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33. Flow in complex river networks simulation through a domain decomposition method

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1. Introduction. Lower river basins are characterized by rivers flowing on flood-plains, usually forming interconnecting networks of streams that frequently interact with lagoons directly or indirectly connecting to the stream reaches. The flood plains both in the Pacific and in the Atlantic coasts of México have experienced, during the last few decades, an accelerated economic development and therefore an appreciable population growth, and some flood-related disasters have occurred in this zones recently. In order to avoid this kind of disasters and the consequent loss of human lives and property, the need to build flood defense infrastructure arises, constituted for instance by levees or dikes, and/or to develop real time flood-warning systems. In any case, computational models are needed to adequately simulate the passage of floods through the river networks. These computational models should take into account the fact that when river reaches flow into or from lagoons, their length is modified according to whether the free surface level in the lagoon is rising or lowering. A model of this kind is presented in this paper. Aldama and Aparicio (1994) [1] presented the fundamentals of this model elsewhere. Here, the complete development is addressed and an application to a real case in the lower Grijalva River is presented.

2. Fundamental equations. The equations on which the model is based are the one-dimensional, free surface Saint-Venant equations: [4]

Continuity

$$B\frac{\partial H}{\partial t} + \frac{\partial UA}{\partial x} = q \tag{2.1}$$

Momentum

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + g\frac{\partial H}{\partial x} + gn^2\frac{U\,|U|}{R^{4/3}} = 0 \tag{2.2}$$

where B is the free surface width; H, free surface elevation or level; U, velocity; A, hydraulic area; q, lateral inflow per unit length; g, acceleration of gravity; n, Manning roughness coefficient; R, hydraulic radius and x and t represent distance and time respectively.

In a channel network such as that shown in fig. 1, two types of flooding areas (heretofore called "lagoons") may be formed: those directly connected to one or more channel reaches, which will be called *interconnecting lagoons* and those receiving or delivering water from or to the river, but not having any influence in the water level of any reach, which will be called *lateral lagoons*.

Interconnecting lagoons will be linked to the corresponding channel reaches by means of a mass conservation equation of the form

$$\frac{\partial V}{\partial t} + \oiint_{sc} \overline{U} \bullet \overline{dA} = 0 \tag{2.3}$$

where V is the lagoon volume, sc is the control surface defined by the lagoon boundaries and the scalar product $\overline{U} \bullet \overline{dA}$ represents the outflow discharge from the lagoon (see fig. 2; note that inflow to the lagoon is negative).

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On the other hand, lateral lagoons are connected to the channel reaches by means of the riverbanks. The unit discharge between river reaches and lateral lagoons will be assumed to be governed by a long-crested weir law:

$$q = C_q h \sqrt{|h|} \tag{2.4}$$

where C_q is a discharge coefficient and h is the net head. C_q is assumed to be a function of the parameter [3]

$$\phi = \frac{|\eta_2 - \eta_1|}{\eta - E} \tag{2.5}$$

where η_2 and η_1 are, respectively, the water surface elevation in the river reach and in the lagoon and E is the elevation of the river bank (see fig. 3). $\eta = \eta_1$ when flow is from lagoon to river and $\eta = \eta_2$ when the river flows into the lagoon. The discharge coefficient is then computed as [3]

$$C_q = \begin{cases} 0.871\sqrt{2g}\phi^{0.478} & for \quad 0 < \phi < 0.1 \\ 0.446\sqrt{2g} & for \quad \phi = 1.0 \\ 0.446\sqrt{2g}\phi^{0.155} & for \quad 0.1 < \phi < 1.0 \end{cases} \tag{2.6}$$

Due to the fact that flow in these conditions occurs in extremely flat terrain, only storage effects are taken into account and no dynamical effects will be considered neither in the interconnecting nor in the lateral lagoons.

3. Transformed equations. River reaches in flat floodplains are frequently confined between lagoons that change in size as floods progress, therefore changing the reach length, which requires solving eqs. (2.1) and (2.2) in variable domains. To avoid the sometimes severe inaccuracies arising from the use of fixed grids in these cases, and following Austria & Aldama [2] and Aldama & Aparicio [1], a coordinate transformation strategy of the following form is employed:

$$\xi = \frac{x - x_r(t)}{x_f(t) - x_r(t)} \tag{3.1}$$

$$\tau = t \tag{3.2}$$

where $x_r(t)$and $x_f(t)$are, respectively, the position of the rear and front of the size-changing river reach and ξ and τ are the transformed coordinates. Applying the coordinate transformation to eqs. (2.1) and (2.2), the following transformed equations are obtained: [1]

