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45 Comparison of two iterative substructuring methods for advection-diffusion problems

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Introduction

In this paper two different methods of domain decomposition for the advection-diffusion-reaction problem are considered. Both methods are analysed on the continuous level. The first approach is an additive nonoverlapping iteration-by-subdomains algorithm with Robin-type transmission conditions at the interface. This method has been well investigated in the last years (cf. [NR95], [LMO00], [Ott99]). We will give a short review of the results.
The second approach is a Schur complement method. The Schur complement is solved by a preconditioned Richardson iteration. As a preconditioner we use a generalised Neumann-Neumann preconditioner, the Robin-Robin preconditioner (cf. [ATNV00], [QV99], [BS00]). The application of the preconditioner requires the solution of a mixed problem with a Robin interface condition in each subdomain.
We apply a two-dimensional Fourier analysis to both methods in order to illustrate the different convergence behaviour of the methods. Finally we summarize the comparison of the two methods.

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with Lipschitz boundary $\partial\Omega$. We consider the following boundary value problem:

$$\left\{ \begin{array}{rcll} Lu := -\epsilon\triangle u + \mathbf{b}\cdot\nabla u + cu &=& f & \text{in } \Omega \\ u &=& 0 & \text{on } \partial\Omega \end{array} \right. \tag{1}$$

with the diffusion coefficient $\epsilon > 0$, a given flow $\mathbf{b} \in (W^{1,\infty}(\Omega))^2$, the source term $f \in L^2(\Omega)$ and the reaction coefficient $c \in L^\infty(\Omega)$. The variational formulation of (1) is given by

$$\text{Find } u \in V := H_0^1(\Omega) \ : \quad a_\Omega(u,v) = l_\Omega(v), \quad \forall v \in V \tag{2}$$

with

$$\begin{aligned} a_G(u,v) &:= \int_G \epsilon\nabla u\nabla v dx + \int_G (c - \frac{1}{2}\nabla\cdot\mathbf{b})uv dx + \frac{1}{2}\int_G (\mathbf{b}\cdot\nabla uv - \mathbf{b}\cdot\nabla vu)\, dx, \\ l_G(v) &:= \int_G fv dx \end{aligned}$$

for a domain $G \subseteq \Omega$. The weak formulation is obtained in the usual way using integration by parts on $\int_\Omega(-\epsilon\triangle u + \frac{1}{2}\mathbf{b}\cdot\nabla u)v dx$. We require the existence of a constant $\alpha > 0$ such that $c - \frac{1}{2}\nabla\cdot\mathbf{b} \geq \alpha > 0$ is satisfied almost everywhere in Ω. Thus we get by virtue of the Lax-Milgram lemma, that there exists a unique solution of (2).

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The Robin-Robin algorithm

Description of the algorithm

We begin by partitioning the domain Ω into N nonoverlapping subdomains $\Omega_1, \ldots, \Omega_N$, i.e. $\overline{\Omega} = \bigcup_{i=1}^{N} \overline{\Omega}_i$ and $\Omega_i \cap \Omega_j = \emptyset$ for $i \neq j$, where each subdomain $\Omega_i,\ i \in \{1, \ldots, N\}$, is itself a Lipschitz domain with piecewise smooth boundary. We denote the interfaces by $\Gamma_i := \partial\Omega_i \setminus \partial\Omega$ and $\Gamma_{ij} := \partial\Omega_i \cap \partial\Omega_j$ for $i \neq j$. We identify Γ_{ij} with Γ_{ji}. For simplicity we assume that the decomposition is stripwise, i.e $\Gamma_{ij} \neq \Gamma_{kl}$ implies $\inf_{\substack{x\in\Gamma_{ij}\\ y\in\Gamma_{kl}}} |x-y| > 0$. Now the main idea of the algorithm is straight forward. In each subdomain Ω_i a local problem with an iterative coupling of the interface values has to be solved. Defining for $i = 1, \ldots N$

$$\phi_i(u) := \epsilon \frac{\partial u}{\partial \mathbf{n}_i} + \left(-\frac{1}{2}\mathbf{b}\cdot\mathbf{n}_i + z_i\right) u$$

with a strictly positive function $z_i \in L^\infty(\Omega_i)$ and the outward normal $\mathbf{n}_i$ we get

