

15th International Conference on  
**Domain Decomposition Methods**

21 -25 July, 2003

Berlin, Germany

**Program & Abstracts**

Revision July 24, 2003

Freie Universität Berlin (FUB)

in cooperation with

Zuse Institute Berlin (ZIB)

Weierstrass Institute for Applied  
Analysis and Stochastics Berlin (WIAS)

and

DFG Research Center  
'Mathematics for key technologies' (FZT 86)

Monday, 21 July

Time	Lecture room			
08:00 – 08:45	<b>Registration (Room 051)</b>			
08:45 – 09:00	<b>Opening</b>			
<b>Plenary talks (Chair: Deuffhard)</b>				
09:00 – 09:45	<b>Brezzi:</b> Nonmatching Grids and Lagrange Multipliers [p. 17]			
09:45 – 10:30	<b>Fischer:</b> Hybrid Schwarz-Multigrid Methods for the Spectral Element Method [p. 18]			
10:30 – 11:00	<b>Coffee break</b>			
<b>Contributed talks</b>				
	Lecture room	Room 005	Room 049	Room 055
11:00 – 11:20	<b>Pavarino:</b> A Parallel Solver for Reaction-Diffusion Systems in Computational Electrophysiology [p. 61]	<b>Khoromskij:</b> Direct Schur Complement Method by Hierarchical Matrix Techniques [p. 57]	<b>Garbey:</b> Multilevel Solutions, Least Square Extrapolation and a Posteriori Error Estimate [p. 55]	<b>Stefanica:</b> Lower Bounds for Overlapping and Nonoverlapping Domain Decomposition Preconditioners for Mortar Element Methods [p. 63]
11:20 – 11:40	<b>Pacull:</b> — cancelled —	<b>Le Borne:</b> Hierarchical Matrices for Convection-Dominated Problems [p. 58]	<b>Marcinkowski:</b> Parallel Performance of a Two-Level ASPIN Algorithm [p. 59]	<b>Sheen:</b> $P_1$ Nonconforming Finite Element Method on Quadrilaterals and its Domain Decomposition Procedure [p. 63]
11:40 – 12:00	<b>Pennacchio:</b> The Mortar Finite Element Method in Computational Electrophysiology [p. 61]	<b>Nabben:</b> A Comparison of Deflation and Coarse Grid Correction Applied to Porous Media Flow [p. 60]	<b>Chen:</b> An Accelerated Block-Parallel Newton Method via Overlapped Partitioning [p. 53]	<b>Cros:</b> Rigid Body Modes within the Framework of Domain Decomposition Methods [p. 54]
12:00 – 12:20	<b>Weber Dos Santos:</b> Preconditioning Techniques for the Bidomain Equations [p. 64]	<b>Szyld:</b> Algebraic Analysis of Schwarz Methods for Singular Systems [p. 63]	<b>Kulkarni:</b> Domain Decomposition Based Two-Level Newton Scheme for non-Linear Problems [p. 58]	<b>Rahman:</b> An Additive Schwarz Method for the Morley Element Approximation of a Non-Linear Biharmonic Equation [p. 62]
12:20 – 12:40	<b>Tagami:</b> Numerical Computations of 3-D Eddy Current Problems by Iterative Domain Decomposition Method [p. 64]	<b>Oudin:</b> Acceleration of the Schwarz Method for Elliptic Problems [p. 60]	<b>Azimi:</b> Geometrical Discretization of the Computational Domain for Computations of Axisymmetric Supersonic Flows [p. 51]	
12:40 – 14:00	<b>Lunch break</b>			

Monday, 21 July

Time	Lecture room			
	<b>Plenary talks (Chair: Widlund)</b>			
14:00 – 14:45	<b>Farhat:</b> Time-Decomposed Parallel Time-Integrators: Theory and Feasibility Studies for Accelerating the Massively Parallel Solution of Fluid, Structure, and Fluid-Structure Problems [p. 17]			
14:45 – 15:30	<b>Klawonn:</b> Dual-Primal FETI Methods for Elasticity [p. 21]			
15:30 – 16:00	<b>Coffee break</b>			
	<b>Minisymposia</b>			
	<b>MS08</b> Domain Decomposition on Nonmatching Grids (Hoppe, Wohlmuth, Kuznetsov) [p. 38]	<b>MS02</b> Discretization Techniques and Algorithms for Multibody Contact Problems (Wohlmuth, Sassi) [p. 26]	<b>MS06</b> Robust Decomposition Methods for Parameter Dependent Problems (Langer, Nepomnyaschikh) [p. 35]	
	<b>Lecture room</b>	<b>Room 005</b>	<b>Room 049</b>	<b>Room 055</b>
16:00 – 16:25	<b>Braess:</b> A Cascadic Multigrid Algorithm for Mortar Elements [p. 38]	<b>Dostal:</b> Optimal Penalty and Scalable FETI Based Algorithms for Numerical Solution of Variational Inequalities [p. 26]	<b>Nepomnyaschikh:</b> Preconditioning for Heterogeneous Problems [p. 35]	
16:25 – 16:50	<b>Maday:</b> Coupling Scalar and Vector Potentials on Nonmatching Grids for Eddy Currents in Moving Conductor [p. 38]	<b>Sassi:</b> A Mixed Finite Element Approximation of 3D Contact Problems with Given Friction: Approximation and the Numerical Realization [p. 26]	<b>Scherer:</b> Weighted Norm-Equivalences for Preconditioning [p. 36]	
16:50 – 17:15	<b>Wieners:</b> Multigrid Analysis for Saddle Point Problems Arising from Mortar Discretizations [p. 38]	<b>Krause:</b> Fast Solving of Contact Problems on Complicated Geometries [p. 27]	<b>Beuchler:</b> A Dirichlet-Dirichlet $DD$ -preconditioner for $p$ -fem [p. 36]	
17:15 – 17:40	<b>Lamichhane:</b> Second Order Lagrange Multiplier Spaces for Mortar Finite Element Discretizations [p. 39]	<b>Sassi:</b> Domain Decomposition Algorithms for Contact Problems [p. 27]	<b>Becirovic:</b> Optimal Extension Operators for High Order Tetrahedral Elements [p. 36]	
17:40 – 18:05	<b>Xu:</b> A Mortar Element Method for a Plate Bending Problem [p. 39]	<b>Vassilevski:</b> Monotone Element Agglomeration $AMG_e$ for Contact Problems [p. 27]	<b>Deng:</b> Folding Process of Thin-Walled Prismatic Columns by Origami Modeling [p. 36]	

Tuesday, 22 July

Time	Lecture room			
	<b>Plenary talks (Chair: Sprekels)</b>			
09:00 – 09:45	<b>Gilg:</b> Industrial Mechatronics - Some Problems from a Mathematical Viewpoint [p. 19]			
09:45 – 10:30	<b>Girault:</b> Combining Domain Decomposition with Other Techniques: Fictitious Domain, Discontinuous Galerkin,... [p. 19]			
10:30 – 11:00	<b>Coffee break</b>			
	<b>Minisymposia</b>			
	<b>MS09</b> FETI and Neumann-Neumann Domain Decomposition Methods (Klawonn, Pierson, Widlund) [p. 41]	<b>MS10</b> Recent Advances for the Parareal in Time Algorithm (Maday) [p. 45]	<b>MS12</b> Trefftz-Methods (Herrera, Yates) [p. 48]	
	<b>Lecture room</b>	<b>Room 005</b>	<b>Room 049</b>	<b>Room 055</b>
11:00 – 11:25	<b>Farhat:</b> An Iterative Domain Decomposition Method for the Solution of a Class of Indefinite Problems in Computational Structural Dynamics [p. 41]	<b>Maday:</b> The Parareal Algorithm: Basics and Combination with Domain Decomposition Iterations [p. 45]	<b>Herrera:</b> A New and More General Version of the Hybrid-Trefftz Finite Element Model, Derived by Application of th-Domain Decomposition [p. 49]	
11:25 – 11:50	<b>Widlund:</b> Selecting Primal Constraints for FETI-DP Algorithms for Linear Elasticity [p. 41]	<b>Bal:</b> On the Analysis and Implementation of the Parareal Algorithm [p. 45]	<b>Yates:</b> Trefftz-Herrera Method: Highly Accurate Numerical Algorithms for Parabolic Equations [p. 49]	
11:50 – 12:15	<b>Kucera:</b> The FETI Based Domain Decomposition Method for Solving 3D-Multibody Contact Problems with Coulomb Friction [p. 41]	<b>Staff:</b> Stability and Convergence of the Parareal Algorithm [p. 45]	<b>Rubio-Acosta:</b> Parallel Implementation of Indirect Collocation Methods [p. 49]	
12:15 – 12:40	<b>Kim:</b> A FETI-DP Method for the Stokes Problems on Nonmatching Grids [p. 42]	<b>Fischer:</b> Investigation of the Parareal Algorithm for Semi-Implicit Incompressible Navier-Stokes Simulations [p. 46]	<b>Diaz-Viera:</b> Trefftz-Herrera Domain Decomposition Method for Biharmonic Equation [p. 50]	
12:40 – 14:00	<b>Lunch break</b>			

