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47 Nonoverlapping Domain Decomposition Algorithms for the System of Euler Equations

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Introduction

We report on our recent efforts concerning the construction of nonoverlapping additive Schwarz type algorithms for the solution of the system of Euler equations for compressible flows. We are specifically concerned with the construction of appropriate interface conditions that improve the convergence rate of the Schwarz algorithm. In Quarteroni and Stolcis[QS95], these transmission conditions are Dirichlet conditions for the characteristic variables corresponding to incoming waves. Such conditions can be qualified as "classical interface conditions" by opposition to more sophisticated formulations such as the "optimized interface conditions " studied in [JNR98] for an advection-diffusion equation. Here, we are interested in extending the principle of optimized interface conditions to the solution of the Euler equations. For this purpose, general type interface operators are introduced in the formulation of the additive Schwarz type algorithm. A convergence analysis is performed in the continuous case by considering the linearized Euler equations. An interface iteration is deduced from the formulation of the Schwarz algorithm in the Fourier space. In [DLN00]-[JNR01], such a convergence analysis has been performed by applying a classical diagonalization method to the operator matrix involved in the problem. In this study, we apply the Smith factorization theory[Gan66] in order to deduce a general form of the interface conditions. Then, the goal is to optimize the convergence rate with respect to certain parameters entering in the definition of these interface conditions. The analysis is limited to a two-subdomain decomposition in vertical strips.

Domain decomposition for the Euler equations

Mathematical model

The conservative form of the Euler equations is given by :

$$\frac{\partial W}{\partial t} + \frac{\partial F_1(W)}{\partial x} + \frac{\partial F_2(W)}{\partial y} = 0 \text{ with } W = \left(\rho \ , \ \rho\vec{V} \ , \ E\right)^T \tag{1}$$

where $W = W(\vec{x}, t)$ is the vector of conservative variables; $\vec{x}$ and t respectively denote the spatial and temporal variables while $\vec{\mathcal{F}}(W) = (F_1(W) \ , \ F_2(W))^T$ is the conservative flux whose components are given by :

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$$
\begin{aligned}
F_1(W) &= \left(\rho u \ , \ \rho u^2 + p \ , \ \rho uv \ , \ u(E+p)\right)^T \\
F_2(W) &= \left(\rho v \ , \ \rho uv \ , \ \rho v^2 + p \ , \ v(E+p)\right)^T
\end{aligned}
$$

In the above expressions, ρ is the density, $\vec{V} = (u, v)^T$ is the velocity vector, E is the total energy per unit of volume and p is the pressure. The pressure is deduced from the other variables using the state equation for a perfect gas $p = (\gamma - 1)(E - \frac{1}{2}\rho \parallel \vec{V} \parallel^2)$ where γ is the ratio of specific heats ($\gamma = 1.4$ for the air). Under the hypothesis that the solution is regular one can also write a nonconservative (or quasi-linear) equivalent form of Eq. (1) :

$$
\frac{\partial W}{\partial t} + A_1(W)\frac{\partial W}{\partial x} + A_2(W)\frac{\partial W}{\partial y} = 0 \tag{2}
$$

where the Jacobian matrices of the flux vectors $F_1(W)$ and $F_2(W)$ (see Dolean[Dol01] for more details). Suppose that we first proceed to an integration in time of (1) using a backward Euler implicit scheme involving a linearization of the flux functions. This operation results in the linearized system :

$$
\mathcal{L}(U) \equiv \frac{\mathrm{Id}}{\Delta t}U + A_1\frac{\partial U}{\partial x} + A_2\frac{\partial U}{\partial y} = f \tag{3}
$$

where $U \equiv W^{n+1} - W^n$ where $W^{n+1} = W(x, (n+1)\Delta t)$, and A_1 (respectively A_2) is a shorthand for $A_1(W^n)$ (respectively $A_2(W^n)$).