The Robin-Robin algorithm - version 1
Solve for $k \geq 1$ and for all $i = 1, \ldots, N$:

$$\begin{cases} Lu_i^k &= f & \text{in } \Omega_i \\ u_i^k &= 0 & \text{on } \partial\Omega \cap \partial\Omega_i \\ \phi_i(u_i^k) &= \theta\phi_i(u_j^{k-1}) + (1-\theta)\phi_i(u_i^{k-1}) & \text{on } \Gamma_{ij},\ j \neq i, \end{cases} \tag{3}$$

where $\theta \in (0, 1]$ is a relaxation parameter and u_i^0 is a given initial guess. An appropriate choice of the parameter z_i is a difficult problem. We will give some suggestions later.
To derive a variational formulation of the algorithm, we first define the spaces V_i by $V_i := V|_{\Omega_i}$ and W_{ij} by $W_{ij} := Tr_{\Gamma_{ij}} V$. Thus W_{ij} consists of traces on Γ_{ij} of functions belonging to V. Taking into account that the decomposition is stripwise, we obtain $W_{ij} = H_{00}^{\frac{1}{2}}(\Gamma_{ij})$, or $W_{ij} = H^{\frac{1}{2}}(\Gamma_{ij})$ if Γ_{ij} is closed.
Using the notation from above we can rewrite the DD-algorithm (3) starting with an initial guess $(\Lambda_{ij}^0)_{i\neq j}$ (cf. Robin-Robin alg. - vers. 2).

The variational algorithm is equivalent to algorithm (3). The well-posedness of the variational formulation has been shown (cf. [Ott99], Theorem 3.2):

Theorem 1 *If $z_i \in L^\infty(\Gamma_i)$ is a strictly positive function for all i then the Robin-Robin algorithm is well defined, i.e. at each iteration step k each subdomain boundary value problem has a unique solution in V_i.*
Moreover if the initial guess Λ_{ij}^0 belongs to $L^2(\Gamma_{ij})$ for all $i \neq j$, then $\Lambda_{ij}^k \in L^2(\Gamma_{ij})$ for all iteration steps k as well.

Convergence results and optimal choice of the parameter

For the remainder of this paper we restrict ourselves to the case of two subdomains. But keep in mind that most of the results remain valid in the multidomain case and can also be applied to the discrete stabilized case (cf. [LMO00]). In this setting we have the following convergence result (cf. [Ott99], Th. 3.3; [LMO00], Th. 3.1):

The Robin-Robin algorithm - version 2

1. given $(\Lambda_{ij}^n)_{i\neq j}$ solve for $i = 1, \ldots, N$

$$\begin{cases} \text{Find } u_i^{n+1} \in V_i \text{ with} \\ \quad a_{\Omega_i}(u_i^{n+1}, v_i) \quad +\langle z_i u_i^{n+1}, v_i\rangle_{\Gamma_i} = \\ \qquad\qquad\qquad\quad l_{\Omega_i}(v_i) + \sum_{j\neq i}\langle \Lambda_{ji}^n, v_i\rangle_{\Gamma_{ij}}, \quad \forall v_i \in V_i \end{cases}$$

2. update $\Lambda_{ij}^{n+1} \in W_{ij}^*$ for $i \neq j$ by

$$\langle \Lambda_{ij}^{n+1}, \phi\rangle_{\Gamma_{ij}} = \theta\langle (z_i + z_j)u_i^{n+1}, \phi\rangle_{\Gamma_{ij}} - \theta\langle \Lambda_{ji}^n, \phi\rangle_{\Gamma_{ij}} + (1-\theta)\langle \Lambda_{ij}^n, \phi\rangle_{\Gamma_{ij}}$$

   for all $\phi \in W_{ij}$.