Tuesday, 22 July

Time	Lecture room			
	<b>Plenary talks (Chair: Kawarada)</b>			
14:00 – 14:45	<b>Kako:</b> Numerical Approximation of Dirichlet-to-Neumann Mapping and its Application to Voice Generation Problem [p. 21]			
14:45 – 15:30	<b>Hiptmair:</b> Domain Decomposition Preconditioners for Edge Elements: A Survey [p. 19]			
15:30 – 16:00	<b>Coffee break</b>			
	<b>Minisymposia</b>			
	<b>MS04</b> Domain Decomposition Methods for Wave Propagation in Unbounded Media (Antoine, Schmidt) [p. 32]	<b>MS05</b> Heterogeneous Domain Decomposition with Applications in Multiphysics (Kornhuber, Quarteroni) [p. 33]	<b>MS03</b> Recent Developments for Schwarz Methods (Gander) [p. 27]	
	<b>Lecture room</b>	<b>Room 005</b>	<b>Room 049</b>	<b>Room 055</b>
16:00 – 16:25	<b>Hohage:</b> New Transparent Boundary Conditions for Coupled Interior/Exterior Wave Propagation Problems [p. 32]	<b>Hoppe:</b> Domain Decomposition Methods in Electrothermomechanical Coupling Problems [p. 34]	<b>Gander:</b> RAS: Understanding Restricted Additive Schwarz [p. 28]	
16:25 – 16:50	<b>Balin:</b> Domain Decomposition and Additive Schwarz Techniques in the Solution of a TE Model of the Scattering by an Electrically Deep Cavity [p. 32]	<b>Nefedov:</b> Subgridding in Finite-Difference Time-Domain Method [p. 34]	<b>Lube:</b> Acceleration of an Iterative Substructuring Method for Singularly Perturbed Elliptic Problems [p. 28]	
16:50 – 17:15	<b>Antoine:</b> On the Construction of Approximate Boundary Conditions for Solving the Interior Problem of the Acoustic Scattering Transmission Problem [p. 33]	<b>Saleri:</b> A Multiphysics Strategy for Free Surface Flows [p. 34]	<b>Gerardo-Giorda:</b> Modified Schwarz Algorithms without Overlap for the Harmonic Maxwell's System [p. 28]	
17:15 – 17:40	<b>Schmidt:</b> Numerical Methods to Realize the Pole Condition Concept [p. 33]	<b>Schieweck:</b> Coupling Fluid Flow with Porous Media Flow [p. 35]	<b>Nataf:</b> Finite Volume Methods on Non-Matching Grids with Arbitrary Interface Conditions [p. 28]	
17:40 – 18:05	<b>Ehrhardt:</b> Approximation, Stability and Fast Calculation of non-Local Boundary Conditions for the Schrödinger Equation [p. 33]	<b>Zunino:</b> Iterative Substructuring Methods for Advection-Diffusion Problems in Heterogeneous Media [p. 35]		

Wednesday, 23 July

Time	Lecture room			
	<b>Plenary talks (Chair: Quarteroni)</b>			
09:00 – 09:45	<b>Valli:</b> The Swiss Carpet Preconditioner: A Simple Parallel Preconditioner of Dirichlet-Neumann Type [p. 22]			
09:45 – 10:30	<b>Langer:</b> Boundary and Finite Element Tearing and Interconnecting Methods. [p. 21]			
10:30 – 11:00	<b>Coffee break</b>			
	<b>Minisymposia</b>			
	<b>MS09</b> FETI and Neumann-Neumann Domain Decomposition Methods (Klawonn, Pierson, Widlund) [p. 41]	<b>MS07</b> Parallel Finite Element Software (Bastian, Wieners) [p. 37]	<b>MS03</b> Recent Developments for Schwarz Methods (Gander) [p. 27]	
	<b>Lecture room</b>	<b>Room 005</b>	<b>Room 049</b>	<b>Room 055</b>
11:00 – 11:25	<b>Stefanica:</b> Parallel FETI Algorithms for Mortars [p. 42]	<b>Banaś:</b> A Model for Parallel Adaptive Finite Element Software [p. 37]	<b>Dolean:</b> A Non-Overlapping Schwarz Type Algorithm for the Resolution of the Euler Equations [p. 29]	
11:25 – 11:50	<b>Rheinbach:</b> Some Computational Results for Dual-Primal FETI Methods for Three Dimensional Elliptic Problems [p. 42]	<b>Pflaum:</b> Parallelization Concepts of the Library EXPDE [p. 37]	<b>Rohde:</b> Overlapping Schwarz Waveform Relaxation for Convection Dominated Nonlinear Conservation Laws [p. 30]	
11:50 – 12:15	<b>Rey:</b> An Hybrid Domain Decomposition Method [p. 43]	<b>Wieners:</b> Distributed Point Objects: A New Concept for Parallel Finite Elements [p. 37]	<b>Martin:</b> Domain Decomposition Methods for Unsteady Convection Diffusion Equation [p. 30]	
12:15 – 12:40	<b>Fragakis:</b> A Family of FETI-Derived Preconditioners for the Primal Substructuring Method: Application to Multiple Right-Hand Side Problems and Implicit Dynamic Analysis [p. 43]	<b>Bastian:</b> Towards a Unified Framework for Finite Element Computations [p. 37]	<b>Rapin:</b> A Stabilized Three Field Domain Decomposition Formulation for Advection-Diffusion Problems and its Iterative Decoupling [p. 30]	
12:40 – 14:00	<b>Lunch break</b>			

**Wednesday, 23 July**

<b>Time</b>	<b>Social event</b>
14:00 – 19:00	<b>Trip to Potsdam</b> (meeting in front of the computer science building 10 minutes in advance)

Thursday, 24 July

Time	Lecture room			
	<b>Plenary talks (Chair: Periaux)</b>			
09:00 – 09:45	<b>Ghattas:</b> Multiscale Methods for Inverse Wave Propagation [p. 18]			
09:45 – 10:30	<b>Joly:</b> Domain Decomposition Approaches to Space-Time Mesh Refinement in Linear Wave Propagation [p. 20]			
10:30 – 11:00	<b>Coffee break</b>			
	<b>Contributed talks</b>			
	Lecture room	Room 005	Room 049	Room 055
11:00 – 11:20	<b>Turek:</b> Adaptivity Concepts and Load Balancing Strategies for a Generalized Parallel Multigrid/Domain Decomposition Solver [p. 64]	<b>Heinkenschloss:</b> Domain Decomposition Preconditioners for the Optimization of Distributed Parameter Systems [p. 56]	<b>Ismail:</b> The Fat Boundary Method: Convergence and Error Analysis [p. 57]	<b>Boursier:</b> Modelling of an Underground Waste Disposal Site by Upscaling and Simulation with Domain Decomposition Method [p. 52]
11:20 – 11:40	<b>Charnpis:</b> Generation of Subdomains and Subdomain Clusters for Domain Decomposition Methods [p. 53]	<b>Periaux:</b> Domain Embedding/Controllability Methods for the Conjugate Gradient Solution of Wave Propagation Problems: Application to Shape Optimization [p. 61]	<b>Daoud:</b> Explicit Implicit Non Overlapping Domain Decomposition Method with Splitting up method for Multi Dimensional Parabolic Problem [p. 54]	<b>Calugaru:</b> A non-Overlapping Domain Decomposition Method to Solve Flow in Discontinuous Porous Media [p. 52]
11:40 – 12:00	<b>Compton:</b> Domain Decomposition and Load Balancing in the AMTRAN Neutron Transport Code [p. 54]	<b>Pieska:</b> Predictor-Corrector Methods for Solving Continuous Casting Problem [p. 62]	<b>Filatov:</b> — cancelled —	<b>Discacciati:</b> Domain Decomposition Methods for Coupling Stokes and Darcy Equations [p. 55]
12:00 – 12:20	<b>Huelsemann:</b> Optimised Data Structures for Efficient Large Scale Parallel Computations [p. 57]	<b>Hientzsch:</b> Domain Decomposition Preconditioners for Spectral Nedelec Elements in Two and Three Dimensions [p. 56]	<b>Li:</b> Robin Transmission Conditions for Overlapping Additive Schwarz Method Applied to Elliptic Problems [p. 59]	<b>Martin:</b> Domain Decomposition Methods for Viscous Shallow Water Equation [p. 59]
12:20 – 12:40	<b>Rychkov:</b> Parallel Distributed Object-Oriented Model of Domain Decomposition [p. 62]	<b>Urazbaeva:</b> System of Queuing Research with the Decomposition Technique and its Application to the Analysis of the Fiber Optic Transmission Network with the DQDB Protocol [p. 64]	<b>Mihai:</b> A Two-Grid Alternate Strip-Based Domain Decomposition Strategy in Two-Dimensions [p. 59]	<b>Krzyzanowski:</b> Domain Decomposition for Discontinuous Galerkin Method with Application to Stokes Flow [p. 58]
12:40 – 14:00	<b>Lunch break</b>			