In the following we are interested in solving the problem (3), associated to a suitable set of boundary conditions, by a nonoverlapping additive Schwarz type algorithm. An algorithm based on transmission conditions at subdomain interfaces that consist in Dirichlet conditions for the characteristic variables corresponding to incoming waves (following a strategy already studied by Quarteroni and Stolcis[QS95]) has been considered in Dolean and Lanteri[DL99]. The main originality of this preliminary study is that in the discrete case the interface conditions are expressed in terms of upwind conservative normal fluxes computed using the approximate Riemann solver of Roe[Roe81]. This choice is before all motivated by the starting point of our study which was given by a flow solver based on a combined finite element/finite volume formulation on unstructured triangular meshes for the spatial discretization. Time integration of the resulting semi-discrete equations is obtained using a linearized backward Euler implicit scheme. As a result, each pseudo time step requires the solution of a sparse linear system for the flow variables, which is the discrete counterpart of (3).

The two-subdomain case

We consider the case of a two-subdomain decomposition with $\Omega_1 = \mathbb{R}_- \times \mathbb{R}$, $\Omega_2 = \mathbb{R}_+ \times \mathbb{R}$ separated by the interface $x = 0$; let $\vec{n} = (1\ ,\ 0)$ denote the normal vector at the interface $x = 0$, directed from Ω_1 to Ω_2. Let :

$$
M_n = \frac{\vec{V}.\vec{n}}{c} = \frac{u}{c} \quad \text{and} \quad M_t = \frac{\vec{V}.\vec{t}}{c} = \frac{v}{c}
$$

respectively denote the normal and the tangential Mach number at the interface $x = 0$. We also have that, at any point of $\Omega_1 \bigcup \Omega_2$, the Mach number can be expressed as $M =$

$\dfrac{\sqrt{u^2+v^2}}{c} = \sqrt{M_n^2 + M_t^2}$. Let $A_{\mathbf{n}} = n_x A_1 + n_y A_2$ for any vector $\vec{n} = (n_x\ ,\ n_y)^T$. Then, it is well known (from the hyperbolic nature of the system of Euler equations) that the matrix $A_{\mathbf{n}}$ is diagonalizable with real eigenvalues :

$$\begin{array}{c} A_{\mathbf{n}} W = T_{\mathbf{n}}(W)\Lambda_{\mathbf{n}}(W)T_{\mathbf{n}}^{-1}(W) \\ \text{with}\quad \Lambda_{\mathbf{n}}(W) = \text{diag}\left(\vec{V}.\vec{n} - c\ ,\ \vec{V}.\vec{n}\ ,\ \vec{V}.\vec{n}\ ,\ \vec{V}.\vec{n} + c\right) \end{array}$$

Let $U_i^{(0)}$ denote the initial appoximation of the solution in subdomain Ω_i. A general formulation of an additive Schwarz type algorithm for computing $U_i^{(p+1)}$ from $U_i^{(p)}$ (where p defines the iteration of the Schwarz algorithm) writes as :

$$\begin{array}{lll} \Omega_1 & : & \left\{ \begin{array}{lcl} \mathcal{L}(U_1^{(p+1)}) & = & f_1 \quad \text{for}\ \ x < 0 \\ \mathcal{B}_1(U_1^{(p+1)}) & = & \mathcal{B}_1(U_2^{(p)}) \quad \text{for}\ \ x = 0 \end{array} \right. \\ \\ \Omega_2 & : & \left\{ \begin{array}{lcl} \mathcal{L}(U_2^{(p+1)}) & = & f_2 \quad \text{for}\ \ x > 0 \\ \mathcal{B}_2(U_2^{(p+1)}) & = & \mathcal{B}_2(U_2^{(p)}) \quad \text{for}\ \ x = 0 \end{array} \right. \end{array} \tag{4}$$

where the $\mathcal{B}_{1,2}$'s are interface operators. Natural (also qualified as " classical ") interface conditions resulting from the variational formulation of the initial and boundary value problem associated to system (1) are given by :

$$\mathcal{B}_1 = A_{\mathbf{n}}^- = T_{\mathbf{n}}\Lambda_{\mathbf{n}}^- T_{\mathbf{n}}^{-1} \quad \text{and} \quad \mathcal{B}_1 = A_{\mathbf{n}}^+ = T_{\mathbf{n}}\Lambda_{\mathbf{n}}^+ T_{\mathbf{n}}^{-1} \tag{5}$$