3. until convergence: $n \mapsto n+1$ and goto step 1.

Theorem 2 *Let be $c - \frac{1}{2}\nabla\cdot\mathbf{b} \geq \alpha > 0$. Furthermore, let be $z = z_1 = z_2$ on $\Gamma := \Gamma_{12}$ and $\Lambda^0 := \Lambda_{12}^0 \in L^2(\Gamma)$. Then the sequences $\{u_i^n\}_n$ for $i = 1, 2$ converge according to*

$$\|u_i^n - u_i\|_{V_i} \to 0, \quad n \to \infty,$$

where $u_i := u|_{\Omega_i}$ is the restriction of the global solution $u \in V$ of (2) onto Ω_i.

Remark 1 *Unfortunately the proof contains no indication about the speed of convergence. With help of Fourier analysis it can be shown that, even in a simple case, the convergence speed is not linear (cf. the section about Fourier analysis).*

If we require some further assumptions, it is possible to give an a-posteriori error estimate. The proof is similiar to [OLM01], [LMO00]. The local error is measured in the ϵ-dependent norm

$$|||u|||_i^2 := \epsilon|u|_{1,\Omega_i}^2 + \|\sqrt{c}u\|_{0,\Omega_i}^2, \qquad \forall u \in V_i, \quad i = 1, 2.$$

Theorem 3 *Let $\nabla\cdot\mathbf{b} = 0$, $c \geq 0$ and $\theta = 1$. Let both subdomains be connected with $\partial\Omega$, i.e. $\partial\Omega \cap \partial\Omega_i \neq \emptyset$ for $i = 1, 2$. Defining $\|\cdot\|_{\infty,\Gamma} := \|\cdot\|_{L^\infty(\Gamma)}$, $\|\cdot\|_{2,\Gamma} := \|\cdot\|_{L^2(\Gamma)}$ we get for $i = 1, 2$ and $j = 3 - i$*

$$|||u_i^{k+1} - u_i|||_i \leq C\epsilon^{-\frac{1}{2}}\|z_i - \frac{1}{2}\mathbf{b}\cdot\mathbf{n}_i\|_{\infty,\Gamma}\|u_2^k - u_1^{k+1}\|_{2,\Gamma} + CL_j\|u_2^k - u_1^{k+1}\|_{W_{12}} \quad (4)$$

where

$$L_j = \sqrt{\epsilon} + C_{F,j}\sqrt{C_{\infty,j}} + B_{\infty,j}\min\{\frac{1}{\sqrt{C_{0,j}}}, \frac{C_{F,j}}{\sqrt{\epsilon}}\},$$

$C_{\infty,i} := \|c\|_{\infty,\Omega_i}$, $B_{\infty,i} := \max_{l=1,2}\|b_l\|_{\infty,\Omega_i}$ and $C_{0,i} := \operatorname{ess\,inf}_{x\in\Omega_i} c(x)$. $C_{F,i}$ is the constant of the Friedrich's inequality and C is a constant independent of ϵ, $\mathbf{b}$ and c.

Remark 2 *The estimate (4) bounds the global error by interface terms, which can be computed without any knowledge about the global solution. Therefore we can simply control the convergence within a practical implementation.*

Now we derive a suitable choice of the parameter z_i from the a-posteriori error estimate using a method which was applied in [OLM01] to the Oseen equations. To be consistent with the case $\epsilon = 0$ we propose

$$z_i = \frac{1}{2}|\mathbf{b}\cdot\mathbf{n}_i| + R \quad \text{with} \quad \lim_{\epsilon\to 0} R = 0 \text{ on } \Gamma_i^0 \cup \Gamma_i^+$$

where $\Gamma_i^0 \cup \Gamma_i^+ := \{x \in \Gamma_i \mid (\mathbf{b}\cdot\mathbf{n}_i)(x) \geq 0\}$ is the non-inflow part. Then equilibration of the terms on the right hand side of (4) motivates for $z = z_1 = z_2$ the choice