$$B(x_f - x_r)\frac{\partial H}{\partial \tau} - B\xi \frac{\partial H}{\partial \xi}\frac{dx_f}{dt} - B(\xi + 1)\frac{\partial H}{\partial \xi}\frac{dx_r}{dt} + \frac{\partial (UA)}{\partial \xi} = q(x_f - x_r) \tag{3.3}$$

$$(x_f - x_r)\frac{\partial U}{\partial \tau} - \xi \frac{\partial U}{\partial \xi}\frac{dx_f}{dt} - (\xi + 1)\frac{\partial U}{\partial \xi}\frac{dx_r}{dt} + U\frac{\partial U}{\partial \xi} + g\frac{\partial H}{\partial \xi} + g(x_f - x_r)n^2 \frac{U\,|U|}{R^{4/3}} = 0 \tag{3.4}$$

4. Domain decomposition and numerical solution. Eqs. (3.3) and (3.4) are solved using an implicit, fractional step scheme [5], which leads to a system of algebraic linear equations of the form [1]

$$[A]^k \{H\}^{k+1} = \{C\}^k \tag{4.1}$$

where $[A]^k$ and $\{C\}^k$ are respectively a matrix and a vector which depend on known values at time level k. Matrix $[A]$ is tridiagonal for a single channel, which makes the solution of eq. (3.1) very efficient, while preserving a second order accuracy. However, in a complex channel network such as that shown in fig. 1, due to interactions between the different reaches, nonzero elements appear outside the three main diagonals, thus making the solution far less efficient. Therefore, Aldama & Aparicio [1] proposed a solution algorithm based on the use of numerical Green's functions consisting of writing eq. (4.1) as

$$[A_R]^k \{H_R\}^{k+1} + B^k_{R,r} \{H_{R,r}\}^{k+1} + B^k_{R,f} \{H_{R,f}\}^{k+1} = \{C_R\}^k \tag{4.2}$$

where $[A]^k$ is a tridiagonal coefficients matrix, $\{H_R\}^{k+1}$ is the unknown water surface elevations vector within the reach, $B^k_{R,r}$ and $B^k_{R,f}$ are scalars and $\{H_{R,r}\}^{k+1} \equiv \left\{H^{k+1}_{R,r}, 0, ..., 0\right\}^T$ and $\{H_{R,f}\}^{k+1} \equiv \left\{0, ..., 0, H^{k+1}_{R,f}\right\}^T$, $H^{k+1}_{R,r}$ and $H^{k+1}_{R,f}$ representing the water surface elevations at the rear and front ends of the reach. The vector of unknowns is decomposed as the sum of a *homogeneous* and an *inhomogeneous* solutions:

$$\{H_R\}^{k+1} = \{H_{R,h}\}^{k+1} + \{H_{R,i}\}^{k+1} \tag{4.3}$$

where the homogeneous solution is defined by

$$[A_R]^k \{H_{R,h}\}^{k+1} = \{C_R\}^k \tag{4.4}$$

and the inhomogeneous solution is given by

$$\{H_{R,i}\}^{k+1} = H^{k+1}_{R,r} \{G_{R,r}\}^{k+1} + H^{k+1}_{R,f} \{G_{R,f}\}^{k+1} \tag{4.5}$$

where $\{G_{R,r}\}^{k+1}$ and $\{G_{R,f}\}^{k+1}$ are *rear* and *front numerical Green's functions*, representing the response of channel reach R to unit variations in the water surface elevations at its rear and front ends and defined respectively by

$$[A_R]^k \{G_{R,r}\}^{k+1} = -B^k_{R,r} \{1, 0, ..., 0\}^T \tag{4.6}$$

and

$$[A_R]^k \{G_{R,f}\}^{k+1} = -B^k_{R,f} \{0, ..., 0, 1\}^T \tag{4.7}$$

On the other hand, the mass conservation equation for interconnecting lagoons (eq. 3) is discretized as

$$A_L \frac{H^{k+1}_c - H^k_c}{\Delta\tau} - \sum_i A_i U_i = 0 \tag{4.8}$$

where A_L is the lagoon surface area, H_c is the free surface elevation at the lagoon and A_i are the hydraulic areas of the river reaches concurring to the lagoon. In the case of lateral lagoons, free surface elevations and therefore stored volumes are computed simply from

$$\frac{A_L}{\Delta\tau} \left(H^{k+1}_c - H^k_c\right) = \sum Q_i \tag{4.9}$$

where total discharge $Q_i = qL_i$, L_i being the *crest* length on the river bank. Note that several river reaches can be connected to the same lateral lagoon. For the sake of simplicity, discharges Q_i are computed explicitly from the previous time step.

