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47. Boundary Point Method in the Dynamic and Static Problems of Mathematical Physics

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1. Introduction. Boundary element method (BEM) is widely used for the numerical solution of integral equations of mathematical physics [1]. For the use of the BEM, the surface of the region should be divided into a finite number of subareas and the unknown functions are approximated by standard (as a rule polynomial) functions in every subarea. After applying the method of moments or the collocation method, the problem is reduced to the solution of a finite system of linear algebraic equations. The components of the matrix of this system are integrals over the subareas (boundary elements) of the surface. In many cases these integrals are singular and the complexity of their calculations depends on the type of approximating functions. In a standard BEM, a great portion of the computer time is spent in calculating these integrals. A non trivial auxiliary problem is dividing an arbitrary surface into a set of boundary elements.

In this work a new numerical method is used for the solution of boundary integral equations of some static and dynamic problems of mathematical physics. In this method actual distributions of unknown functions on the surface of the region is approximated by Gaussian functions located on the planes tangent to the boundary surface at some homogeneous set of surface nodes. The idea to use these functions for the solution of a wide class of integral equations of mathematical physics belongs to V. Maz'ya. The theory of approximation by Gaussian functions was developed in the works of V. Maz'ya [5] and V. Maz'ya and G. Shmidt [6].

In the method developed bellow we will use the following result of the mentioned authors. Let $u(x)$ be a scalar function in d-dimensional space $\mathbf{R}^d$. If $u(x)$ and its first derivative are bounded, $u(x)$ may be approximated by the following series

$$u(x) \approx u_h(x) = \sum_{m \in \mathbf{Z}^d} u_m \varphi(x - h\mathbf{m}), \quad \varphi(x) = \frac{1}{(\pi D)^{d/2}} \exp\left(-\frac{|x|^2}{Dh^2}\right). \tag{1.1}$$

Here $\mathbf{m} \in \mathbf{Z}^d$ is a d-dimensional vector with integer components, $h\mathbf{m}$ are the coordinates of the nodes of this approximation and h is the distance between the neighboring nodes, $u_m = u(h\mathbf{m})$ is the value of the function $u(x)$ at node $x = h\mathbf{m}$, D is a non-dimensional parameter. It is demonstrated in [2,3] that the following estimation holds

$$|u(x) - u_h(x)| \le ch \, \|\nabla u\| + |u(x)| R(D), \quad R(D) = O(\exp(-\pi^2 D)). \tag{1.2}$$

Here $||\nabla u||$ is the norm in the space of continuous functions, $c = O(1)$. If h is sufficiently small the error of the approximation (1.1) may be made negligible by the appropriate choosing of the parameter D ($D = O(1)$). The properties of this approximation were studied in detail in [5, 6].

The use of these functions for the solution of the integral equations of mathematical physics has two main advantages. First, the action of the integral operators of the problems on these functions in many cases is a combination of few standard functions. The latter may be simply tabulated, kept in the computer memory and then used for the solution of any similar problem for regions of arbitrary geometry. As a result, the time for the calculation of the matrix of the linear system obtained after the discretization of the problem, is essentially reduced in comparison with a standard BEM. It is also important that only the coordinates

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of the surface nodes and the surface orientations at the nodes are necessary for the surface description in the present method. The method was called by V. Maz'ya the Boundary Point Method (BPM) and in the latter the boundary points (nodes) play the role of boundary elements of the conventional BEM. Note that the problem of covering an arbitrary smooth surface by a homogeneous set of nodes is simpler than the detailed description of the geometry of all the boundary elements that is necessary for the application of any traditional BEM to the solution of surface integral equations.

Here we develope the method for the solution of 2D problems of elasticity and for 3D electromagnetic wave diffraction problems. The numerical results are compared with exact solutions existent in the literature.

