Preface

The annual International Conference on Domain Decomposition Methods for Partial Differential Equations has been a major event in Applied Mathematics and Engineering for the last fifteen years. The proceedings of the Conferences have become a standard reference in the field, publishing seminal papers as well as the latest theoretical results and reports on practical applications.

The Fourteenth International Conference on Domain Decomposition Methods, was hosted by the Universidad Nacional Autónoma de México (UNAM) at Hacienda de Cocoyoc in Morelos, Mexico, January 6-12, 2002. It was organized by Ismael Herrera, Institute of Geophysics, of the National Autonomous University of Mexico (UNAM). He was assisted by a Local Organizing Committee headed by Robert Yates, with the active participation of Gustavo Ayala-Milian, Martin Diaz and Gerardo Zenteno.

This was the sixth of the meetings in this nearly annual conference to be hosted in the Americas, but the first such outside of the United States. It was stimulating and rewarding to have the participation of many practicing scientists and graduate students from Mexico's growing applied mathematics community. Approximately one hundred mathematicians, engineers, physical scientists, and computer scientists from 17 countries spanning five continents participated. This volume captures 52 of the 78 presentations of the Conference.

Since three parallel sessions were employed at the conference in order to accommodate as many presenters as possible, attendees and non-attendees alike may turn to this volume to keep up with the diversity of subject matter that the topical umbrella of "domain decomposition" inspires throughout the community. The interest of so many authors in meeting the editorial demands of this proceedings volume demonstrates that the common thread of domain decomposition continues to justify a regular meeting. "Divide and conquer" may be the most basic of algorithmic paradigms, but theoreticians and practitioners alike continue to seek — and find — incrementally more effective forms, and value the interdisciplinary forum provided by this proceedings series.

Domain decomposition is indeed a basic concept of numerical methods for partial differential equations (PDE's) in general, although this fact is not always recognized explicitly. It is enlightening to interpret many numerical methods for PDE's as domain decomposition procedures and, therefore, the advances in Domain Decomposition Methods are opening new avenues of research in this general area. This is exhibited in this volume. In particular, using a continuous approach an elegant general theory of domain decomposition methods (DDM's) is explained, which incorporates direct and a new class of indirect methods in a single framework. This general theory interprets DDM's as procedures for gathering a target of information, on the internal boundary -'the sought information'-, that is chosen beforehand and is sufficient for defining well-posed local problems in each one of the subdomains of the partition. There are two main procedures for gathering the 'sought information': the *direct method*, which applies local solutions of the original differential equation, and the *indirect method*, which uses local solutions of the adjoint differential equation. Several advantages of the 'indirect method' are exhibited.

Besides inspiring elegant theory, domain decomposition methodology satisfies the architectural imperatives of high-performance computers better than methods operating only on the finest scale of the discretization and over the global data set. These imperatives include: concurrency on the scale of the number of available processors, spatial data locality, temporal data locality, reasonably small communication-to-computation ratios, and reasonably infrequent process synchronization (measured by the number of useful floating-point operations performed between synchronizations). Spatial data locality refers to the proximity of the addresses of successively used elements, and temporal data locality refers to the proximity in time of successive references to a given element.

Spatial and temporal locality are both enhanced when a large computation based on nearest-neighbor updates is processed in contiguous blocks. On cache-based computers, subdomain blocks may be tuned for workingset sizes that reside in cache. On message-passing or cache-coherent nonuniform memory access (cc-NUMA) parallel computers, the concentration of gridpoint-oriented computations — proportional to subdomain volume — between external stencil edge-oriented communications — proportional to subdomain surface area, combined with a synchronization frequency of at most once per volume computation, gives domain decomposition excellent parallel scalability on a per iteration basis, over a range of problem size and concurrency. In view of these important architectural advantages for domain decomposition methods, it is fortunate, indeed, that mathematicians studied the convergence behavior aspects of the subject in advance of the wide availability of these cost-effective architectures, and showed how to endow domain decomposition iterative methods with algorithmic scalability, as well.

Domain decomposition has proved to be an ideal paradigm not only for execution on advanced architecture computers, but also for the development of reusable, portable software. Since the most complex operation in a Schwarz-type domain decomposition iterative method — the application of the preconditioner — is logically equivalent in each subdomain to a conventional preconditioner applied to the global domain, software developed for the global problem can readily be adapted to the local problem, instantly presenting lots of "legacy" scientific code for to be harvested for parallel implementations. Furthermore, since the majority of data sharing between subdomains in domain decomposition codes occurs in two archetypal communication operations — ghost point updates in overlapping zones between neighboring subdomains, and global reduction operations, as in forming an inner product — domain decomposition methods map readily onto optimized, standardized message-passing environments, such as MPI.