In this way, eqs. (4.4), (4.6) and (4.7) are tridiagonal systems and eqs. (4.3) and (4.5), along with the mass conservation equation (4.8) for each interconnecting node, lead to a sparse but relatively small system of equations in terms of the water surface elevation at the node.

Therefore, with the above outlined procedure, three small tridiagonal systems for each channel and a small sparse system for the interconnection nodes are solved, which makes the overall solution considerably more efficient than the large, nonbanded system which would otherwise arise.

5. Boundary conditions. Every channel reach within the network is connected to a node with any of the several possible boundary conditions. The most common boundary conditions are known upstream or downstream discharge, specified upstream or downstream water level or interconnecting lagoon. In the first case, the known discharge is susbstituted into equation (3.3) an a two-term equation is obtained. In the second case, the known level is directly substituted into equation (4.1) and in the third case, equation (4.8) is used to couple the lagoon level with the corresponding channel reach level, either at the reach rear or front.

6. Application. The numerical model described above was integrated in a computational system called *Trans-R* and was applied to the uncontrolled part of the lower Grijalva river basin in the Southeastern region of México (see fig. 1). This river flows from a mountainous zone into a considerably flat region and finally into the Gulf of México. Four major tributaries can be identified as part of the river network: the La Sierra, Pichucalco, Teapa and Puyacatengo rivers. A main concern in this case is the city of Villahermosa, located downstream of the confluence of these major tributaries as shown in fig. 1. With a fast-growing population of about 400,000 inhabitants, the city and its surroundings are subject to flooding caused by the intense precipitations frequently produced by cyclones. A high population growth index produces a severe urban pressure on the Grijalva River and its naturally flooding lagoons, therefore requiring a real-time forecasting system and quantitative aids for the urban growth planning process. For the analysis, about 200 topographic maps of the zone were used, and some *ad hoc* topographic surveys were performed, from which channel sections for the whole network were obtained, and 21 lagoons were identified, including two interconnecting lagoons (lagoons 3 and 8) and 19 lateral lagoons (see fig. 1).

Data for boundary conditions were provided by four gauging stations in each of the major tributaries located at the boundary between the mountains and the floodplains and one at the downstream end of the considered region, called *Gaviotas*, where Villahermosa city is located.

Several flooding events were considered. Due to lack of space, only three of them will be shown here. Figures 4 to 6 show the May 1970 flood; in fig. 4 the measured hydrographs at each of the five gauging stations are shown. Only the stage-discharge relationship at the Gaviotas Station was used for the simulation as boundary condition and the measured hydrograph was reserved for comparison purposes. In figures 5 and 6 some of the results are shown. Figure 5 shows a comparison between measured and computed hydrographs at the Gaviotas gauging station. A reasonable agreement is observed. In fig. 6 a sequence of the flood progress in plan view is shown. It can be seen that two interconnecting lagoons are flooded in the first place (fig. 6 b); in fig. 6 c, one interconnecting lagoon is totally flooded and one lateral lagoon is affected. In fig. 6 d, the flood hydrograph has started to recede, one lagoon has disappeared and another has started to do so. In fig. 6 e, this interconnecting lagoon has completely disappeared. Figures 7 and 8 show recorded and simulated limnigraphs for the May-August, 1967 and September-October, 1999 flows at the Gaviotas Station. The latter event produced extensive flooding and damages in Villahermosa City and vicinity. Good agreement is observed.

7. Conclusions. A numerical model for transient flow simulation in complex river networks with interconnecting and lateral flooding lagoons has been developed. The model uses a coordinate transformation, which allows the channel-interconnecting lagoon interaction simulation and numerical Green's functions to decompose the domain and efficiently solve the problem in the whole river reaches-lagoons hydraulic system. Lateral lagoons connected to the river through the riverbanks are also taken into account in the model. Application to the lower Grijalva River network shows good agreement between computed and measured hydrographs and limnigraphs at the basin outlet.