In the particular case $\vec{n} = (1\ ,\ 0)$ we have that $T(W) \equiv T_{\mathbf{n}}(W)$ with :

$$T(W) = \begin{pmatrix} 1 & 0 & 1 & 1 \\ u - c & 0 & u & u + c \\ v & c\sqrt{2} & v & v \\ \dfrac{u^2+v^2}{2} - cu + \dfrac{c^2}{\gamma - 1} & vc\sqrt{2} & \dfrac{u^2+v^2}{2} & \dfrac{u^2+v^2}{2} + cu + \dfrac{c^2}{\gamma - 1} \end{pmatrix}$$

By considering the approach adopted by Kroner[Kro91], we can use the matrix $T(W)$ to obtain a symmetrized form of the system (3) :

$$\widetilde{\mathcal{L}}(\widetilde{U}) \equiv \frac{\text{Id}}{\Delta t}\widetilde{U} + \widetilde{A}_1 \frac{\partial \widetilde{U}}{\partial x} + \widetilde{A}_2 \frac{\partial \widetilde{U}}{\partial y} = \widetilde{f} \tag{6}$$

where $\widetilde{U} = T^{-1}U$ and :

$$\begin{array}{lclcl} \widetilde{A}_1(W) & = & T^{-1}(W)A_1(W)T(W) & = & \text{diag}(u - c\ ,\ u\ ,\ u\ ,\ u + c) \\ \widetilde{A}_2(W) & = & T^{-1}(W)A_2(W)T(W) & & \text{is a symmetric matix} \end{array}$$

Smith factorization

The first step consists in applying a Laplace transform in the x direction (the Laplace variable is denoted by λ) and a Fourier transform in the y direction (the Fourier variable is denoted by k) to system (6). The transformed system writes $A(\lambda, k)\widehat{W} = \hat{f}$. The expression of the transformed matrix $A(\lambda, k)$ is given in Dolean[Dol01]. An important result of the Smith factorization theory[Gan66] is that the polynomial matrix $A(\lambda, k)$ can be factorized as :

$$A(\lambda, k) = E(\lambda, k) D_s(\lambda, k) F(\lambda, k)$$

where $D_s(\lambda, k)$ represents the Smith diagonal form of $A(\lambda, k)$; $E(\lambda, k)$ (respectively $F(\lambda, k)$) is a permutation matrix that operates on the lines (respectively the columns) of $A(\lambda, k)$. In the present case, we obtain :

$$D_s(\lambda, k) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \mathcal{G}(\lambda, k) & 0 \\ 0 & 0 & 0 & \mathcal{G}(\lambda, k)\mathcal{L}(\lambda, k) \end{pmatrix} \tag{7}$$

where :

$$\begin{cases} \mathcal{L}(\lambda, k) & = & -(c^2 - u^2)\lambda^2 + 2u(\beta + ikv)\lambda + c^2k^2 + (\beta + ikv)^2 \\ \mathcal{G}(\lambda, k) & = & \lambda u + (\beta + ikv) \end{cases} \tag{8}$$

and :

$$F(\lambda, k) = \begin{pmatrix} \beta & \lambda & ik & 0 \\ 0 & F_{22} & F_{23} & (\gamma - 1)\beta^2 \\ 0 & F_{32} & F_{33} & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{9}$$

Smith form of the Schwarz algorithm

Let $W = (w_1\ ,\ w_2\ ,\ w_3\ ,\ w_4)^T$ denote the vector of conservative variables and $\overline{W} = FW$ the corresponding vector of Smith variables. The equations within each subdomain can be rewritten as :

$$D_s\overline{W} = \bar{f} \quad \Leftrightarrow \quad \begin{cases} \beta w_1 + \lambda w_2 + ikw_3 = \bar{f}_1 \\ F_{22}w_2 + F_{23}w_3 + (\gamma - 1)\beta^2 w_4 = \bar{f}_2 \\ \mathcal{G}\bar{w}_s \equiv \mathcal{G}(F_{32}w_2 + F_{33}w_3) = \bar{f}_3 \\ \mathcal{G}\mathcal{L}\bar{w}_2 = \bar{f}_4 \end{cases} \tag{10}$$