The same arguments for reuse of existing serial methods in a parallel environment can be made for Schur-type or substructuring forms of domain decomposition, although in the substructuring case, there are additional types of operations to be performed on interfaces that are absent in the undecomposed original problem. Of course, treatment of the interface problem is where the art continues to undergo development, as the overall convergence depends upon this aspect when the subdomain problems are solved exactly.

Finally, it should be noted that domain decomposition is often a natural paradigm for the modeling community. Physical systems are often decomposed into two or more contiguous subdomains based on phenomenological considerations, such as the importance or negligibility of viscosity or reactivity, or any other feature, and the subdomains are discretized accordingly, as independent tasks. This physically-based domain decomposition may be mirrored in the software engineering of the corresponding code, and leads to threads of execution that operate on contiguous subdomain blocks, which can either be further subdivided or aggregated to fit the granularity of an available parallel computer, and have the correct topological and mathematical characteristics for scalability.

The organization of the present proceedings differs from that of previous volumes in that many of the papers are grouped into minisymposia, which provides a finer-grained topical grouping.

These proceedings will be of interest to mathematicians, computer scientists, and computational scientists, so we project its contents onto some relevant classification schemes below.

American Mathematical Society (AMS) 2000 subject classifications (`http://www.ams.org/msc/`) include:

65C20 Numerical simulation, modeling

65F10 Iterative methods for linear systems

65F15 Eigenvalue problems

65M55 Multigrid methods, domain decomposition for IVPs

65N30 Finite elements, Rayleigh-Ritz and Galerkin methods, finite methods

65N35 Spectral, collocation and related methods

65N55 Multigrid methods, domain decomposition for BVPs

65Y05 Parallel computation

68N99 Mathematical software

Association for Computing Machinery (ACM) 1998 subject classifications (`http://www.acm.org/class/1998/`) include:

D2 Programming environments, reusable libraries

F2 Analysis and complexity of numerical algorithms

G1 Numerical linear algebra, optimization, differential equations

G4 Mathematical software, parallel implementations, portability

J2 Applications in physical sciences and engineering

Applications for which domain decomposition methods have been specialized in this proceedings include:

fluids Stokes, Navier-Stokes, multiphase flow, dynamics of arteries, pipes, and rivers

materials phase change, composites

structures linear and nonlinear elasticity, fluid-structure interaction

other electrostatics, obstacle problems

For the convenience of readers coming recently into the subject of domain decomposition methods, a bibliography of previous proceedings is provided below, along with some major recent review articles and related special interest volumes. This list will inevitably be found embarrassingly incomplete. (No attempt has been made to supplement this list with the larger and closely related literature of multigrid and general iterative methods, except for the books by Hackbusch and Saad, which have significant domain decomposition components.)

1. P. Bjørstad, M. Espedal and D. E. Keyes, eds., *Proc. Ninth Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Ullensvang, 1997), Wiley, New York, 1999.

2. T. F. Chan and T. P. Mathew, *Domain Decomposition Algorithms*, Acta Numerica, 1994, pp. 61-143.

3. T. F. Chan, R. Glowinski, J. Périaux and O. B. Widlund, eds., *Proc. Second Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Los Angeles, 1988), SIAM, Philadelphia, 1989.

4. T. F. Chan, R. Glowinski, J. Périaux, O. B. Widlund, eds., *Proc. Third Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Houston, 1989), SIAM, Philadelphia, 1990.

5. T. Chan, T. Kako, H. Kawarada and O. Pironneau, eds., *Proc. Twelfth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations* (Chiba, 1999), DDM.org, Bergen, 2001.

6. N. Débit, M. Garbey, R. Hoppe, D. Keyes, Y. Kuznetsov and J. Périaux, eds., *Proc. Thirteenth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations* (Lyon, 2000), CINME, Barcelona, 2002.

7. C. Farhat and F.-X. Roux, *Implicit Parallel Processing in Structural Mechanics*, Computational Mechanics Advances **2**, 1994, pp. 1–124.

8. R. Glowinski, G. H. Golub, G. A. Meurant and J. Périaux, eds., *Proc. First Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Paris, 1987), SIAM, Philadelphia, 1988.

9. R. Glowinski, Yu. A. Kuznetsov, G. A. Meurant, J. Périaux and O. B. Widlund, eds., *Proc. Fourth Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Moscow, 1990), SIAM, Philadelphia, 1991.

10. R. Glowinski, J. Périaux, Z.-C. Shi and O. B. Widlund, eds., *Eighth International Conference of Domain Decomposition Methods* (Beijing, 1995), Wiley, Strasbourg, 1997.