$$\begin{aligned} z_i &= \frac{1}{2}|\mathbf{b}\cdot\mathbf{n}_i| + \max_{j=1,2}\frac{L_j}{\epsilon^{-\frac{1}{2}}} \\ &= \frac{1}{2}|\mathbf{b}\cdot\mathbf{n}_i| + \sqrt{\epsilon}\max_{j=1,2}\left(\sqrt{\epsilon} + C_{F,j}\sqrt{C_{\infty,j}} + B_{\infty,j}\min\{\frac{1}{\sqrt{C_{0,j}}}, \frac{C_{F,j}}{\sqrt{\epsilon}}\}\right). \quad (5) \end{aligned}$$

Remark 3 *Further suggestions and numerical experiments concerning an appropriate choice of z_i can be found in [LMO00]. Other approaches like the technique of absorbing boundary conditions [NR95] or asymptotic analysis for singularly perturbed problems [Ott99] yield similiar results. In the last section additional variants are derived from Fourier analysis.*

The Robin-Robin preconditioner for the Schur complement equation

The Schur complement equation

It is possible to reduce the original problem (1) on Ω to an interface problem on $\Gamma := \bigcup_{i=1}^N \Gamma_i$. This is a very natural way to transform the global problem into local problems on the subdomains Ω_i (cf. [QV99]).
First we introduce two operators, which extend functions from the interface Γ_i to the subdomain Ω_i. Defining the trace spaces $W := Tr_\Gamma V$ and $W_i := H_{00}^{\frac{1}{2}}(\Gamma_i)$ the $a_{\Omega_i}(\cdot,\cdot)$-extension $Tr_i^{-1}w_i$ for a $w_i \in W_i$ is given by $Tr_i^{-1}w_i = y_i \in V_i$ with

$$a_{\Omega_i}(y_i, v_i) = 0, \quad \forall v_i \in H_0^1(\Omega_i) \quad \text{and} \quad Tr_{\Gamma_i}(y_i) = w_i. \quad (6)$$

Remark 4 *If y_i is regular enough, this implies that y_i satisfies*

$$(Ly_i)(x) = 0 \text{ in } \Omega_i, \quad y_i = w_i \text{ on } \Gamma_i, \quad y_i = 0 \text{ on } \partial\Omega_i \cap \partial\Omega.$$

Analogously the extension operator Tr_i^{-*} for a $w_i \in W_i$ is defined by $Tr_i^{-*}w_i = y_i$ with

$$a_{\Omega_i}(v_i, y_i) = 0 \quad \forall v_i \in H_0^1(\Omega_i), \quad \text{and} \quad Tr_{\Gamma_i}(y_i) = w_i. \quad (7)$$

The extensions are well-posed (cf. [QV99], ch. 5.1):

Lemma 1 *Equations (6) and (7) have unique solutions. They satisfy the a-priori estimates*

$$\|Tr_i^{-1}w_i\|_{1,\Omega_i} \leq C\|w_i\|_{W_i} \quad \textit{and} \quad \|Tr_i^{-*}w_i\|_{1,\Omega_i} \leq C\|w_i\|_{W_i}$$

for all $w_i \in W_i$ and $i = 1, \ldots N$.

Decomposing $u, v \in V$ into $u = Tr^{-1}(Tr_\Gamma u) + \sum_{i=1}^N u_i^0$, $v = Tr^{-*}(Tr_\Gamma v) + \sum_{i=1}^N v_i^0$ with $u_i^0, v_i^0 \in H_0^1(\Omega_i)$ for all $i = 1, \dots N$ and $Tr^{-1} = Tr_i^{-1}$, $Tr^{-*} = Tr_i^{-*}$ on subdomain Ω_i, the Schur complement equation can be derived from (2). The *Schur complement equation* is given by