Because of the structure of the matrix D_s it is sufficient to work with two Smith variables, w_2 and w_s, the other ones being obtained from the relations (11). Let $(E_i^{(p)})(x) = (W_i^{(p)} -$

$W_i)(x) = (e_1^i\ ,\ e_2^i\ ,\ e_3^i\ ,\ e_4^i)^T$ be the error vector in the subdomain Ω_i after the iteration p of the Schwarz algorithm. Using the change of variables $\overline{E} = FE$, the Schwarz algorithm is given by :

$$
\begin{aligned}
\Omega_1 \quad &: \quad \begin{cases} \mathcal{G}\left((\bar{e}_s^1)^{(p+1)}\right) \text{ and } \mathcal{GL}\left((\bar{e}_2^1)^{(p+1)}\right) \quad \text{for } x<0 \\ \mathcal{B}(\overline{E}_1^{(p+1)})_j = \mathcal{B}(\overline{E}_2^{(p)})_j \quad \text{for } \ x=0 \ \text{ and } \ \lambda_j(A_1)<0 \end{cases} \\
\Omega_2 \quad &: \quad \begin{cases} \mathcal{G}\left((\bar{e}_s^2)^{(p+1)}\right) \text{ and } \mathcal{GL}\left((\bar{e}_2^2)^{(p+1)}\right) \quad \text{for } x>0 \\ \mathcal{B}(\overline{E}_2^{(p+1)})_j = \mathcal{B}(\overline{E}_1^{(p)})_j \quad \text{for } \ x=0 \ \text{ and } \ \lambda_j(A_1)>0 \end{cases}
\end{aligned}
\tag{11}
$$

where $\mathcal{B} = \mathcal{B}(\lambda, k)$ is a 4×2 matrix corresponding to the last two columns of the 4×4 matrix $T^{-1}(W)F^{-1}(\lambda, k)$. From now, we assume that the flow in subsonic i.e. $M < 1$. By taking into account the sign of the eigenvalues we obtain :

$$
\begin{aligned}
\Omega_1 \quad &: \quad \left\{ b_{11}(\bar{e}_2^1)^{(p+1)} = b_{11}(\bar{e}_2^2)^{(p)} + b_{12}(\bar{e}_s^2)^{(p)} \right. \\
\Omega_2 \quad &: \quad \begin{cases} b_{21}(\bar{e}_2^2)^{(p+1)} + b_{22}(\bar{e}_s^2)^{(p+1)} = b_{21}(\bar{e}_2^1)^{(p)} \\ b_{31}(\bar{e}_2^2)^{(p+1)} + b_{32}(\bar{e}_s^2)^{(p+1)} = b_{31}(\bar{e}_2^1)^{(p)} \\ b_{41}(\bar{e}_2^2)^{(p+1)} + b_{42}(\bar{e}_s^2)^{(p+1)} = b_{41}(\bar{e}_2^1)^{(p)} \end{cases}
\end{aligned}
\tag{12}
$$

On the other hand, the local solutions are explicitly given by :

$$
\bar{e}_2^1 = \alpha_1 e^{\lambda_{\mathcal{L}_1} x} \ , \ \ \bar{e}_2^2 = \alpha_2 e^{\lambda_{\mathcal{G}} x} + \alpha_3 e^{\lambda_{\mathcal{L}_2} x} \ , \ \ \bar{e}_s^2 = \alpha_4 e^{\lambda_{\mathcal{G}} x}
\tag{13}
$$

where $\lambda_{\mathcal{G}}$ and $\lambda_{\mathcal{L}_{1,2}}$ are the eigenvalues of the Fourier symbols $\Lambda_{\mathcal{G}}$ and $\Lambda_{\mathcal{L}_{1,2}}$ that factorize the operators $\mathcal{G}$ and $\mathcal{L}$ i.e. $\mathcal{G} = \partial_x - \Lambda_{\mathcal{G}}$ and $\mathcal{L} = (\partial_x - \Lambda_{\mathcal{L}_1})(\partial_x - \Lambda_{\mathcal{L}_2})$.