11. W. Hackbusch, *Iterative Methods for Large Sparse Linear Systems*, Springer, Heidelberg, 1993.

12. I. Herrera, R. Yates and M. Diaz, *General Theory of Domain Decomposition: Indirect Methods*, Numerical Methods for Partial Differential Equations, **18(3)**, pp 296-322, 2002.

13. D. E. Keyes, T. F. Chan, G. A. Meurant, J. S. Scroggs and R. G. Voigt, eds., *Proc. Fifth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations* (Norfolk, 1991), SIAM, Philadelphia, 1992.

14. D. E. Keyes, Y. Saad and D. G. Truhlar, eds., *Domain-based Parallelism and Problem Decomposition Methods in Science and Engineering*, SIAM, Philadelphia, 1995.

15. D. E. Keyes and J. Xu, eds. *Proc. Seventh Int. Conf. on Domain Decomposition Methods for Partial Differential Equations* (PennState, 1993), AMS, Providence, 1995.

16. C.-H. Lai, P. Bjørstad, M. Cross and O. Widlund, eds., *Proc. Eleventh Int. Conf. on Domain Decomposition Methods for Partial Differential Equations* (Greenwich, 1999), DDM.org, Bergen, 2000.

17. P. Le Tallec, *Domain Decomposition Methods in Computational Mechanics*, Computational Mechanics Advances **2**, 1994, pp. 121–220.

18. J. Mandel, ed., *Proc. Tenth Int. Conf. on Domain Decomposition Methods in Science and Engineering* (Boulder, 1998), AMS, Providence, 1999.

19. L. Pavarino and A. Toselli, *Recent Developments in Domain Decomposition Methods*, Volume 23 of *Lecture Notes in Computational Science & Engineering*, Springer Verlag, Heidelberg, 2002.

20. A. Quarteroni and A. Valli, *Domain Decomposition Methods for Partial Differential Equations*, Oxford, 1999.

21. A. Quarteroni, J. Périaux, Yu. A. Kuznetsov and O. B. Widlund, eds., *Proc. Sixth Int. Conf. on Domain Decomposition Methods in Science and Engineering* (Como, 1992), AMS, Providence, 1994.

22. Y. Saad, *Iterative Methods for Sparse Linear Systems*, PWS, Boston, 1996.

23. B. F. Smith, P. E. Bjørstad and W. D. Gropp, *Domain Decomposition: Parallel Multilevel Algorithms for Elliptic Partial Differential Equations*, Cambridge Univ. Press, Cambridge, 1996.

24. B. I. Wolmuth, *Discretization Methods and Iterative Solvers Based on Domain Decomposition*, Volume 17 of *Lecture Notes in Computational Science & Engineering*, Springer Verlag, Heidelberg, 2001.

25. J. Xu, *Iterative Methods by Space Decomposition and Subspace Correction*, SIAM Review **34**, 1991, pp. 581-613.

We also mention the homepage for domain decomposition on the World Wide Web, `www.ddm.org`, maintained by Professor Martin Gander of McGill University. This site features links to conference, bibliographic, and personal information pertaining to domain decomposition, internationally.

Previous proceedings of the International Conferences on Domain Decomposition were published by SIAM, AMS, John Wiley and Sons and CIMNE. This time the publisher has been the National University of Mexico (UNAM), with the assistance of Impretei S.A. de C.V.

We wish to thank the members of the International Scientific Committee, and in particular the Chair, Ronald H.W. Hoppe, for their help in setting the scientific direction of the Conference. We are also grateful to the organizers of the mini-symposia for attracting high-quality presentations. The timely production of these Proceedings would not have been possible without the cooperation of the authors and the anonymous referees. We would like to thank them all for their graceful and timely response to our various demands.

The organizers of the Conference would like to acknowledge the sponsors of the Conference, namely UNAM through its Institute of Geophysics, the Instituto Nacional de Tecnología del Agua (IMTA) and the newly created Sociedad Mexicana de Métodos Numéricos en Ingeniería y Ciencia Aplicada (SMMNICA). Thanks are also due to Roland Glowinski and Yuri A. Kuznetsov, for their participation in the American Committee of the Conference, and to Alvaro Aldama, Fabian Garcia-Nocetti, Jaime Urrutia-Fucugauchi, Francisco Sanchez-Bernabe and Carlos Signoret-Poillon, for their participation in the Local Organizing Committee. Finally, we would like to express our appreciation to Ms. Marthita Cerrilla, the Secretary of the Conference, who made all the organizational details run smoothly, together with Martin Diaz and Ernesto Rubio, the Technical Editors of these Proceedings, who finalized the formatting of the papers in LATEX and prepared the whole book for printing.

Ismael Herrera
Mexico City, Mexico

David E. Keyes
Norfolk, USA

Olof B. Widlund
New York, USA

Robert Yates
Mexico City, Mexico

June 2003