15th International Conference on Domain Decomposition Methods
July 21-25, 2003, Berlin, Germany

Direct Schur Complement Method by Hierarchical Matrix Techniques

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Abstract: A class of hierarchical matrices ($\mathcal{H}$-matrices) allows the data-sparse approximation to integral and more general nonlocal operators (say, the elliptic Green function and Poincaré-Steklov operators) with almost linear complexity. We consider the $\mathcal{H}$-matrix-based approximation to the Schur complement on the interface [HKhKr1:03] corresponding to the finite element discretisation of an elliptic operator $\mathcal{L}$ with piecewise constant coefficients in $\mathbb{R}^2$. As with the standard Schur complement domain decomposition methods, we split the elliptic inverse $\mathcal{L}^{-1}$ as a sum of local inverses associated with subdomains (this can be implemented in parallel), and the corresponding Poincaré-Steklov operator on the interface. We focus on the data-sparse approximation to the Poincaré-Steklov operator and its inverse. Using the hierarchical formats based on weakened admissibility criteria (cf. [HKhKr1:02]) we elaborate the *approximate Schur complement inverse* in an explicit form that is proved to have a linear-logarithmic cost $O(N_\Gamma \log^q N_\Gamma)$, where N_Γ is the number of degrees of freedom on the interface. In the case of piecewise constant coefficients, the local Schur complements are approximated by the explicit BEM representations. We also prove the asymptotically optimal error estimate in the case of piecewise linear finite elements. In the case of variable coefficients, our method manifests a linear-logarithmic complexity in the discrete problem size N_Ω.
Numerical examples confirm the almost linear cost of our direct Schur complement method. In particular, for the discrete Laplacian on 255×255 and 511×511 grids with 6×6 decomposition we have $N_\Gamma = 2525$ and $N_\Gamma = 5085$, respectively. The elapsed CPU times to compute the explicit Schur complement inverses on the interface with the relative error $1.1e-03$ are $29.0sec$ and $74.0sec$, correspondingly (SUN 6800).

[1] W. Hackbusch, B.N. Khoromskij and R. Kriemann. *Hierarchical Matrices Based on Weak Admissibility Criterion.* Preprint MPI MIS 2, Leipzig, 2003; Computing (submitted).

[2] W. Hackbusch, B.N. Khoromskij and R. Kriemann. *Direct Schur Complement Method by Hierarchical Matrix Techniques.* Preprint MPI MIS, Leipzig, 2003 (in progress).

Type of contribution: Talk
Location: Room 005, **Time:** Monday, 21 July, 11:00

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