$$\text{Find } \bar{u} \in W \ : \quad \langle S\bar{u}, \bar{v}\rangle = \sum_{i=1}^N \langle S_i\bar{u}, \bar{v}\rangle = \langle F, \bar{v}\rangle, \quad \forall \bar{v} \in W \tag{8}$$

with

$$\langle S_i\bar{u}, \bar{v}\rangle = a_{\Omega_i}(Tr_{\Gamma_i}^{-1}\bar{u}, Tr_{\Gamma_i}^{-*}\bar{v}) \quad \text{and} \quad \langle F, \bar{v}\rangle = \sum_{i=1}^N l_{\Omega_i}(Tr_{\Gamma_i}^{-*}\bar{v}).$$

It can be proved, that the *Steklov-Poincaré operator* S is continuous and coercive on W (cf. [QV99], ch. 5.1). Thus we have proven the following:

Lemma 2 *There exists a unique solution $\bar{u} \in W$ of (8). Furthermore, if $u \in V$ is a solution of (2) then $u|_\Gamma$ is a solution of (8).*

The Robin-Robin Preconditioner

Here the Schur complement equation (8) is solved by a preconditioned Richardson iteration:

$$\lambda_{k+1} = \lambda_k + \theta T(F - S\lambda_k), \quad k \in \mathbb{N} \tag{9}$$

with an initial guess $\lambda_0 \in W$, a relaxation parameter $\theta > 0$ and a preconditioner T.
From now on we consider again the case of two subdomains. The Robin-Robin preconditioner is thus given by a sum of weighted inverses of local Steklov-Poincaré operators (cf. [QV99], [BS00], [ATNV00]):

$$T = \sigma_1 S_1^{-1} + \sigma_2 S_2^{-1}$$

with $\sigma_1, \sigma_2 \geq 0$. The operator T is continuous and coercive for $\sigma_1 + \sigma_2 > 0$. Thus, by virtue of the Lax-Milgram Lemma, T^{-1} exists and is also continuous and coercive (cf. [QV99], p. 108). Unfortunately up to now linear convergence can only be proven in the diffusion dominated case for two subdomains (cf. [QV99], ch. 5.1).
It is interesting that this method can also be interpreted as an iteration-by-subdomains method (cf. [BS00]). To see this note that (9) can be written equivalently as: given $\lambda_{k-1} \in W$, solve for $i = 1, 2$ the Dirichlet problems and the mixed Dirichlet-Robin problems

$$\left\{\begin{array}{rcll} Lw_i^k &=& f & \text{in } \Omega_i \\ w_i^k &=& 0 & \text{on } \partial\Omega_i \setminus \Gamma \\ w_i^k &=& \lambda^{k-1} & \text{on } \Gamma \end{array}\right. \qquad \left\{\begin{array}{rcll} Ly_i^k &=& 0 & \text{in } \Omega_i \\ y_i^k &=& 0 & \text{on } \partial\Omega_i \setminus \Gamma \\ \Phi_i(y_i^k) &=& \Phi_i(w_i^k) & \\ &+& \Phi_i(w_j^k) & \text{on } \Gamma \end{array}\right. \tag{10}$$

with $\Phi_i(y) := \epsilon\frac{\partial y}{\partial \mathbf{n}_i} - \frac{1}{2}\mathbf{b}\cdot\mathbf{n}_i y$ on Γ. Finally, update the interface function by

$$\lambda^k := \lambda^{k-1} + \theta\left(\sigma_1 y_1^k|_\Gamma + \sigma_2 y_2^k|_\Gamma\right).$$

Fourier Analysis

In this section we consider the special case that the flow $\mathbf{b}$ and the reaction term c are constants. The domain Ω is given by $(0, L) \times (0, 1)$ and is divided into $\Omega_1 = (0, A) \times (0, 1)$ and $\Omega_2 = (A, L) \times (0, 1)$ (cf. figure (a)). Now we carry out a Fourier analysis for both methods.