Thursday, 24 July

Time	Lecture room			
	<b>Plenary talks (Chair: Xu)</b>			
14:00 – 14:45	<b>Espedal:</b> Parallel Simulation of Multiphase/Multicomponent Flow Models [p. 17]			
14:45 – 15:30	<b>Bänsch:</b> Finite Element Methods for Curvature Driven Problems [p. 17]			
15:30 – 16:00	<b>Coffee break</b>			
	<b>Minisymposia</b>			
	<b>MS08</b> Domain Decomposition on Nonmatching Grids (Hoppe, Wohlmuth, Kuznetsov) [p. 38]	<b>MS01</b> Collaborating Subdomains for Multi-Scale Multi-Physics Modelling (Chow, Lai) [p. 23]	<b>MS11</b> Space Decomposition and Subspace Correction Methods for Linear and Nonlinear Problems (Tai, Xu) [p. 46]	
	<b>Lecture room</b>	<b>Room 005</b>	<b>Room 049</b>	<b>Room 055</b>
16:00 – 16:25	<b>Heinrich:</b> Nitsche-Type Mortaring for Elliptic Problems with Singularities and Boundary Layers [p. 39]	<b>Cai:</b> ASPIN for Incompressible Navier-Stokes Equations [p. 23]	<b>Badea:</b> On a Multilevel Schwarz Method for the Constraint Minimization of non-Quadratic Functionals [p. 46]	
16:25 – 16:50	<b>Kuznetsov:</b> Mixed Finite Element Methods for Diffusion Equations on Nonmatching Grids [p. 39]	<b>Knopp:</b> Iterative Substructuring Methods for Indoor Air Flow Simulation [p. 23]	<b>Kornhuber:</b> Hierarchical Decomposition of Domains with Fractures [p. 47]	
16:50 – 17:15	<b>Rahman:</b> Additive Schwarz Method for the Mortar Crouzeix-Raviart Element [p. 40]	<b>Swim:</b> Fluid-Structure Interaction with Nonconforming Finite Elements [p. 24]	<b>Tai:</b> Nonlinear Positive Intepolation Operators for Analysis with Multilevel Grids [p. 47]	
17:15 – 17:40	<b>Gantner:</b> Multilevel Additive Schwarz Preconditioner for Nonconforming Mortar Finite Element Methods [p. 40]	<b>Lai:</b> Some Effective Techniques of Nonlinear Solvers for Black-Oil Modelling [p. 24]	<b>Xu:</b> Anisotropic grid adaptation and multigrid methods [p. 47]	
17:40 – 18:05	<b>Seshaiyer:</b> Non-Conforming Finite Element Methods for Nonmatching Grids Tuned to Parallel Implementation [p. 40]		<b>Garrido:</b> A Convergent Algorithm for Time Parallelization Applied to Reservoir Simulation [p. 47]	
17:40 – 19:00	<b>Poster session (Room 046, see next page)</b>			
18:30 – 22:00	<b>Conference dinner (Pavilion)</b>			

## Poster session, Thursday, 17:40 – 19:00, Room 046

- **Cai:** A Restricted Additive Schwarz Method for the Bidomain Model of Cardiac Excitation [p. 67]
- **Cai:** Domain Decomposition Methods for A PDE Constrained Optimization Problem [p. 67]
- **Cai:** Simulations of Branching Blood fluids on Parallel Computers [p. 67]
- **Cho:** Domain Decomposition Preconditioning for Elliptic Problems with Jumps in the Coefficients [p. 67]
- **Deng:** Perturbation Analysis and its Approximation by FEM for Coupled Systems Between Structure and Acoustic Field [p. 68]
- **Fahimuddin:** PLATON - An Environment for Coupling Optimisation and Simulation Codes [p. 68]
- **Horak:** Scalability of FETI Based Algorithms for Variational Inequalities [p. 68]
- **Keyes:** Optimization of PDE-constrained Systems in the Terascale Optimal PDE Simulations Project [p. 68]
- **Keyes:** Scalable Solvers in the Terascale Optimal PDE Simulations Project [p. 68]
- **Keyes:** Terascale Optimal PDE Solvers: Project Overview [p. 69]
- **Sary:** Parallel Iterative Solvers in Geomechanics [p. 69]
- **Steckel:** MpCCI: A Tool for the Simulation of Coupled Problems, eg. Using Domain Decomposition [p. 69]
- **Vlach:** Signorini Problem with a Solution Dependent Coefficient of Friction (Model with Given Friction): Approximation and the Numerical Realization [p. 69]

The posters are on display in Room 046 throughout the whole conference.

Friday, 25 July

Time	Minisymposia			
	<b>MS09</b> FETI and Neumann-Neumann Domain Decomposition Methods (Klawonn, Pierson, Widlund) [p. 41]	<b>MS01</b> Collaborating Subdomains for Multi-Scale Multi-Physics Modelling (Chow, Lai) [p. 23]	<b>MS03</b> Recent Developments for Schwarz Methods (Gander) [p. 27]	
	<b>Lecture room</b>	<b>Room 005</b>	<b>Room 049</b>	<b>Room 055</b>
09:00 – 09:25	<b>Dryja:</b> A Dual-Primal FETI Method with Face Constraints for Mortar Discretization of Elliptic Problems [p. 44]	<b>Anthonissen:</b> Local Defect Correction Techniques Applied to a Combustion Problem [p. 24]	<b>Roux:</b> Approximation of Optimal Interface Boundary Conditions for Two-Lagrange Multiplier FETI Method [p. 30]	
09:25 – 09:50	<b>Proskurowski:</b> A FETI-DP Method for the Mortar Discretization of Elliptic Problems with Discontinuous Coefficients [p. 44]	<b>Garbey:</b> Heterogeneous Domain Decomposition for Boundary Layer Problems [p. 25]	<b>Vandewalle:</b> Optimized Overlapping Schwarz Methods for Parabolic PDEs with Time-Delay. [p. 31]	
09:50 – 10:15	<b>Toselli:</b> FETI and Neumann-Neumann Preconditioners for hp Finite Element Approximations on Anisotropic Meshes: Algorithms and Theory [p. 44]	<b>Veldman:</b> Interaction Laws in Viscous-Inviscid Coupling [p. 25]	<b>Laayouni:</b> Non-Overlapping Optimized Domain Decomposition Methods in Spherical Coordinates [p. 31]	
10:15 – 10:40	<b>Vasseur:</b> FETI and Neumann-Neumann Preconditioners for hp Finite Element Approximations on Anisotropic Meshes: Numerical Validation [p. 44]	<b>Chow:</b> Electronic Packaging and Reduction in Modelling Time Using Domain Decomposition [p. 25]	<b>Japhet:</b> A New Cement to Glue Non-conforming Grids with Robin Interface Conditions: The Finite Element Case [p. 31]	
10:40 – 11:15	<b>Coffee break</b>			
	<b>Lecture room</b>			
	<b>Plenary talk (Chair: Hoppe)</b>			
11:15 – 12:00	<b>Schütte:</b> Domain Decomposition for Metastable Dynamical Systems [p. 22]			
12:00 – 12:20	<b>Closing Ceremony</b>			
12:20 – 14:00	<b>Lunch</b>			



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## 1 Abstracts of Invited Talks

EBERHARD BÄNSCH

### Finite Element Methods for Curvature Driven Problems

**Location:** Lecture Room, **Time:** Thursday, 24 July, 14:45

We present two classes of free boundary problems involving curvature terms. The first class is fluid flow with a free surface subject to surface tension. The second one arises in a totally different context and describes instabilities in mechanically stresses epitaxially thin (solid) films.

A proper variational formulation of the curvature terms along with a semi-implicit time discretization yield effective finite element methods for these type of problems.

- [1] E. Bänsch: *Finite element discretization of the Navier–Stokes equations with a free capillary surface*. Numer. Math. 88, 203-235 (2001)
- [2] E. Bänsch, P. Morin, R. H. Nochetto: *Finite Element Methods for Surface Diffusion*. Preprint no. 805, WIAS–Berlin (2003), to appear in in Proceedings FBP2002, Trento.
- [3] E. Bänsch, P. Morin, R. H. Nochetto: *Surface Diffusion of Graphs: Variational Formulation, Error Analysis, and Simulation*. Submitted.

FRANCO BREZZI, L.D. MARINI

### Nonmatching Grids and Lagrange Multipliers

**Location:** Lecture Room, **Time:** Monday, 21 July, 9:00

The talk will survey old and recent issues on the use of Lagrange multipliers when enforcing interelement “Dirichlet” boundary conditions in nonoverlapping Domain Decomposition Methods with nonmatching grids.

MAGNE S. ESPEDAL, E. ØIAN, IZASKUN GARRIDO

### Parallel Simulation of Multiphase/Multicomponent Flow Models

**Location:** Lecture Room, **Time:** Thursday, 24 July, 14:00

The simulation of flow in porous media is a computationally demanding task. Thermodynamical equilibrium calculations and complex, heterogeneous geological structures normally gives a multiphysics-multidomain problem to solve. Thus, efficient solution methods are needed. The research simulator Athene is a 3D, multiphase, multicomponent, porous media flow simulator. A parallel version of the simulator was developed based on a non-overlapping domain decomposition strategy, where the domains are defined a-priori from e.g. geological data. Selected domains are refined with locally matching grids, giving a globally non-matching grid.

Similarly, algorithms for parallel processing in time is tested.

Adding communication functionality enables the original serial version to run on each sub-domain in parallel. Motivated by the need for larger time steps, a more implicit formulation of the mass transport equations was formulated. We will present the extensions of the existing parallel framework to include this formulation. Further, as the Message Passing Interface (MPI) is used for communication, the simulator is highly portable. Through realistic numerical experiments, we explore the overall capabilities of the simulator and the new formulation on platforms ranging from commercial super-computers to heterogeneous networks of PC workstations.

Numerical results are reported.

CHARBEL FARHAT

### Time-Decomposed Parallel Time-Integrators: Theory and Feasibility Studies for Accelerating the Massively Parallel Solution of Fluid, Structure, and Fluid-Structure Problems

**Location:** Lecture Room, **Time:** Monday, 21 July, 14:00

A methodology for squeezing the most out of massively parallel processors when solving partial differential evolution equations by implicit schemes is presented. Its key components are related to those of

the parareal scheme proposed by Lions and his co-workers and which offers a unique opportunity for parallelizing the time-loop of a PDE solver. Hence, these components include a preferred implicit ODE solver, a decomposition of the time domain into time-slices, independent time-integrations in each time-slice of the semi-discrete equations, and the propagation of solution jumps on a coarse time-grid. A Newton-based mathematical framework is presented for analyzing this methodology and gaining further insight into its limitations and possibilities. The original coarsening operator is generalized to address multi-stage time-integrators as these are often used for the solution of the ODEs arising from the semi-discretization of second-order hyperbolic problems. For such problems, a spurious and show-stopper resonance phenomenon is identified highlighted, and addressed. Error and stability analyses are presented. Feasibility and parallel efficiency analyses are also performed to identify both the physical and simulation parameters that make or break the desired objective of reducing the total CPU time of a PDE solver by parallelizing its time-loop a la parareal. The main findings are illustrated with results for fluid, structure, and fluid-structure model and real problems. Finally, conclusions of practical as well as theoretical interests are formulated.