2. Integral equations of the second boundary value problem of elasticity. Let an elastic body occupy region V in 3D or 2D-space with closed boundary S. The material of the body is homogeneous with elastic moduli tensor $\mathbf{C}$ (C_{ijkl}). The stress tensor in the body can be presented in the form

$$\sigma_{ij}(x) = \int_S S_{ijkl}(x - x')n_k(x')b_l(x')dS'. \tag{2.1}$$

Here $x(x_1, x_2, x_3)$ is a point of the medium with Cartesian coordinates x_1, x_2, x_3, summation with respect to repeated indexes is implied. The kernel of integral operators in Eq.(2.1) has form

$$S_{ijpq}(x) = -C_{ijkl} \bigtriangledown_k \bigtriangledown_m G_{ls}(x) C_{mspq} - C_{ijpq}\delta(x), \tag{2.2}$$

where $G_{ls}(x)$ is the Green function of the infinite medium with elastic moduli $\mathbf{C}$. Tensor $\sigma(x)$ in Eq.(2.1) satisfies the system of equations of continuum mechanics: $\nabla \cdot \sigma(x) = 0$, $\varepsilon^{(e)}(x) = \mathbf{C}^{-1} \cdot \sigma(x)$, Rot $\varepsilon^{(e)}(x) = \mathbf{0}$, $\bigtriangledown = \mathbf{e}_i \partial/\partial x_i$ is the vector gradient, $\mathbf{e}_i$ $(i = 1, 2, 3)$ are unit vectors of the axes x_i. A dot $(\cdot)$ is the scalar product, $\delta(x)$ is Dirac's delta-function. Thus, tensor $\sigma(x)$ in Eq.(2.1) gives us the solution of the second boundary value problem of elasticity if it satisfies the boundary conditions at the surface of the body

$$\sigma(x) \cdot \mathbf{n}(x)|_S = \mathbf{f}(x), \tag{2.3}$$

where $\mathbf{f}(x)$ is the vector of surface forces.

The integral equation for vector $\mathbf{b}(x)$ in Eq.(2.1) follows from the boundary condition (2.3) and takes the form

$$\int_S T_{ij}(x, x') \cdot b_j(x')dS' = f_i(x), \tag{2.4}$$

$$T_{ij}(x, x') = n_k(x) S_{kijl}(x - x') n_l(x).$$

The kernel of the integral operators in Eq.(2.4) has a high singularity and should be understood in the sense of some regularizations.

3. Numerical solution in 2D-case. Let us consider the plane problem of elasticity for homogeneous and isotropic elastic body. The body occupies a closed region Ω in 2D-space with the bourder Γ. The solution of this problem may be found in the form similar to Eq.(2.1)

$$\sigma_{ij}(x) = \int_\Gamma S_{ijkl}(x - x')n_k(x')b_l(x')\delta(\Gamma')dx'. \tag{3.1}$$

Here $\delta(\Gamma)$ is delta-function concentrated on the contour Γ, integration in this formula is spread over 2D-space. For the numerical solution of Eq.(2.3) let us chose a set of nodes $x^{(i)}$ on boundary Γ of the body with equal distances h between neigboring nodes.

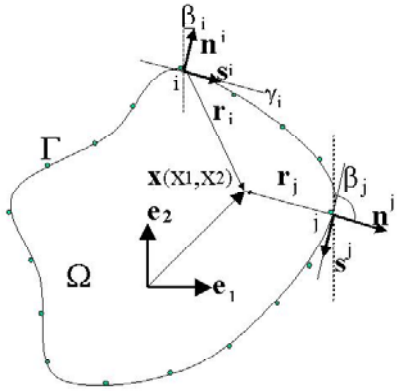

Figure 3.1: The local and global coordinate systems

Then let us change the potential (3.1) concentrated on Γ for the sum of potentials concentrated on the tangent lines γ^i at every node i (see Fig.(3.1)). Thus, density $n_k(x)b_l(x)\delta(\Gamma)$ of the potential in Eq.(3.1) is approximated by the equation

$$n_k(x)b_l(x)\delta(\Gamma) = \sum_i n_k^{(i)} b_l^{(i)} \varphi^i(x)\gamma^i(x), \quad \varphi^i(x) = \frac{1}{\sqrt{\pi D}} \exp\left(-\frac{|x-x^{(i)}|^2}{Dh^2}\right). \tag{3.2}$$