The Robin-Robin algorithm

Via separation of variables we obtain the following representation of the error $e_i^k := u_i^k - u|_{\Omega_i}$ of the k-th step:

$$\begin{aligned} e_1^k(x,y) &= \exp\left(\frac{\mathbf{b}\cdot(x,y)}{2\epsilon}\right)\sum_{l=1}^{\infty} F_{1,l}^k \sinh(\nu_l x)\sin(l\pi y), \\ e_2^k(x,y) &= \exp\left(\frac{\mathbf{b}\cdot(x,y)}{2\epsilon}\right)\sum_{l=1}^{\infty} F_{2,l}^k \sinh(\nu_l (L-x))\sin(l\pi y) \end{aligned}$$

where $\nu_l^2 := \frac{|\mathbf{b}|^2}{4\epsilon^2} + \frac{c}{\epsilon} + l^2\pi^2$. (cf. GASTALDI ET. AL. [GGQ96]). Inserting the boundary condition on $\Gamma = \Gamma_{12}$ yields in the case of $\theta = 1$ the recursion formulas

$$F_{i,l}^k = K_l^{RR} F_{i,l}^{k-2}$$

for $i = 1, 2$ where

$$K_l^{RR} = \frac{(-z + \epsilon\nu_l \coth(\nu_l A))\,(z - \epsilon\nu_l \coth(\nu_l (L-A)))}{(z + \epsilon\nu_l \coth(\nu_l A))\,(-z - \epsilon\nu_l \coth(\nu_l (L-A)))}.$$

F. NATAF and F. ROGIER [NR95] perform a similiar analysis for the case of infinite strips with Fourier transform techniques. They require exact boundary condition for the first Fourier mode to yield the following choice for the free parameter:

$$z = \frac{1}{2}\sqrt{b_1^2 + 4\epsilon c}. \tag{11}$$

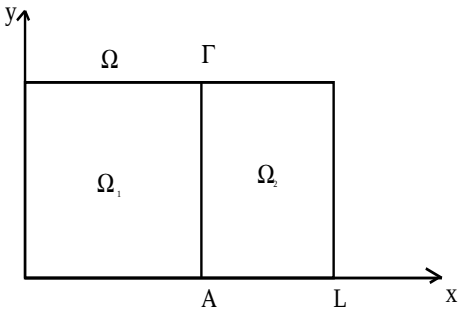
Analogously, assuming exact boundary conditon for the first Fourier mode, we get

$$z = \frac{1}{2}\sqrt{|\mathbf{b}|^2 + 4\epsilon c + 4\pi^2\epsilon^2}, \tag{12}$$

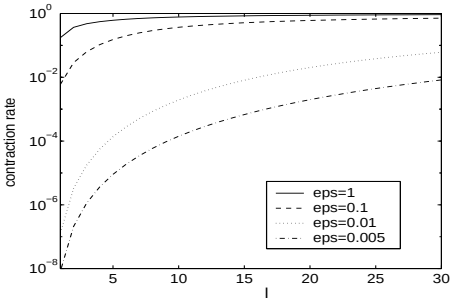
where the term $\coth(\nu_1 A) \approx 1$ is neglected. In [JNR01] the choice (11) is improved by adding additional interface terms in the tangential direction. Then the constants are determined by minimizing the convergence ratio for a certain range of wave numbers. Numerical experiments of the choice (5) resulting from the a-posteriori estimate show, that this choice also damps the lower wave numbers very well (cf. [LMO00]).

With help of the recursion formulas it can be shown directly, that the algorithm converges for this special domain decomposition and positive constant z. The convergence rate, however, is in general not linear.

We illustrate the contraction rates $|K_l^{RR}|$ in figure (b) for different ϵ, the choice (12) and the parameters $L = 1$, $A = 0.1$, $\mathbf{b} = (1, 1)^t$, $c = 1$. We observe that the contraction rates $|K_l^{RR}|$ tend to 1 for $l \to \infty$. Thus higher modes are reduced slower. Further we recognize that the algorithm works well for the case of small ϵ.