Generalized interface conditions

Using the relation $(\bar{e}_s^2)^{(p+1)} = \dfrac{b_{21}}{b_{22}}\left((\bar{e}_2^1)^{(p)} - (\bar{e}_2^2)^{(p+1)}\right)$ we can rewrite the interface iterations (12) as:

$$
\begin{aligned}
\Omega_1 \quad &: \quad \left\{ b_{11}b_{22}(\bar{e}_2^1)^{(p+1)} = (b_{11}b_{22} - b_{21}b_{12})(\bar{e}_2^2)^{(p)} + b_{21}b_{12}(\bar{e}_2^1)^{(p-1)} \right. \\
\Omega_2 \quad &: \quad \begin{cases} (b_{31}b_{22} - b_{21}b_{32})(\bar{e}_2^2)^{(p+1)} = (b_{31}b_{22} - b_{21}b_{32})(\bar{e}_2^1)^{(p)} \\ (b_{41}b_{22} - b_{21}b_{42})(\bar{e}_2^2)^{(p+1)} = (b_{41}b_{22} - b_{21}b_{42})(\bar{e}_2^1)^{(p)} \end{cases}
\end{aligned}
\tag{14}
$$

In order to obtain a general form of the iterations we introduce the operators $\mathcal{B}_i = p_i(k)\partial_x^2 + q_i(k)\partial_x + r_i(k)\ i = 1, 4$ and we consider the Schwarz algorithm :

$$\begin{array}{ll}\Omega_1 & : \left\{\begin{array}{lll}\mathcal{L}(e_1^{(p+1)}) & = & 0 \quad \text{for} \quad x<0 \\ \mathcal{B}_1(e_1^{(p+1)}) & = & (\mathcal{B}_1+\mathcal{B}_2)(e_2^{(p)})-\mathcal{B}_2(e_1^{(p-1)}) \quad \text{for} \quad x=0\end{array}\right. \\ \Omega_2 & : \left\{\begin{array}{lll}\mathcal{L}(e_2^{(p+1)}) & = & 0 \quad \text{for} \quad x>0 \\ \mathcal{B}_{3,4}(e_2^{(p+1)}) & = & \mathcal{B}_{3,4}(e_1^{(p)}) \quad \text{for} \quad x=0\end{array}\right.\end{array} \tag{15}$$

where $p_i(k)$, $q_i(k)$, $r_i(k)$ are polynomials in ik. Then, our strategy consists in several steps (see Dolean[Dol01] for more details). First, we derive a new form of the interface conditions by generalizing the expressions of $p_i(k)$, $q_i(k)$, $r_i(k)$. Second, we construct the interface operator $\mathcal{B}$. Finally, we retrieve the interface conditions in physical variables by using the matricial relation $SW=\mathcal{B}F^{-1}W$. The interface conditions in physical variables are obtain from the the matrix S that generalizes the matrix T^{-1} :

$$S(W)=\begin{pmatrix} \dfrac{\sigma+1}{2} & -\dfrac{1}{2}\dfrac{(\sigma-1)(u+c)}{c-u} & 0 & -\dfrac{1}{2}\dfrac{u(\sigma-1)}{c-u} \\ -\dfrac{\sigma+\delta-1}{2} & \dfrac{(\sigma+\delta-1)(c+u)+2(c-u)}{2(c-u)} & 0 & \dfrac{(\sigma+\delta-1)u}{2(c-u)} \\ s_{31} & s_{32} & \alpha & s_{34} \\ s_{41} & s_{42} & 0 & s_{44}\end{pmatrix}$$

with :