Here $\mathbf{n}^{(i)}$ is the external normal vector to Γ at the node $x^{(i)}$, $\gamma^i(x)$ is delta function concentrated in the tangent line γ^i to Γ in the node $x^{(i)}$, h is the distance between neighboring nodes, $D = 2$. The vectors $\mathbf{b}^{(i)}$ in every node should be found from the solution of the problem and are the main unknowns of the method. If we substitute Eq.(3.2) into Eq.(3.1) the latter is converted into the sum of potentials concentrated in the tangent lines γ^i

$$\begin{aligned} \sigma_{ij}(x) &= \int_\Gamma S_{ijkl}(x-x')n_k(x')b_l(x')\delta(\Gamma')dx' \approx \sum_i I_{ij}^{(i)}(x)\cdot b_j^{(i)}, \\ I_{ijl}^{(i)}(x) &= \int_{\gamma_i} S_{ijkl}(x-x')n_k(x')\varphi^s(x')\gamma^i(x')dx'. \end{aligned} \tag{3.3}$$

Let us introduce the local coordinate systems (s, z) connected with the nodes; $(\mathbf{s}^{(i)}, \mathbf{n}^{(i)})$ are the unit vectors of axis s directed along tangent line γ^i and of axis z directed along normal to Γ at the node $x^{(i)}$ (Fig.(3.1)). In this basis vector $\mathbf{b}^{(i)}$ in Eq.(3.1) has the form

$$\mathbf{b}^{(i)} = b_s^{(i)}\mathbf{s}^{(i)} + b_n^{(i)}\mathbf{n}^{(i)}. \tag{3.4}$$

After substituting Eq.(3.2) into Eq.(3.3) and calculating the integrals we go to the fol-

lowing expression of the tensor $\mathbf{I}^{(i)}$ in the local basis of the i-th node (see [4] for details)

$$\begin{aligned}
\mathbf{I}^{(i)}(s,z) &= -4\mu_0\kappa_0\left[J_{11}(s,z)\mathbf{t}_{11}^{(i)}+J_{12}(s,z)\mathbf{t}_{12}^{(i)}+J_{21}(s,z)\mathbf{t}_{21}^{(i)}+J_{22}(s,z)\mathbf{t}_{22}^{(i)}\right],\\
J_{11}(s,z) &= \frac{h}{4p^2}\,\mathrm{sign}(\zeta)\left[2\mathrm{sign}(\eta)j_1(|\zeta|,|\eta|)-\eta j_2(|\zeta|,|\eta|)\right],\\
J_{12}(s,z) &= \frac{h}{4p^2}\left[j_3(|\zeta|,|\eta|)-|\eta|j_4(|\zeta|,|\eta|)\right],\\
J_{21}(s,z) &= \frac{h}{4p^2}\eta j_2(|\zeta|,|\eta|),\quad J_{22}(x,y)=\frac{h}{4p^2}\left[j_3(|\zeta|,|\eta|)+|\eta|j_4(|\zeta|,|\eta|)\right],\\
\mathbf{t}_{11}^{(i)} &= \mathbf{s}^{(i)}\otimes\mathbf{s}^{(i)}\otimes\mathbf{s}^{(i)},\quad \mathbf{t}_{22}^{(i)}=\mathbf{n}^{(i)}\otimes\mathbf{n}^{(i)}\otimes\mathbf{n}^{(i)}\\
\mathbf{t}_{12}^{(i)} &= \mathbf{s}^{(i)}\otimes\mathbf{s}^{(i)}\otimes\mathbf{n}^{(i)}+\mathbf{s}^{(i)}\otimes\mathbf{n}^{(i)}\otimes\mathbf{s}^{(i)}+\mathbf{n}^{(i)}\otimes\mathbf{s}^{(i)}\otimes\mathbf{s}^{(i)},\\
\mathbf{t}_{21}^{(i)} &= \mathbf{s}^{(i)}\otimes\mathbf{n}^{(i)}\otimes\mathbf{n}^{(i)}+\mathbf{n}^{(i)}\otimes\mathbf{s}^{(i)}\otimes\mathbf{n}^{(i)}+\mathbf{n}^{(i)}\otimes\mathbf{n}^{(i)}\otimes\mathbf{s}^{(\mathbf{i})}.
\end{aligned}$$