PAUL F. FISCHER, JAMES W. LOTTES

### **Hybrid Schwarz-Multigrid Methods for the Spectral Element Method**

**Location:** Lecture Room, **Time:** Monday, 21 July, 9:45

The spectral element method (SEM) is a high-order weighted residual technique that, because of its minimal numerical dissipation and dispersion, is well-suited to the simulation of transitional flows in complex domains. Globally, elements are unstructured, with interelement coupling enforced through standard matching of nodal interface values. Locally, functions are represented as tensor-products of  $N$ th-order Lagrangian interpolants based upon Gauss-Lobatto (GL) quadrature points. For problems having smooth solutions, such as the incompressible Navier-Stokes equations, the SEM converges exponentially fast with the local order  $N$ .

The two-level hierarchy of the spectral element discretization provides a natural route to domain decomposition with several benefits. The loose  $C^0$  interelement coupling implies that the stencil depth does not increase with approximation order, so that interprocessor communication is minimal. The local tensor-product structure allows matrix-vector products to be recast as cache-efficient *matrix-matrix* products, and allows for fast local subdomain solves with a cost that is equal to forward operator evaluation. Finally, the high-order polynomial expansions provide a readily available sequence of nested grids (obtained through successive reductions in polynomial degree) for use in multilevel solvers.

This talk presents recent developments in spectral element multigrid (SEMG) methods. Our point of departure is the original work of Rønquist and Patera, and Maday and Muñoz, who developed variational SEMG for the two-dimensional Poisson problem using intra-element prolongation/restriction operators coupled with Jacobi smoothing. The high-aspect-ratio cells present in the tensor-product GL grid are a well-known source of difficulty in spectral multigrid methods that have drawn much attention over the past decade. In the present work, we employ Schwarz-based smoothers that employ fast tensor-product-based solvers. This approach bypasses the problem of high-aspect-ratio cells at a cost equivalent to a matrix-vector product. We show that it is important to weight the additive Schwarz solutions by the inverse of the counting matrix to obtain adequate smoothing. We also introduce a new LCS (local-coarse-strip) domain-decomposition method that shows very good performance with low complexity. We compare SEMG with other multi-level solvers, including two-level additive Schwarz and deflation. Finally, we present extensions beyond the two-dimensional Poisson problem to Helmholtz and Navier-Stokes applications in general domains.

OMAR GHATTAS

### **Multiscale Methods for Inverse Wave Propagation**

**Location:** Lecture Room, **Time:** Thursday, 24 July, 9:00

Toward our goal of modeling strong earthquakes in seismic regions, we are interested in determining mechanical properties of sedimentary basins (such as the greater Los Angeles Basin) and descriptions of earthquake sources from seismograms of past earthquakes. This gives rise to very large inverse problems of recovering the coefficients and source of the elastic wave equation from boundary observations of the response. Our current forward simulations involve 100 million finite elements; over the next several years the desired increase in resolution and growth in basin size will require an order of magnitude increase in number of unknowns. Inversion of such forward models provides a major challenge for inverse methods.

It is imperative that these methods be able to scale algorithmically to  $O(10^9)$  grid points, to highly-resolved (e.g. grid-based) elastic material models of large seismic basins, and to parallel architectures with thousands of processors.

I will discuss prototype multiscale parallel algorithms for the earthquake material and source inversion problem. Tikhonov and total variation regularization treat ill-posedness associated with rough components of the model, while multiscale grid/frequency continuation addresses multiple minima associated with smooth components. Parallel inexact Gauss-Newton-Krylov methods are used to solve the optimality conditions. CG matrix-vector products are computed via checkpointed adjoints, which involve forward and adjoint wave equation solutions at each iteration. Preconditioning is via limited memory BFGS updates, initialized with approximate inverses of an approximation to the Gauss-Newton Hessian. Experience on problems with up to several million grid points suggests near mesh-independence of Newton and CG iterations, good parallel efficiency, and distinct speedups over a quasi-Newton method. However, significant difficulties remain, and I will conclude with a discussion of these, along with possible avenues for addressing them.

This work is joint with V. Akcelik and J. Bielak at Carnegie Mellon, G. Biros at Courant, and other members of the Quake Project.

ALBERT B. GILG

### **Industrial Mechatronics - Some Problems from a Mathematical Viewpoint**

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 9:00

An increasing number of industrial problems cannot be treated adequately by isolating pure mechanical, electronics or software problems. Such hybrid product problems have initiated the Mechatronics engineering discipline. It challenges traditional academic research areas. Bridging these areas, e.g. by data exchange formats, shows to be insufficient - even at the abstract level of modelling. More promising attempts are based on initiating and conducting interdisciplinary project activities, bringing together engineering, physics, computer science and mathematics expertise. This fully coincides with typical guidelines of industrial frameworks. We concentrate here to some engineering questions and their impact to mathematics. Our application projects stems from power generation and distribution with its challenges in shape optimization of turbomachinery, isolating devices and thermo-acoustic problems. We present actual progress in these areas based on improved and new mathematical algorithms. Checking for the original engineering design tasks we detect so far mathematically neglected overall goals of optimization like multi-objective optimization and design robustness. Here straight forward optimization may even give misleading answers. We demonstrate this pitfall in a case study targeting robust designs.

VIVETTE GIRAULT

### **Combining Domain Decomposition with Other Techniques: Fictitious Domain, Discontinuous Galerkin,...**

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 9:45

In this talk, I describe the numerical analysis of simple problems (Laplace equation, heat equation, Stokes system) where non-overlapping domain decomposition is combined with other methods. When standard finite-element methods are used, the matching condition in the domain decomposition is imposed weakly by a Lagrange multiplier. When discontinuous Galerkin methods are used, the matching condition is imposed explicitly by adding jump terms.

The results presented here are part of work in common with R. Glowinski, H. Lopez, B. Riviere and M. Wheeler.

RALF HIPTMAIR

### **Domain Decomposition Preconditioners for Edge Elements: A Survey**

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 14:45

Magnetoquasistatic models in electromagnetism give rise to linear  $H(\mathbf{curl}, \Omega)$ -elliptic variational problems. It is an important observation that those belong to a larger family of boundary value problems (BVP), which also includes the conventional  $H^1(\Omega)$ -elliptic variational problems. This suggests that numerical techniques like domain decomposition preconditioners that have been successfully applied to discretized second-order elliptic BVPs can be adapted to the  $H(\mathbf{curl}, \Omega)$ -elliptic setting. This is largely true, provided that  $\mathbf{curl}$ -conforming edge elements, which are the natural counterpart of Lagrangian finite elements, are used for Galerkin discretization.

As far as the analysis and design of subspace correction schemes is concerned, in the  $H(\mathbf{curl}, \Omega)$ -elliptic case new difficulties are encountered due to the presence of a large kernel of the  $\mathbf{curl}$ -operator. Obviously, it thwarts the strong ellipticity of the differential operators, and, thus, it requires a special treatment in theoretical investigations: the subspace splitting has to take it into account explicitly. The crucial tool is Helmholtz-type decompositions of  $H(\mathbf{curl}, \Omega)$  into irrotational vectorfields  $H^0$  and a complement space  $H^\perp$  with enhanced regularity properties. It is an exceptional feature of edge elements that they allow for discrete scalar potentials in the space of Lagrangian finite elements. Thus, the treatment of  $H^0$  can eventually rely on established techniques borrowed from  $H^1(\Omega)$ -conforming theory. On the other hand, the higher smoothness of functions in  $H^\perp$  paves the way to stable splittings of this component.

In this talk I am going to review how these ideas have been applied to lay the theoretical foundation of overlapping and non-overlapping domain decomposition preconditioners for discrete  $H(\mathbf{curl}, \Omega)$ -elliptic problems on conforming meshes. The overlapping case has been pioneered in [4,6] based on theoretical studies of multigrid schemes [3,1]. An extension to spectral edge elements is presented in [2]. Theory for a non-overlapping domain decomposition method has recently been presented [5]. By and large, asymptotic condition number estimates that parallel those for Lagrangian finite elements and second-order elliptic boundary value problems are obtained. However, robustness with respect to jumps in the coefficients has remained elusive even for the non-overlapping schemes.

- [1] D. ARNOLD, R. FALK, AND R. WINTHER, *Multigrid in  $H(\text{div})$  and  $H(\mathbf{curl})$* , Numer. Math., 85 (2000), pp. 175–195.
- [2] B. HIENZSCH, *Overlapping Schwarz preconditioners for spectral nédélec elements for a model problem in  $H(\mathbf{curl})$* , Technical Report TR2002-834, Department of Computer Science, Courant Institute of Mathematical Sciences, New York University, New York, 2002.
- [3] R. HIPTMAIR, *Multigrid method for Maxwell's equations*, SIAM J. Numer. Anal., 36 (1999), pp. 204–225.
- [4] R. HIPTMAIR AND A. TOSELLI, *Overlapping and multilevel Schwarz methods for vector valued elliptic problems in three dimensions.*, in Parallel Solution of Partial Differential Equations, P. Bjorstad and M. Luskin, eds., no. 120 in IMA Volumes in Mathematics and its Applications, Springer, Berlin, 1999, pp. 181–202.
- [5] Q. HU AND J. ZOU, *A non-overlapping domain decomposition method for Maxwell's equation in three dimensions*, SIAM J. Numer. Anal. (2003). To appear.
- [6] A. TOSELLI, *Overlapping Schwarz methods for Maxwell's equations in three dimensions*, Numer. Math., 86 (2000), pp. 733–752.