Fig. 1. Lower Grijalva river network

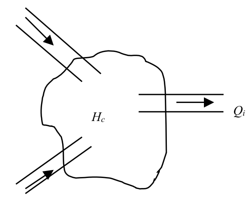

Fig. 2. Interconnecting lagoon

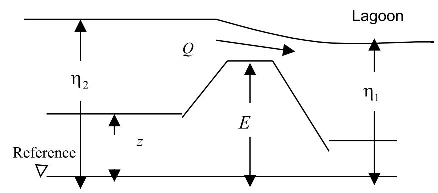

Fig. 3. Lateral lagoon

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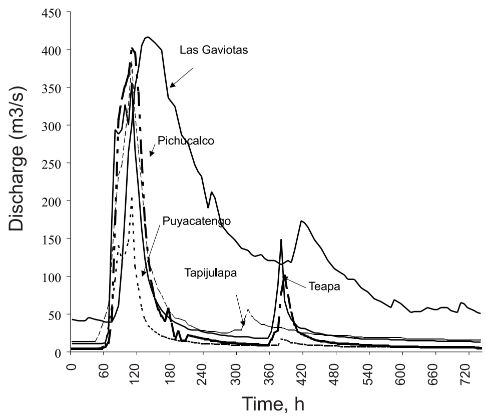

Fig. 4. Recorded Hydrographs, May, 1970

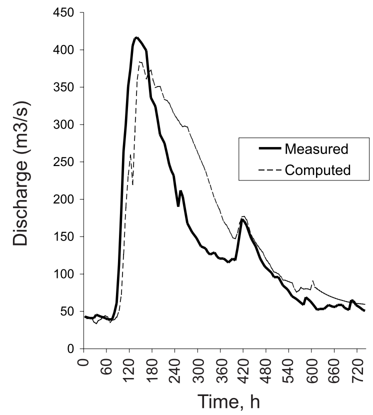

Fig. 5. Measured and computed hydrographs at the Gaviotas gauging station, May, 1970