(a) The domain Ω



(b) The contraction rate $|K_l^{RR}|$

Robin-Robin preconditioner for the Schur complement equation

Next we examine the preconditioned Richardson iteration of the Schur complement equation. With help of the differential interpretation of the algorithm it is also possible to apply Fourier analysis.
Denoting the error at the k-th step by $\tilde{e}_i^k := w_i^k - u|_{\Omega_i}$, where w_i^k is the solution of the Dirichlet problem in (10), the following representation can be derived in a similiar manner described for the Robin-Robin algorithm:

$$\begin{aligned}\tilde{e}_1^k(x,y) &= \exp\left(\frac{\mathbf{b}\cdot(x,y)}{2\epsilon}\right)\sum_{l=1}^{\infty} C_{1,l}^k \sinh(\nu_l x)\sin(l\pi y)\\ \tilde{e}_2^k(x,y) &= \exp\left(\frac{\mathbf{b}\cdot(x,y)}{2\epsilon}\right)\sum_{l=1}^{\infty} C_{2,l}^k \sinh(\nu_l (L-x))\sin(l\pi y).\end{aligned}$$

Inserting again the boundary conditions on Γ yields

$$C_{2,l}^{k+1} = \frac{\sinh(\nu_l A)}{\sinh(\nu_l (L-A))} C_{1,l}^{k+1} \quad \text{and} \quad C_{1,l}^{k+1} = K_l C_{1,l}^k$$

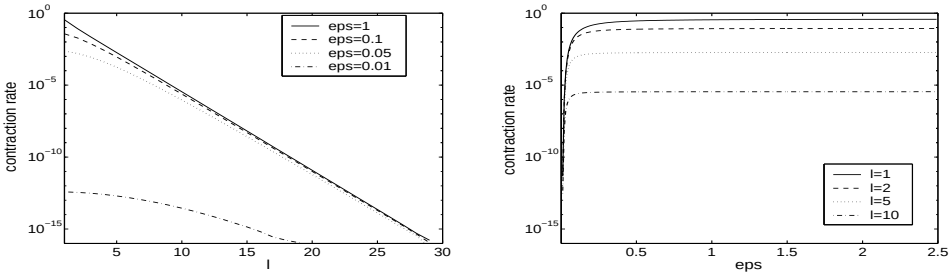
with

$$K_l = 1 - \theta\{\sigma_1 + \sigma_2 + \sigma_1 \coth(\nu_l (L-A))\tanh(\nu_l A) + \sigma_2 \coth(\nu_l A)\tanh(\nu_l (L-A))\}.$$

Thus, again, the convergence behaviour depends on the contraction rates $|K_l|$. In figure (c) and (d) the contraction rates are illustrated for the following choice of the parameters: $L = 1$, $A = 0.1$, $\mathbf{b} = (1,1)^t$, $c = 1$, $\sigma_1 = \sigma_2 = \frac{1}{4}$, $\theta = 1$. In contrast to the Robin-Robin algorithm we can state that the contraction rates $|K_l|$ tend to 0 for $l \to \infty$. This allows us to prove linear convergence in the H^1-norm for this special case. Further we observe that for $\epsilon \to 0$ the contraction rates K_l tend to 0.

Comparison of the two methods

First we consider the convergence behaviour. Numerical experiments indicate that convergence of the Schur complement method is linear, but up to now, it is not proved in the general



(c)+(d) Contraction rates $|K_l|$ for different ϵ resp. l

case. Conversely, it can be shown, that the Robin-Robin method converges, but the convergence is in general not linear. In the case of the Robin-Robin algorithm one need compute only one local problem in each subdomain per iteration step. The other method needs the computation of two local problems per iteration step. Thus the Robin-Robin method is easier to implement. A problem of the Robin-Robin algorithm is an appropriate choice of the free parameter z_i. Numerical experiments have shown that the algorithm is sensitive to the choice of z_i.

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