PATRICK JOLY

### **Domain Decomposition Approaches to Space-Time Mesh Refinement in Linear Wave Propagation**

**Location:** Lecture Room, **Time:** Thursday, 24 July, 9:45

In order to treat complex geometries or geometrical details in wave diffraction problems in time domain, a possible approach is to use local mesh refinements. Moreover, it is highly desirable to be able to treat non matching grids (this is even needed if one uses FDTD like schemes in each grid). A first idea consists in using only spatial refinement. However, when a uniform time step is used, it is the finest mesh that will impose the time step because of the stability condition. There are two drawbacks with this: first, the computational costs will be increased and, second, the ratio  $c\Delta t/h$  ( $h$  is the space step,  $\Delta t$  the time step and  $c$  the wave velocity) on the coarse grid will be much smaller than its optimal value, which will generate dispersion errors. A way to avoid these problems is thus to use a local time step  $\Delta t$ , related to  $h$  in order to keep the ratio  $c\Delta t/h$  constant. This solution however raised other practical and theoretical problems that are much more intricate than in the case of a simple spatial refinement, in particular in terms of stability.

The solutions suggested for Maxwell's equations in the electromagnetic literature are primarily based on interpolation techniques (in time and/or in space) especially designed to guarantee the consistency of the scheme at the coarse grid-finegrid interface. Unfortunately, the resulting schemes appear to be very difficult to analyze and may suffer from some instability phenomena.

Recently, we developed alternative solutions to these interpolation procedure. These methods, that we shall present for Maxwell's equations, are applicable to a large class of "abstract" variational problems including acoustics, elastodynamics, fluid-structure interaction. They are based on the principle of domain

decomposition techniques and consists essentially in ensuring a priori the numerical stability of the scheme via the conservation of a discrete energy our alternative solution. One can distinguish two kinds of methods, depending on the choice of the variational formulation inside each grid : one uses an interface Lagrange multiplier, the other does not. In my talk, I shall present these methods in a constructive way. We shall also give some elements about their convergence analysis.

TAKASHI KAKO

**Numerical Approximation of Dirichlet-to-Neumann Mapping and its Application to Voice Generation Problem**

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 14:00

In this talk, we treat the numerical method for the Helmholtz equation in unbounded region with simple cylindrical or spherical shape outside some bounded region. Numerical methods for the Helmholtz equation for this problem are based on the domain decomposition method and we treat the approximation of the artificial boundary condition given by the DtN mapping. We adopt the finite element method for this problem and apply the results to the computation of resonance phenomena related to the voice generation of human being.

AXEL KLAWONN, OLOF B. WIDLUND

**Dual-Primal FETI Methods for Elasticity**

**Location:** Lecture Room, **Time:** Monday, 21 July, 14:45

Iterative substructuring methods with Lagrange multipliers are considered for the elliptic system of linear elasticity. The algorithms belong to the family of dual-primal FETI methods introduced for elliptic problems in the plane by Farhat, Lesoinne, Le Tallec, Pierson, and Rixen and later extended to three dimensional problems by Farhat, Lesoinne, and Pierson.

In dual-primal FETI methods, certain primal continuity constraints on the displacement variables are enforced throughout the iteration, as in a primal iterative substructuring method, while Lagrange multipliers are used to handle the remaining, dual constraints which are fully satisfied only at the convergence of the iteration. The primal constraints are chosen such that certain local problems as well as a global problem are invertible.

In recent joint work with Maksymilian Dryja, an extended family of dual-primal FETI algorithms was designed for scalar elliptic problems in three dimensions and strong bounds for their condition numbers were developed. In particular, the bounds grow only polylogarithmically as a function of the dimension of the subdomain problems and they are independent of the number of subdomains and possible jumps in the coefficients across the interface.

Essential changes to the convergence theory and the algorithms are required for the elliptic system of linear elasticity. In this talk, recent results on the convergence theory for dual-primal FETI methods for linear elasticity will be presented. The bounds are of the same kind as for the scalar case. A special focus will be on the design of fast algorithms with as few primal constraints as possible.

ULRICH LANGER, OLAF STEINBACH

**Boundary and Finite Element Tearing and Interconnecting Methods.**

**Location:** Lecture Room, **Time:** Wednesday, 23 July, 9:45

We introduce the Boundary Element Tearing and Interconnecting (BETI) methods as boundary element counterparts of the well-established Finite Element Tearing and Interconnecting (FETI) methods.

Since Finite Element Methods (FEM) and the Boundary Element Methods (BEM) have certain complementary properties, it is sometimes very useful to couple these discretization techniques and to benefit from both worlds. This concerns not only the treatment of unbounded domains (BEM), but also the right handling of singularities (BEM), moving parts (BEM), source terms (FEM), non-linearities (FEM) etc. Thus, combining our BETI techniques with the FETI methods gives new, quite attractive tearing and interconnecting parallel solvers for large scale coupled boundary and finite element equations.

There is an unified framework for coupling, handling, and analyzing both methods. In particular, the FETI methods can benefit from preconditioning components constructed by boundary element techniques. This is especially true for sparse versions of the boundary element method which are also considered in the talk. Sparse approximation techniques such as the multipole method avoid fully populated matrices arising in classical boundary element methods.

There are many application of coupled boundary and finite element tearing and interconnecting methods

including potential equations with large coefficient jumps, elasticity problems in structural mechanics, Maxwell equations in electromagnetics etc. Our numerical results confirm the efficiency and the robustness predicted by our analysis.

\* This work has been supported by the Austrian Science Fund ‘Fonds zur Förderung der wissenschaftlichen Forschung (FWF)’ under grant SFB F013 ‘Numerical and Symbolic Scientific Computing’ and the German Research Foundation ‘Deutsche Forschungsgemeinschaft (DFG)’ under the grant SFB 404 ‘Multifield Problems in Continuum Mechanics’

CHRISTOF SCHÜTTE

### **Domain Decomposition for Metastable Dynamical Systems**

**Location:** Lecture Room, **Time:** Friday, 25 July, 11:15

Many complex systems in climate dynamics, materials science, or biomolecular modelling exhibit metastability: the state space of the system can be decomposed into some dominant metastable sets. The exit of the dynamics from these sets is a rare event: they represent the longest temporal scales of the system, while the dynamics visits all parts of the interior of each sets on significantly shorter time spans. It has recently been demonstrated that such metastable decompositions can be computed by means of the so-called transfer operator of the nonlinear dynamical system under consideration. In this approach the metastable sets are deeply related to dominant eigenmodes of the transfer operator, i.e., can be identified via a linear problem. The talk will introduce this approach, discuss the relation to domain decomposition, and demonstrate the application of domain decomposition techniques to transfer operators that are generated by partial differential operators.

ALBERTO VALLI

### **The Swiss Carpet Preconditioner: A Simple Parallel Preconditioner of Dirichlet-Neumann Type**

**Location:** Lecture Room, **Time:** Wednesday, 23 July, 9:00

We present a new type of preconditioner, that is based on a vertex-oriented decomposition of the computational domain  $\Omega$ . This means that we consider a decomposition of  $\Omega$  given by  $\Omega = \Omega_1 \cup \Omega_2$ , where  $\Omega_1$  is the union of  $M$  disjoint subdomains, and  $\Omega_2$  is a thin connected subdomain, composed by only one layer of elements of width  $h$ , and having a common boundary with all the  $M$  connected components of  $\Omega_1$ .

A Dirichlet-Neumann preconditioner is proposed, where the Neumann step is held in  $\Omega_2$  (and the Dirichlet step is performed in parallel in all the  $M$  connected components of  $\Omega_1$ ).

The condition number of the preconditioned Schur matrix is proved to be of order  $H/h$ , where  $H$  is basically the diameter of all the connected components of  $\Omega_1$ . Though this asymptotic behaviour is not optimal, the preconditioning step is quite cheap, as the domain  $\Omega_2$  is very thin: therefore, the global computational cost of this algorithm is shown to be competitive with the cost of more complex optimal preconditioner, based on a similar multi-domain decomposition (e.g., the balancing Neumann–Neumann preconditioner with coarse grid correction). The performances of the proposed domain decomposition algorithm are illustrated by some numerical results for the Laplace operator and for more general (symmetric) second order elliptic equations; additional results for more complex partial differential equations in fluid dynamics are also presented. These results have been obtained jointly with A. Quarteroni and M. Sala.

## 2 Abstracts of Minisymposia Talks

### MS01 Collaborating Subdomains for Multi-Scale Multi-Physics Modelling

Organized by: PETER CHOW, CHOI-HONG LAI

Many physical problems such as solidification process, aeroacoustics, groundwater simulation, and electronic packaging involve the localisation of the physics and their transient variability across the geometric domain. While it is impossible to address all of the problems mentioned, it is important to understand various options of coupling subdomains in relation to the multi-physics problems. In particular it is important to address how subdomains may be collaborated in achieving a fast global solution to the original problem based on distributed computing.