Here $p^2=(h^2D)/4$, $\zeta=s/p$, $\eta=z/p$, $\kappa_0=(\lambda_0+\mu_0)/((\lambda_0+2\mu_0),\lambda_0,\mu_0$ are Lamé parameters of the material.

The four functions $j_i(\zeta,\eta)$ in these equations are connected with function Erf(ξ) (the error function) by the equations

$$j_1(\zeta,\eta)+ij_3(\zeta,\eta)=iF_1\left(\frac{\eta+i\zeta}{2}\right),\qquad j_2(\zeta,\eta)+ij_4(\zeta,\eta)=iF_2\left(\frac{\eta+i\zeta}{2}\right),\tag{3.5}$$

$$\begin{aligned}
F_1(z) &= \frac{1}{2\pi}\left[1-\sqrt{\pi}z\exp(z^2)\,(1-\ \mathrm{Erf}(z))\right],\\
F_2(z) &= \frac{1}{2\pi}\left[-z+\frac{\sqrt{\pi}}{2}(1+2z^2)\exp(z^2)\,(1-\ \mathrm{Erf}(z))\right].
\end{aligned}\tag{3.6}$$

The system of linear algebraic equations for the components $(b_s^{(i)},b_n^{(i)})$ of the vectors $\mathbf{b}^{(i)}$ in the local bases can be obtained from the boundary conditions (2.3) that will be satisfied in all the nodes (the collocation method). Let us introduce vector $\mathbf{X}$ of the unknowns that is connected with the components $b_s^{(m)},b_n^{(m)}$ by the relations

$$\begin{aligned}
\mathbf{X} &= \|X_j\|,\qquad j=1,2,3,...,2M,\\
X_{2m-1} &= b_s^{(m)},\qquad X_{2m}=b_n^{(m)},\qquad m=1,2,3,...,M
\end{aligned}\tag{3.7}$$

Here M is the total number of nodes. The vector-column $\mathbf{F}$ defines the forces that act in the nodes

$$\begin{aligned}
\mathbf{F} &= \|F_j\|,\qquad j=1,2,3,...,2M,\\
F_{2m-1} &= -\frac{f_s^{(m)}}{4\mu_0\kappa_0},\qquad F_{2m}=-\frac{f_n^{(m)}}{4\mu_0\kappa_0},\qquad m=1,2,3,...,M.
\end{aligned}\tag{3.8}$$

Here $f_s^{(m)}$, $f_n^{(m)}$ are the values of the forces applied at the nodes that are known from the boundary conditions.

The equation for the vector $\mathbf{X}$ follows from the boundary conditions at the nodes and takes the form

$$\sum_{j=1}^{2N}B_{ij}X_j=F_i,\qquad j=1,2,3,...,2M.\tag{3.9}$$

Here the components of the matrix $\mathbf{B}=\|B_{ij}\|$ are defined in [3] and expressed via the standard functions $j_1,...,j_4$ defined in Eqs.(3.5, 3.6). The computer time of calculation of these functions is very small.