$$\left\{\begin{array}{lll} s_{31} & = & -\dfrac{1}{2\sqrt{2}}\dfrac{-(\alpha_1-1)u+c(\delta+\sigma-1))(ikv+\beta)}{ikcu} \\ s_{32} & = & \dfrac{1}{2\sqrt{2}}\dfrac{(\alpha_1-1)u(c-u)+c(c+u)(\delta+\sigma-1))(ikv+\beta)}{ikcu(c-u)} \\ s_{34} & = & \dfrac{1}{2\sqrt{2}}\dfrac{(\alpha_1-1)(c-u)+c(\delta+\sigma-1))(ikv+\beta)}{ikcu(c-u)} \\ s_{41} & = & \dfrac{c(\delta+\sigma-\alpha_2)+(\alpha_2-1)u}{u} \\ s_{42} & = & -\dfrac{(u+c)(c(\delta+\sigma-\alpha_2)+(\alpha_2-1)u)}{u(c-u)} \\ s_{44} & = & -\dfrac{c(\delta+\sigma-\alpha_2)+\alpha_2 u-c}{c-u}\end{array}\right.$$

In Dolean[Dol01] simple interface conditions (without derivatives) are derived from the expression of the convergence rate associated to the iterations (15). These conditions are obtained by setting $\alpha_2=1$ and $\delta=1-\sigma$. This results in interface conditions that depend on the parameter σ only.

Table 1: Nonoverlapping additive Schwarz type algorithm
Classical interface conditions versus generalized interface conditions

M_n	OPT0	OPT1	M_∞	OPT0	OPT1
0.1 and $M_t = 0.0$	20	20	0.3 and $M_t = 0.0$	24	19
0.6 and $M_t = 0.0$	27	17	0.1 and $M_t = 0.1$	24	21
0.3 and $M_t = 0.2$	24	28	0.6 and $M_t = 0.4$	32	18
0.6 and $M_t = 0.7$	25	21	0.8 and $M_t = 0.5$	42	21

Numerical results

Space and time discretization methods

The spatial discretization method adopted here combines the following elements (see Dolean and Lanteri[DL99] for more details) : (1) a finite volume formulation on triangular meshes together with upwind schemes for the discretization of the convective fluxes; (2) an extension to second order accuracy that relies on the MUSCL (Monotonic Upstream Schemes for Conservation Laws) introduced by van Leer[Lee79] and extended to unstructured triangular meshes by Fezoui and Stoufflet[FS89]. Time integration of the resulting semi-discrete equations is obtained using a linearized backward Euler implicit scheme[FS89]. As a result, each pseudo time step requires the solution of a sparse linear system for the flow variables. In this study, a nonoverlapping domain decomposition algorithm is used for advancing the solution at each implicit time step.

Numerical results

We present here a set of preliminary results of numerical experiments that are concerned with the evaluation of the influence of the interface conditions on the convergence of the nonoverlapping additive Schwarz type algorithm of the form (4). The computational domain is given by the rectangle $[0\ ,\ 2] \times [0\ ,\ 1]$. The numerical investigation is limited to the resolution of the linear system resulting from the first implicit time step using a Courant number CFL=100. A slipping condition ($\vec{V}.\vec{n} = 0$) is applied on the lower ($y = 0$) and upper ($y = 1$) walls; an inflow (respectively outflow) condition is applied on the left $x = 0$ (respectively right $x = 10$) boundary. Table 1 summarizes the number of Schwarz iterations required to reduce the initial linear residual by a factor 10^{-10} for different values of the reference Mach number. The underlying triangular mesh is a regular one deduced from a finite difference grid containing 4000 nodes (200×20). In this table, OPT0 stands for the classical interface conditions while OPT1 corresponds to the algorithm based on the generalized interface conditions.

Conclusions

In this work we were interested in the acceleration of the convergence of a nonoverlapping additive Schwarz type algorithm by modifying the transmission conditions applied to the subdomain interfaces. We built generalized zero order interface conditions using Smith theory of diagonalizing polynomial matrices. The numerical experiments confirmed at least qualitatively the behaviour in accordance with the theory even if from the discrete point of view we couldn't reproduce identically the results obtained in the continuous case. The preliminary results are very encouraging as the lead to a very good convergence rate for certain Mach numbers.

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