The minisymposium will discuss the collaboration between subdomains in electronic packaging which is a multi-scale multi-physics problem. The collaboration between subdomains is done using classical non-overlapping subdomains and the corresponding acceleration techniques. The main aim is to reduce total engineer's modelling time in an engineering process. Other topics involved in this minisymposium include coupling of various physical properties in many fluid dynamics involving nonlinear Navier-Stokes solvers. A list of tentative speakers and areas of the talks is given below.

XIAO-CHUAN CAI, FENG-NAN HWANG

#### ASPIN for Incompressible Navier-Stokes Equations

**Location:** Room 005, **Time:** Thursday, 24 July, 16:00

A nonlinear additive Schwarz preconditioned inexact Newton method (ASPIN) was introduced recently by Cai and Keyes for solving large sparse nonlinear systems of equations obtained from the discretization of nonlinear partial differential equations.

In this talk, we discuss some recent development of ASPIN for solving the steady state incompressible Navier-Stokes equations in the velocity-pressure formulation. The sparse nonlinear system is obtained by using a  $Q_1 - Q_1$  Galerkin least squares finite element discretization on two dimensional unstructured meshes. The key idea of ASPIN is that we find the solution of the original system  $F(u) = 0$  by solving a nonlinearly preconditioned system  $G(u) = 0$  that has the same solution

as the original system, but with more balanced nonlinearities. The nonlinear preconditioners are based on the solution of the Navier-Stokes equations defined on the overlapping subdomains with some proper boundary conditions. In this talk we present some numerical results obtained on parallel computers for two challenging CFD benchmark problems: a driven cavity flow problem and a back facing step problem.

We compare our approach with some inexact Newton method with different choices of forcing terms. The numerical results show that ASPIN is more robust than the traditional inexact Newton method for high Reynolds number flows as well as for large number of processors.

TOBIAS KNOPP, GERT LUBE, RALF GRITZKI, MARKUS ROESLER

#### Iterative Substructuring Methods for Indoor Air Flow Simulation

**Location:** Room 005, **Time:** Thursday, 24 July, 16:25

The numerical simulation of turbulent indoor air flows is performed using iterative substructuring methods. We consider the  $k/\epsilon$  model based on the nonstationary, incompressible Navier-Stokes problem [4]. The resolution of turbulent layers in the near-wall region is avoided using an improved wall function concept [3, 2].

For quasi-stationary flows we apply the semidiscretization in time with the simplest discontinuous Galerkin method. After proper linearization in each time step, the iterative process requires the fast solution of linearized Navier-Stokes problems and of advection-diffusion-reaction problems. These subproblems are discretized using stabilized FEM together with a shock-capturing technique for the transport equations for the temperature and turbulence quantities  $k$  and  $\epsilon$ , cf. [1, 5]. For the linearized problems we apply an iterative substructuring technique which couples the subdomain problems via Robin-type transmission conditions. Proper chosen parameter functions of these transmission conditions allow a considerable acceleration of this well-posed convergent method [6]. The applicability of

the approach is shown for some benchmark problems, including comparison with experimental data [7], and realistic ventilation problems [3, 2].

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- [3] KNOPP, T.: *Finite element simulation of buoyancy-driven turbulent flows*, Ph.D. Thesis, University of Göttingen, 2003
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EDWARD SWIM, PADMANABHAN SESHAIYER

### **Fluid-Structure Interaction with Nonconforming Finite Elements**

**Location:** Room 005, **Time:** Thursday, 24 July, 16:50

We investigate the interaction between a viscous incompressible fluid and a structure whose deformation defines the interface between the two. Direct numerical solution of the highly nonlinear equations governing even the most simplified models of fluid-structure interaction requires that both the flow field and the domain shape be determined as part of the solution since neither is known a priori. To accomplish this, previous algorithms have decoupled the solid and fluid mechanics, solving for each separately and converging iteratively to a solution which satisfies both.

In this talk, we describe a non-conforming hp finite element method which solves the problem simultaneously on each subdomain. Mortar finite elements are used to construct approximate solutions of the corresponding partial differential equations on the fluid and structure domains as well as the flexible boundary between them. The local approximation within each subdomain is designed using stable hp finite elements, where both mesh refinement and degree enhancement are combined to increase accuracy.

CHOI-HONG LAI, JIANWEN CAO

### **Some Effective Techniques of Nonlinear Solvers for Black-Oil Modelling**

**Location:** Room 005, **Time:** Thursday, 24 July, 17:15

In this paper, we discuss how to improve the performance of solving the set of nonlinear equations originated from a fully implicit scheme for multiphase flow in porous media on Beuwolf PCs cluster.

A hybrid nonlinear method is suggested which combines global convergent inexact newton method and rank-2 correction BFGS quasi-newton algorithm.

In order to improve the computational efficiency of the nonlinear solver, parallel strategy is adopted based on distributed memory HPC by means of using DDM method, and each newtonian iteration step uses the so-

called inner-outer nested algorithm with a proper preconditioner. As one of the most important part, the preconditioner uses multipurpose oblique projection correction strategy which based on iterative algorithms to solve a small linear system of  $(P^T AP)z = r$  involved. This preconditioner involves several preconditioning such as AMG, Relaxed ILU, upscaling, DDM, Constraint Residual Precondition etc.

Numerical results display the performance and scalability on 4 - 128 CPUs Beuwolf Linux Cluster.

MARTIJN ANTHONISSEN

### **Local Defect Correction Techniques Applied to a Combustion Problem**

**Location:** Room 005, **Time:** Friday, 25 July, 9:00

We consider a discretization method for elliptic boundary value problems introduced by Hackbusch. In this technique, called the *local defect correction* (LDC) method, the discretization on the *composite grid* is based on a combination of standard discretizations on several uniform grids with different grid sizes that cover different parts of the domain. At least one grid, the coarse grid, should cover the entire domain, and its grid size should be chosen in agreement with the relatively smooth behavior of the solution outside the high activity areas. Apart from this *global coarse grid*, one or several *local fine grids* are used which are also uniform. Each of the local grids covers only a (small) part of the domain and contains a high activity region. The grid sizes of the local grids are chosen in agreement with the behavior of the continuous solution in that part of the domain.

The LDC method is an iterative process: a basic global discretization is improved by local discretizations de-

finied in subdomains. The update of the coarse grid solution is achieved by adding a defect correction term to the right hand side of the coarse grid problem.

We extend the LDC algorithm by successively adding adaptivity, multilevel refinement, domain decomposition and regridding. The final result will be a technique for discretizing and solving boundary value problems on a composite grid found by adaptive grid refinement given a code for solving a boundary value problem on a tensor-product grid in a rectangular domain.

We apply our proposed adaptive multilevel LDC algorithm to a Bunsen flame problem.

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MARC GARBEY, OMAR DIA, Y. JOBIC

### Heterogeneous Domain Decomposition for Boundary Layer Problems

**Location:** Room 005, **Time:** Friday, 25 July, 9:25

In this paper, we propose an heterogeneous domain decomposition solver for Navier-Stokes flow in pipe. The domain of computation is decomposed into a regular domain with Cartesian grids and several boundary layer domains that fit the boundaries with local orthogonal meshes. The domain decomposition is motivated by the physic and/or singular perturbation analysis for large high Reynolds numbers. We have then very different type of meshes between the regular domain and the boundary layer domain. The numerical efficiency of the domain decomposition is the consequences of few factors such as:

1. each sub-domain can use a fast solver that takes full advantage of either the stretching of the mesh

in one space direction for boundary layer domains, or the regular data structure with Cartesian grids used for the main part of the flow.

2. simplicity of the implementation, grid generation, and memory allocation due to the use of the additive Schwarz method for the iteration process between overlapping non matching grids.
3. fast convergence of the domain decomposition algorithm thanks to the use of an acceleration procedure to speed up the convergence of the Schwarz method.

We will discuss the numerical accuracy and robustness of this heterogeneous domain decomposition technique.

ARTHUR E. P. VELDMAN, E.G.M. COENEN

### Interaction Laws in Viscous-Inviscid Coupling

**Location:** Room 005, **Time:** Friday, 25 July, 9:50

A classical example of domain decomposition is found in aerodynamic boundary-layer theory. About one century ago Prandtl proposed to split an aerodynamic flow field at high Reynolds number into a thin viscous boundary layer and an inviscid external-flow region. The two regions are typically coupled by exchanging information on pressure and boundary-layer displacement effects (in DD terms: Dirichlet-Neuman coupling).

Following Prandtl, the boundary-layer equations are solved with pressure prescribed from the inviscid-flow solution. However, as discussed already at length by Goldstein in 1948, this concept fails as soon as flow separation occurs. Only in the late 60-ies and 70-ies the first understanding came up on why this failure occurs. Subsequently, this has led to alternative approaches, such

as the semi-inverse method and the quasi-simultaneous method, that do not suffer from Goldstein's singularity. In the presented research, the quasi-simultaneous method has been analysed and brought back to its essentials. The result is a method that is very close to Prandtl's original concept – any existing Prandtl boundary-layer code can be modified with modest effort into our approach – yet it has no problem to cope with flow separation. The ideas behind the method are easily generalized to other instances of domain decomposition.

Results on convergence behaviour and numerical robustness of the method will be shown for two- and three-dimensional flow past indented plates and airfoils.