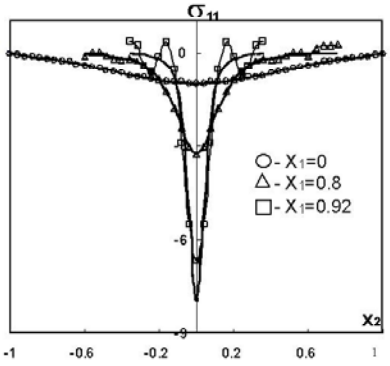

Figure 3.2: Distribution of stresses in a disk subjected with two concentrated forces.

Let us consider a numerical example. An elastic disk of unit radius, $R = 1$, subjected by two concentrated forces applied along its diameter. The distributions of normal stress $\sigma(x_1, x_2)$ in various intersections orthogonal to the direction of the force application are presented in Fig.3.2. Solid lines are exact solutions; dashed lines correspond to 60 nodes homogeneously distributed along the boundary. It is seen that the error of the numerical solution is essential only in a small vicinity of the points of application of the forces.

4. The integral equation of the problem of electromagnetic wave diffraction on a perfectly conducting screen. Let a monochromatic electromagnetic wave of frequency ω propagate through a homogeneous and isotropic medium, and Ω be a smooth, perfectly conducting surface embedded in this medium. The electric field $\mathbf{E}(x)$ in the medium with such a surface may be presented in the form

$$\begin{aligned} \mathbf{E}(x) &= \mathbf{E}^0(x) + \mathbf{E}^s(x), \quad \mathbf{E}^s(x) = -i\frac{4\pi c}{k_0}\int_\Omega \mathbf{K}(x-x')\cdot\mathbf{J}(x')d\Omega', \\ \mathbf{K}(x) &= \bigtriangledown\otimes\bigtriangledown g(x) + k_0^2 g(x)\mathbf{1}. \end{aligned} \tag{4.1}$$

Here $\mathbf{1}$ is the second-rank unit tensor, c is the wave velocity, $k_0 = \omega/c$. $\mathbf{E}^0(x)$ is an incident field that is assumed to be a plane monochromatic wave: $\mathbf{E}^0(x) = \mathbf{e}\exp(-i\mathbf{k}_0\cdot x)$, $\mathbf{k}_0 = k_0\mathbf{m}$, $|\mathbf{m}| = 1$, $\mathbf{k}_0$ is the wave vector, and $\mathbf{e}$ is the polarization vector of this wave. The kernel $\mathbf{K}(x)$ of the integral operator in Eq.(4.1) is the second derivative of Green function $g(x)$ of Helmholtz's operator, and in the 3D-case $g(x)$ takes the form $g(x) = (e^{-ik_0 r})/(4\pi r)$, $r = |x|$.

The density $\mathbf{J}(x)$ of the potential in Eq.(4.1) is the surface current generated on Ω by incident field $\mathbf{E}^0(x)$. Vector $\mathbf{J}(x)$ belongs to Ω and satisfies the integral equation

$$i\frac{4\pi c}{k_0}\int_\Omega \mathbf{U}(x,x')\cdot\mathbf{J}(x')d\Omega' = \theta(x)\cdot\mathbf{E}^0(x), \qquad \theta(x) = \mathbf{1} - \mathbf{n}(x)\otimes\mathbf{n}(x), \qquad x\in\Omega. \tag{4.2}$$

$$\mathbf{U}(x,x') = \theta(x)\cdot\mathbf{K}(x-x')\cdot\theta(x').$$

The integral on the left hand side of Eq.(4.2) has a strong singularity and should be understood in terms of some regularization (see [3]).