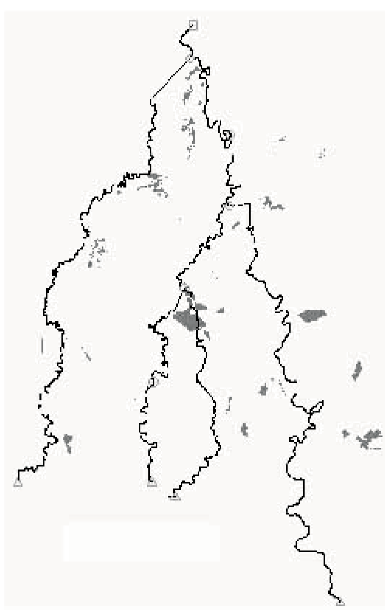

Fig. 6a. Plan view, initial conditions

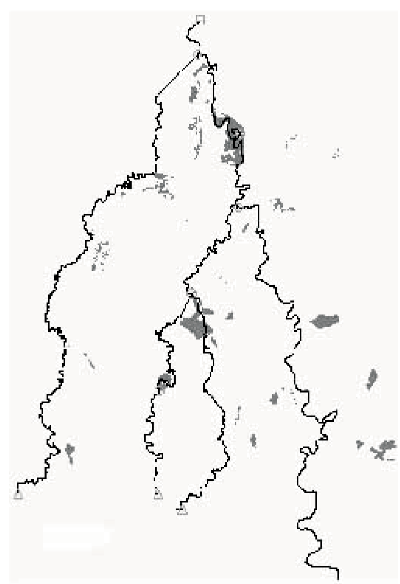

Fig. 6b. Plan view, time = 126 h

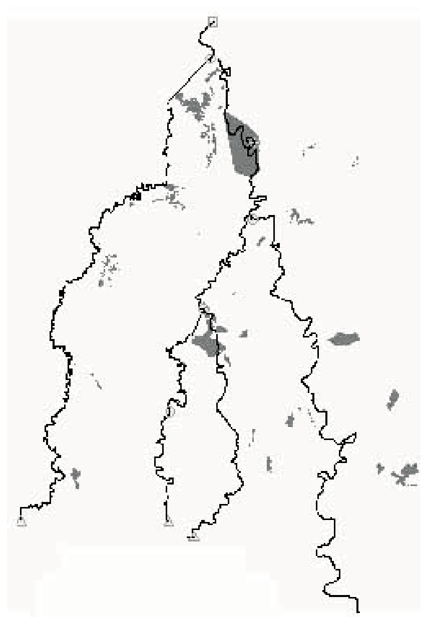

Fig. 6c. Plan view, time = 204 h

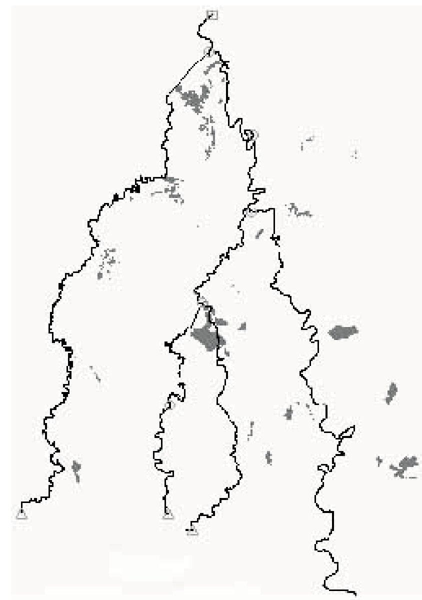

Fig. 6d. Plan view, time = 738 h

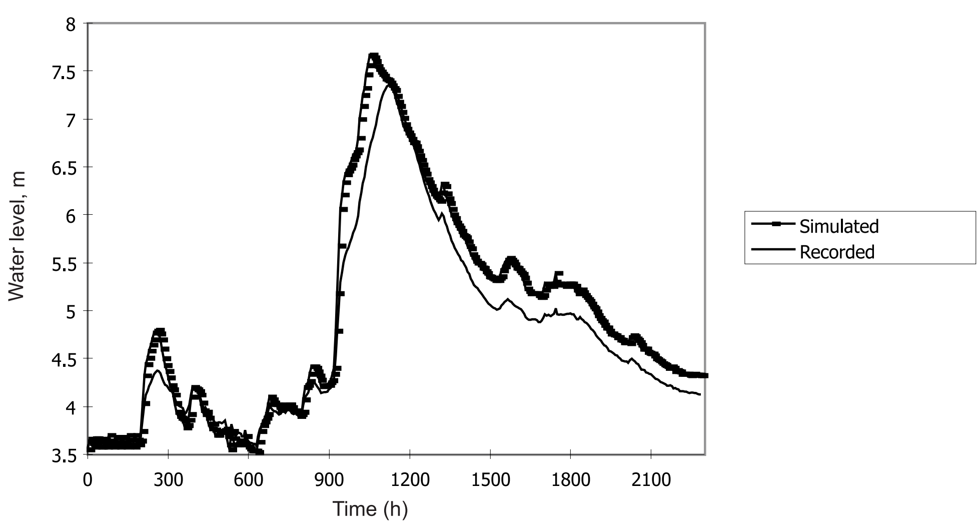

Fig. 7. Limnigraphs, May-August, 1967

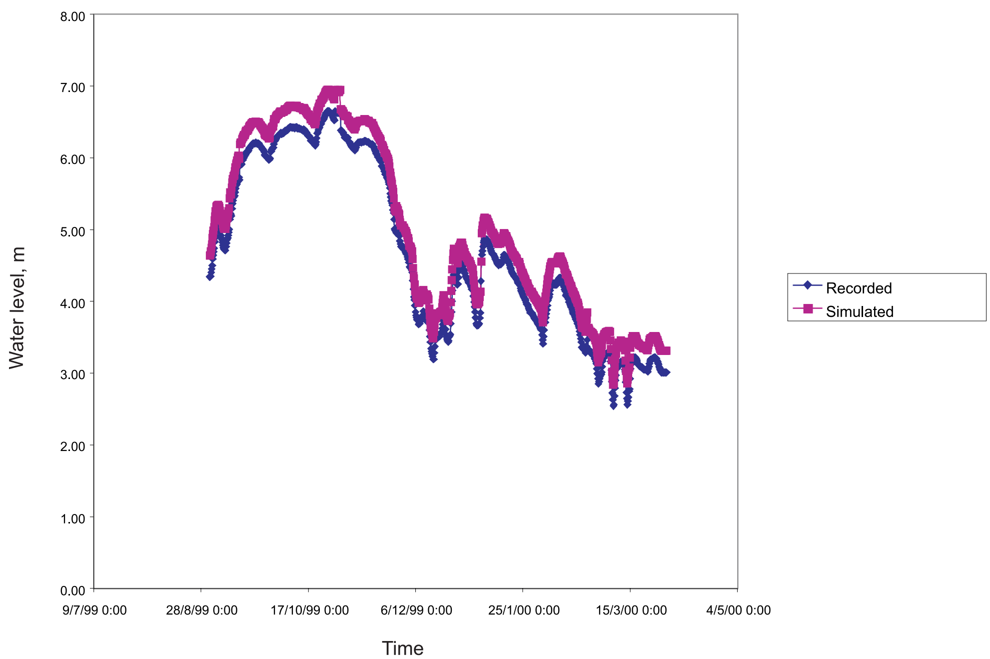

Fig. 8. Limnigraphs, September-October, 1999