PETER CHOW, CHOI-HONG LAI

### Electronic Packaging and Reduction in Modelling Time Using Domain Decomposition

**Location:** Room 005, **Time:** Friday, 25 July, 10:15

'Small' is exquisite and cool to the consumers of electronic products, but it throws out enormous technical challenges that need to be overcome by designers and engineers. Elements such as health and safety compliance, power and heat management, and usability are

commonly top on the list of issues. The primary technical challenges [chow] are: 1) High density of components leads to an increase of model complexity that needs to address not just the processes of thermal cooling, electromagnetic radiation and solid structure but the in-

teraction and interference between the processes, i.e. Multi-Physics. 2) Creating highly intricate and highly detailed geometry and mesh models with parts of dissimilar scales, such as the entire electronic components in a laptop computer and exposure analysis of electronic devices on an entire human body and tissues. 3) Size of computational demands million plus cells/elements models are common in industrial simulations and models with tens of millions of elements are appearing, but still the resolution is not enough in applications such as electromagnetic and thermal cooling designs. 4) The market demands and speed to market adds to the ever-increasing pressures of shortening the modelling time and number of design cycles.

Whilst domain decomposition has been successfully applied to areas such as parallel solvers and preconditioners, coupling of different numerical methods and physical models, it has not been considered for the entire simulation process chain in order to achieve a comprehensive

reduction in modelling time. An early concept of using domain decomposition methods in the reduction of modelling time can be found in [1]. This paper gives a rigorous approach of the concept and uses the framework of defect equations in coupling [2]. Algorithms developed in this paper concentrate on problems with geometrical multi-scale at the macroscopic mathematical models. Numerical experiments, including mono-phase and multi-phase, are examined with efficiency of the algorithm being linked to the overall modelling time.

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## MS02 Discretization Techniques and Algorithms for Multibody Contact Problems

Organized by: BARBARA WOHLMUTH, TAOUFIK SASSI

Domain decomposition techniques on non-matching grids provide a flexible and efficient tool for the numerical approximation of non linear multibody contact problems with friction. Generalized mortar methods based on Lagrange multipliers can be used to discretize contact problems. The resulting variational inequalities are defined on a discrete convex set. The non-penetration condition is realized in terms of suitable weak integral conditions. Here, a priori error estimates will be considered as well as iterative solvers based on domain decomposition techniques. Different concepts as monotone multigrid methods, non linear Dirichlet–Neumann algorithms and FETI techniques will be discussed and analyzed.

ZDENEK DOSTAL, D. HORÁK

### Optimal Penalty and Scalable FETI Based Algorithms for Numerical Solution of Variational Inequalities

**Location:** Room 005, **Time:** Monday, 21 July, 16:00

We shall first briefly review the FETI methodology proposed by C. Farhat and F.-X. Roux adapted for solution of variational inequalities. Using the classical results by Mandel and Tezaur related to the scalability of FETI for solution of linear elliptic boundary value problems, we shall show how to exploit their results to reduce the discretized variational inequality to the bound and equality constrained quadratic problem with the condition number of the Hessian independent on the discretization parameter  $h$ . Then we shall show that such problems may be solved efficiently by recently proposed algorithms for solution of quadratic programming problems with the rate of convergence in terms of the spectral condition of the Hessian of the cost function. In particular, it follows that if we impose the equality constraints by the

penalty method, we can solve our penalized problem to the prescribed relative precision in a number of iterations that does not depend on  $h$ . Since we managed to prove that a prescribed bound on the relative feasibility error of the solution may be achieved with the value of the penalty parameter that does not depend on the discretization parameter  $h$ , we conclude that these results may be used to develop scalable algorithms for numerical solution of elliptic variational inequalities. We give results of numerical experiments with parallel solution of both coercive and semicoercive model problems discretized by up to more than eight million of nodal variables to demonstrate numerically optimality of the penalty and scalability of the algorithm presented.

TAOUFIK SASSI, JAROSLAV HASLINGER

### A Mixed Finite Element Approximation of 3D Contact Problems with Given Friction: Approximation and the Numerical Realization

**Location:** Room 005, **Time:** Monday, 21 July, 16:25

This contribution deals with a mixed variational formulation of 3D contact problems with the simplest model involving friction. This formulation is based on a dualization of the set of admissible displacements and the regularization of the nondifferentiable term. Displace-

ments are approximated by piecewise linear elements while the respective dual variables by piecewise constant functions on a dual partition of the contact zone. The rate of convergence is established provided that the solution is smooth enough. The numerical realization of

such problems will be discussed and results of several model examples will be shown.

ROLF KRAUSE, O. SANDER

### **Fast Solving of Contact Problems on Complicated Geometries**

**Location:** Room 005, **Time:** Monday, 21 July, 16:50

The efficient solution of contact problems involving friction in three space dimension is a non-trivial task. The mathematical modeling of the non-penetration condition and of the frictional effects at the interface gives rise to a non-linear and non-differentiable energy functional which has to be minimized. In our talk, we present a non-linear monotone multigrid method, by means of which one-sided frictional contact problems can be solved with optimal complexity. For the case of two sided contact problems, the information transfer at the interface is realized in terms of Mortar methods. Our method uses a combination of dual Basis functions for the Lagrange Multiplier space and a suitable transformation of the arising discrete system based on work by Wohlmuth and Krause. Thus the proposed method is also applicable in case of non-matching triangulations

at the interface.

We also apply our monotone method to problems on complicated geometries. To this end, we combine the multigrid method with boundary parametrizations, by means of which the possibly complex geometry of the computational domain can be successively regained starting from a simple and coarse initial mesh. We also show how these parametrization techniques can be applied in order to realize the information transfer between two bodies coming into contact by means of Mortar methods in the case of curvilinear boundaries. Numerical results in three space dimensions illustrate the efficiency and accuracy of our method. In particular, we consider the case of curvilinear contact boundaries in three space dimensions.

TAOUFIK SASSI, LAURENT BAILLET, GUY BAYADA, JALILA SABIL

### **Domain Decomposition Algorithms for Contact Problems**

**Location:** Room 005, **Time:** Monday, 21 July, 17:15

We propose and we prove the convergence of a Neumann-Dirichlet and a Neumann-Neumann algorithms in order to approximate a Signorini problem between two elastic bodies. The Neumann-Neumann algorithm is a parallel one, in which we have to solve a

Dirichlet problem then a Neumann one simultaneously on each domain. Primary interest of these algorithms is to retain the natural interface between the two bodies as a numerical interface for the domain decomposition.

PANAYOT VASSILEVSKI

### **Monotone Element Agglomeration AMG<sub>e</sub> for Contact Problems**

**Location:** Room 005, **Time:** Monday, 21 July, 17:40

In this talk we will present an application of the recently developed sparse matrix element topology [1] to create agglomerated elements coarsened away from a (contact) boundary. Based on thus constructed hierarchy of agglomerated elements one then uses the AMG<sub>e</sub> in the form proposed in [2] to construct coarse finite element spaces to be used in multilevel schemes. A main feature is that the contact boundary is present (not coarsened) on all coarse levels, hence the constraints are straightforward to satisfy. We consider two multilevel schemes – a classical FAS and a subspace minimization scheme. Both, when combined with monotone smoothers, such as the projected Gauss–Seidel, lead to overall monotone multilevel algorithms. Some numerical illustration will be presented as well.

Major part of the talk is based on a joint work with Ana

Iontcheva [3].

This work was performed under the auspices of the U. S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract W-7405-Eng-48.

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## **MS03 Recent Developments for Schwarz Methods**

Organized by: MARTIN J. GANDER

The Schwarz methods are the oldest domain decomposition methods, but over the last decade several interesting developments happened. This mini-symposium gives an overview of recent results and new directions of research for Schwarz methods. It includes Schwarz methods with new transmission conditions, optimized Schwarz methods, restricted Schwarz methods, Schwarz waveform relaxation methods for evolution and delay differential equations and Schwarz methods for systems of equations.

MARTIN J. GANDER, EVRIDIKI EFSTATHIOU

**RAS: Understanding Restricted Additive Schwarz**

**Location:** Room 049, **Time:** Tuesday, 22 July, 16:00

Recently a variant of the additive Schwarz (AS) preconditioner, the restricted additive Schwarz (RAS) preconditioner has been introduced, and numerical experiments showed that RAS converges faster and requires less communication than AS. We show in this talk how RAS, which is defined at the matrix level, can be in-

terpreted as an iteration at the continuous level of the partial differential equation. This interpretation reveals why RAS converges faster than classical AS and shows that RAS coincides with a parallel version of the alternating Schwarz method introduced by Lions.

GERT LUBE, TOBIAS KNOPP, GERD RAPIN

**Acceleration of an Iterative Substructuring Method for Singularly Perturbed Elliptic Problems**

**Location:** Room 049, **Time:** Tuesday, 22 July, 16:25

Iterative substructuring methods with transmission conditions of Robin type allow the parallel solution of elliptic equations, e.g. of advection-diffusion problems [1]. The implementation of such methods is relatively easy but the convergence speed is basically influenced by a properly optimized transmission condition, for a nice overview see [2]. For symmetric elliptic problems, it is shown in [2] that the simplest variant of the non-overlapping method with Robin transmission condition can be dramatically accelerated if a critical parameter of the transmission condition is adapted in a cyclic way (or even in a multilevel version).

First we reconsider the method of [2] for the singularly perturbed symmetric case. The optimization of the transmission condition is more complicated for advection-diffusion problems. In [3] we extracted from an a-posteriori estimate some information on a proper design of the critical parameter of the Robin transmis-

sion condition. The method provides a fast reduction of the interface error in the first steps; but, at a certain level (related to the discretization error), the convergence slows down. Inspired by the approach in [2], we investigate whether a proper adaptation of the critical parameter allows to accelerate the convergence speed.