5. Numerical solution of the diffraction problems. Integral equation (4.2) may be presented in the form

$$i\frac{4\pi c}{k_0}\int \mathbf{U}(x,x')\cdot\mathbf{J}(x')\Omega(x')dx' = \theta(x)\cdot\mathbf{E}^0(x), \qquad x\in\Omega, \tag{5.1}$$

where $\Omega(x)$ is delta-function concentrated on the surface Ω and integration in this equation is spread over 3D-space. Let us cover the scattering surface by a set of nodes $x^{(i)}$ $(i = 1, 2, ..., M)$ with approximately the same distances between neighboring nodes, and ω_i be the tangent plane to Ω at the i-th node. For the application of the BPM the actual current distribution on Ω is changed for the following sum

$$\mathbf{J}(x)\Omega(x) \approx \sum_i \mathbf{J}^{(i)}\varphi_i(x)\omega_i(x), \quad \varphi_i(x) = \frac{1}{\pi D}\exp\left(-\frac{|x-x^{(i)}|}{Dh^2}\right). \tag{5.2}$$

Here $\omega_i(x)$ is delta-function concentrated in the plane ω_i and $\mathbf{J}^{(i)}$ is the vector of this plane. The approximation of the scattered field $\mathbf{E}^s(x)$ in Eq.(4.1) takes the form

$$\mathbf{E}^s(x) = -i\frac{4\pi c}{k_0}\int \mathbf{K}(x-x')\cdot\mathbf{J}(x')\Omega(x')dx' \approx -i4\pi c\sum_i \mathbf{I}^{(i)}(x)\cdot\mathbf{J}^{(i)}, \tag{5.3}$$

$$\mathbf{I}^{(i)}(x) = \frac{1}{k_0}\int \mathbf{K}(x-x')\varphi_i(x')\omega_i(x')dx'. \tag{5.4}$$

Let us introduce a local Cartesian basis $\left(\mathbf{e}_1^{(i)}, \mathbf{e}_2^{(i)}, \mathbf{e}_3^{(i)}\right)$ with the origin at the i-th node (the unit vector $\mathbf{e}_3^{(i)}$ coincides with the normal $\mathbf{n}^{(i)}$ to ω_i at point $x^{(i)}$). In this local coordinate system the scalar product $\mathbf{I}^{(i)}(x)\cdot\mathbf{J}^{(i)}$ in Eq.(5.3) in the local basis of the i-th node takes the form

$$\mathbf{I}^{(i)}(x)\cdot\mathbf{J}^{(i)} = \frac{4}{D\kappa_0}\left[F_1(\kappa_0,\eta,\zeta)\mathbf{1} + 2F_2(\kappa_0,\eta,\zeta)\mu\otimes\mu + \right.$$
$$\left. +2\ \mathrm{sign}(\zeta)F_3(\kappa_0,\eta,\zeta)\mathbf{n}\otimes\mu\right]\cdot\mathbf{J}^{(i)}, \qquad \mathbf{n} = \mathbf{e}_3^{(i)}, \quad \kappa_0 = k_0 h_1,$$

$$\eta = \frac{1}{h_1}\left(x_1^2 + x_2^2\right)^{1/2}, \quad \zeta = \frac{x_3}{h_1}, \quad \mu = \frac{x_1\mathbf{e}_1^{(i)} + x_2\mathbf{e}_2^{(i)}}{h_1\eta}, \quad h_1 = \frac{D^{1/2}}{2}h.$$

Here the three functions $F_i(\kappa_0,\eta,\zeta)$ are the following one dimensional integrals

$$\begin{aligned}
F_1(\kappa_0,\eta,\zeta) &= \frac{1}{8\pi}\int_0^\infty \left[(2\kappa_0^2-\kappa^2)J_0(\kappa\eta) - \right. \\
&\quad \left. -\kappa^2 J_2(\kappa\eta)\right]\exp\left[-k^2 - |\zeta|\beta(\kappa,\kappa_0)\right]\frac{\kappa d\kappa}{\beta(\kappa,\kappa_0)}, \\
F_2(\kappa_0,\eta,\zeta) &= \frac{1}{8\pi}\int_0^\infty J_2(\kappa\eta)\exp\left[-k^2 - |\zeta|\beta(\kappa,\kappa_0)\right]\frac{\kappa^3 d\kappa}{\beta(\kappa,\kappa_0)}, \\
F_3(\kappa_0,\eta,\zeta) &= \frac{1}{8\pi}\int_0^\infty J_1(\kappa\eta)\exp\left[-k^2 - |\zeta|\beta(\kappa,\kappa_0)\right]\kappa^2 d\kappa,
\end{aligned} \tag{5.5}$$

