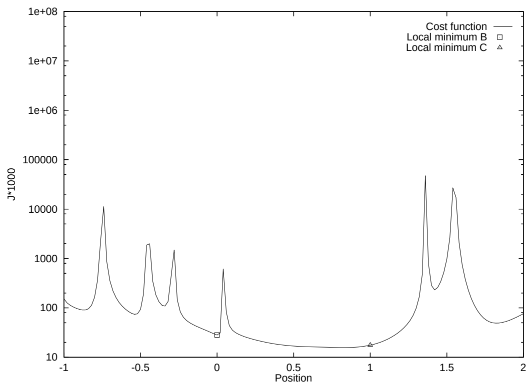

Figure 7: A cross section of the cost manifold connecting local minimizers B and C

A reason of the existence of several local maximizers shown in figures 6, 7 and 8 may be that each local maximizer corresponds to the resonance frequencies of sound

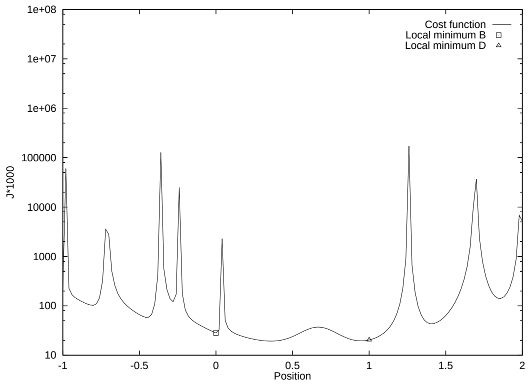

Figure 8: A cross section of the cost manifold connecting local minimizers B and D

propagation in coupled media. The eigenfrequencies of coupled media are sensitive to the interfacial boundary between them. We checked similar phenomena in such a case with a simpler geometry through numerical experiments, in which numerical eigenfrequencies coincided with theoretical ones. In order to provide evidence for such a conjecture, computations of eigenfrequencies to the domain with the related interfacial boundary remain.

Finally, it should be emphasized that MS-FOM has not found out all local minimizers but some of them, however, it counted up the local minimizers very efficiently and it was able to find out the reliable global minimizer. This fact shows the usefulness of our search strategy such as *repeating up-down procedures* and *rearrangement of starting points* mentioned in the previous section, which makes it possible to investigate the perspective of the cost manifold.

Conclusions

The shape design of the interfacial boundary in order to minimize the amplitude of a reflected wave was discussed by use of an algorithm based on MS-FOM. Since the optimization problem with respect to sound propagation includes resonance structure, the cost manifold has very complicated shapes. The numerical results show that the algorithm works well for such problems by avoiding the influence of resonance phenomena.

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