where $\beta(\kappa,\kappa_0) = \sqrt{\kappa^2-\kappa_0^2}$ if $\kappa > \kappa_0$, and $\beta(\kappa,\kappa_0) = i\sqrt{\kappa_0^2-\kappa^2}$ if $\kappa < \kappa_0$; $J_n(z)$ is the Bessel function of order n. For small values of the arguments η, ζ $(\rho = \left(\eta^2+\zeta^2\right)^{1/2} \le 10)$ these integrals may be simply tabulated and kept in the computer memory. For $\rho > 10$ these integrals have simple asymptotic expressions presented in [2].

Using approximation (5.3) one can reduce the integral equation (5.1) to the system of linear algebraic equations which unknowns are the components of the vector $\mathbf{J}$ in the local bases connected with the nodes. This linear system may be written in the canonical form

$$BX = F, \tag{5.6}$$

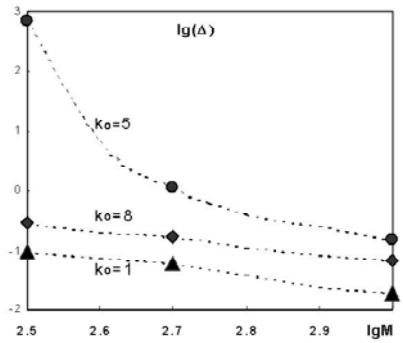

Figure 5.1: Error of the numerical solution for a sphere.

were the elements of the square matrix B of dimension $(2M \times 2M)$ are defined via standard functions F_1, F_2, F_3 in Eq.(5.5) and the components of vectors X and F are

$$\begin{aligned} X_i &= J_1^{(i)}, \quad i \le M; \quad X_i = J_2^{(i-M)}, \quad i > M; \\ F_i &= E_1^0(x^{(i)}), \quad i \le M; \quad F_i = E_2^0(x^{(i-M)}), \quad i > M. \end{aligned}$$

Here M is the total number of nodes. B in Eq.(5.6) is a dense matrix with maximal terms concentrated near the main diagonal. For a homogeneous distribution of the nodes on Ω matrix B is symmetric with the same elements in the main diagonal.

Let us consider a spherical surface Ω of unit radius ($a = 1$) when an analytical solution of the problem may be constructed. For the application of the BPM, a homogeneous set of nodes on Ω was generated by the algorithm described in [2]. In Fig.5.1 the dependences of relative error Δ of the numerical solutions on the number of surface nodes M are presented for $k_0 a = 1; 5; 8$.

$$\Delta = \frac{\int_\Omega (|\mathbf{J}_*| - |\mathbf{J}|)^2 d\Omega}{\int_\Omega |\mathbf{J}|^2 d\Omega}. \tag{5.7}$$

Here $\mathbf{J}_*$ is a numerical solution of Eq.(5.6), $\mathbf{J}$ is an exact current distribution.

6. Conclusion. The use of Gaussian approximating functions proposed in [5, 6] for the solution of boundary integral equations has two main advantages: the simplicity of preparation of the initial data (the coordinates of surface nodes and surface orientations at the nodes), and a short time for the construction of the matrix of the linear system obtained after the discretization of the problem. The accuracy of the method depends on the density of surface nodes. The method may be applied to a wide class of the problems of mathematical physics that are reduced to surface pseudo-differential equations. In particular, the problems of electrostatics, static elasticity and elasto-plasticity, the problems of elastic wave diffraction on inclusions and cracks [3], etc., may be successfully solved with the help of the developed version of BPM.

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