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LUCA GERARDO-GIORDA

**Modified Schwarz Algorithms without Overlap for the Harmonic Maxwell's System**

**Location:** Room 049, **Time:** Tuesday, 22 July, 16:50

The earliest non-overlapping Schwarz algorithm for the time-harmonic Maxwell's system was introduced by B. Després *et al.* in [1]. We show that the original method, based on an interface transmission condition of radiation type, is convergent only for propagative modes. We then consider a modified interface condition, where we add a tangential second order operator of **curl-curl** type to the original Després' condition, and we prove that such algorithm is convergent for all modes. The resulting discrete system is then solved by means of a Krylov subspace method, using the Schwarz algorithm

as a preconditioner, and some numerical results are also addressed.

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FRÉDÉRIC NATAF, I. FAILLE, L. SAAS, F. WILLIEN

**Finite Volume Methods on Non-Matching Grids with Arbitrary Interface Conditions**

**Location:** Room 049, **Time:** Tuesday, 22 July, 17:15

Many works have been carried out on conforming grids either for finite element methods (see e.g. [5]), or mixed finite element methods (see e.g. [2]) and to a smaller extent for finite volume methods (see e.g. [6]).

Another important related issue is the associated linear solver which is very often based on domain decomposition methods. It is then very convenient to be able

to use not only Dirichlet or Neumann boundary at the subdomain interfaces but arbitrary conditions as well (e.g. Robin boundary conditions), see [8], [7] or [2].

So, there is a need for a discretization method which allows for non matching grids and arbitrary boundary conditions at the subdomain interfaces without losing the accuracy of the original discretization scheme. This

question has been addressed in [4] and [1]. The main drawback of these papers is that the discrete solution depends on the parameters of the boundary condition used in the domain decomposition solver at the interface between the subdomains.

The main novelty of the submitted work is to introduce and analyse a method which enables to decouple in an efficient way the problem of the discretization scheme on non-matching grids and the use of arbitrary conditions at the subdomain interfaces. The analysis is valid for a convection-diffusion equation. Numerical comparisons with other discretization schemes (e.g. the standard TPFA (two point flux approximation) method standard in the petroleum community) will be shown.

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VICTORITA DOLEAN, FRÉDÉRIC NATAF

### **A Non-Overlapping Schwarz Type Algorithm for the Resolution of the Euler Equations**

**Location:** Room 049, **Time:** Wednesday, 23 July, 11:00

In a previous paper [1] we reported on numerical experiments with a non-overlapping domain decomposition method that has been specifically designed for the calculation of steady compressible inviscid flows governed by the two-dimensional Euler equations. We have also studied this method from the theoretical standpoint [3] and by using different type of interface conditions [2]. The proposed method relies on the formulation of an additive Schwarz algorithm which is based on interface conditions that are Dirichlet conditions for the characteristic variables corresponding to incoming waves (often referred as *natural* or *classical* interface conditions), thus taking into account the hyperbolic nature of the Euler equations. The convergence of the additive Schwarz algorithm has been analyzed in two- and three-dimensional cases by considering the linearized equations and applying a Fourier analysis. We found out that besides the fact that the algorithm is always convergent, surprisingly, there exists flow conditions for which the asymptotic convergence rate is equal to zero. Moreover, this behaviour is independent of the space dimension.

For the discretized mesh we use an non-overlapping element-based partitioning where the elements are the finite volume cells. This corresponds to a “minimum” overlap partition of the set of all nodes of the mesh as seen in [4]. Therefore even if we considered a non-overlapping decomposition from the continuous point of view, by studying the discrete counterpart of the “non-overlapping” additive Schwarz type algorithm considered before on a regular geometry we find out that the

result corresponds rather to an “overlapping” expression of the continuous problem. This proves that the decomposition as well as the discrete scheme used impose a naturally “overlapping” discrete formulation of the continuous non-overlapping algorithm. In this work we will build a real non-overlapping discrete formulation of the algorithm which will lead to local well posed problems and will improve the convergence for certain flow regimes.

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CHRISTIAN ROHDE, MARTIN J. GANDER

### Overlapping Schwarz Waveform Relaxation for Convection Dominated Nonlinear Conservation Laws

**Location:** Room 049, **Time:** Wednesday, 23 July, 11:25

Schwarz waveform relaxation methods have been successfully applied to solve a wide class of linear and nonlinear evolution problems. On the theoretical side the algorithm has been analyzed for, e.g., the heat equation or the wave equation including even nonlinear source terms. Less is known for problems with strong nonlinearities. In the talk we consider an instance of this class: convection dominated problems with nonlinear fluxes. New phenomena for these problems are genuinely nonlinear waves like viscous shocks and rarefactions. First we present a superlinear convergence result

on bounded time intervals. In particular we observe that the convergence rate gets better with decreasing viscosity parameter. This is in agreement with the fact that the algorithm converges in a finite number of steps for the hyperbolic limit problem. Surprisingly the situation changes for a corresponding steady state equation: The (linear) convergence rate becomes arbitrarily bad if viscosity tends to 0. We will show that this striking discrepancy is caused by nonlinear fluxes. The results are illustrated with numerical experiments.

VÉRONIQUE MARTIN

### Domain Decomposition Methods for Unsteady Convection Diffusion Equation

**Location:** Room 049, **Time:** Wednesday, 23 July, 11:50

Classical Schwarz methods have been first applied to stationary problems and for overlapping subdomains. The first idea was developed by Schwarz who introduced an alternating algorithm with Dirichlet transmission conditions at the interface. More recently, new types of transmission conditions have been introduced which apply also to non-overlapping problems: the simplest conditions are of Robin type *i.e.* they involve normal derivatives. And it is possible to choose these transmission conditions such as to minimize the convergence rate. This strategy proved to be very useful for many steady problems as for instance convection diffu-

sion, Euler or Helmholtz equations.

In this talk we are interested in solving time dependent equations, formulating algorithms directly for the original problem without first discretizing in time, contrary to the classical approach.

We apply this method to the convection diffusion equation in two dimensions and we write an algorithm with optimized transmission conditions. We prove the well-posedness and the convergence of this algorithm and show numerical results which illustrate the efficiency of the method.

GERD RAPIN, GERT LUBE

### A Stabilized Three Field Domain Decomposition Formulation for Advection-Diffusion Problems and its Iterative Decoupling

**Location:** Room 049, **Time:** Wednesday, 23 July, 12:15

Based on the ideas of [1] a stabilized three field formulation for advection-diffusion problems is presented. Due to the stabilization it is possible to choose the discrete function spaces almost arbitrary. Stability and an optimal a priori estimate are proven for the scheme. By minimizing the right hand side of the a priori estimate a suitable choice of the stabilization parameter is given. Numerical tests confirm our results.

Following the line of [3] the scheme is decoupled by an alternating Schwarz method for nonmatching grids. The resulting iterative scheme is similar to the algorithms in [2,4,5]. In these papers the key problem is the optimal choice of some acceleration parameters. Here the parameters are determined by the a priori estimate in a very intrinsic way. Finally some numerical experiments corresponding to the choice of the acceleration parameters are presented.

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- [5] C. Japhet, F. Nataf, and F. Rogier. *The Optimized Order 2 Method. Application to Convection-Diffusion Problems.* Future Generation Computer Systems., Vol. 18, Elsevier Science, 2000. no. 1.

FRANÇOIS-XAVIER ROUX

### Approximation of Optimal Interface Boundary Conditions for Two-Lagrange Multiplier FETI Method

**Location:** Room 049, **Time:** Friday, 25 July, 9:00

Interface boundary conditions are the key ingredient to design efficient domain decomposition methods. Without a global preconditioner, convergence cannot be obtained for any method in a number of iterations less than the number of subdomains minus one in the case of a one-way splitting. For the two-Lagrange multiplier FETI method, this optimal convergence can be obtained with generalized Robin type boundary conditions associated with an operator equal to the Schur complement of the outer domain.

In practice however this optimal condition cannot be implemented since the Schur complement is too expensive to compute exactly. Furthermore, the Schur complement is a dense matrix on each interface and even if it were computed, using it would create a very large increase of the bandwidth of the local stiffness matrix. Hence the issue is how to build a sparse approximation of the Schur complement that is not expensive

to compute and that gives good convergence for the two-Lagrange multiplier FETI method. Different approaches based on approximate factorization or inverse computation of the inner stiffness matrix have been tested. None of them is actually general and robust.

A new approach based on the computation of the exact Schur complement for a small patch around each interface node appears to be a very efficient method for designing approximations of the complete Schur complement that give very good convergence for many different kinds of problems. Furthermore this approach can be easily implemented without any other information than the local stiffness matrix in each subdomain.

In this paper, the results obtained with this new method will be presented for various applications in structural analysis for both static and time harmonic kinds of problems and in acoustics.

STEFAN VANDEWALLE, MARTIN J. GANDER

**Optimized Overlapping Schwarz Methods for Parabolic PDEs with Time-Delay.**

**Location:** Room 049, **Time:** Friday, 25 July, 9:25

Parabolic delay partial differential equations model physical systems for which the evolution does not only depend on the present state of the system but also on the past history. Such models are found, for example, in population dynamics and epidemiology, where the delay is due to a gestation or maturation period, or in numerical control, where the delay arises from the processing in the controller feedback loop.

In the first part of the talk we will study the analytical properties of the solutions of parabolic delay PDEs. Two model problems will be considered in particular: the heat equation with a fixed delay term, and the heat equation with a distributed delay in the form of an in-

tegral over the past. It will be shown that the dynamics of delay PDEs is fundamentally different from that of regular time-dependent PDEs without time delay.

Next, we will study the numerical solution of the above model problems with overlapping Schwarz methods. The considered methods are of waveform relaxation type: they compute the local solution in each subdomain over many time-levels before exchanging boundary information to neighbouring subdomains. We analyse the effect of the overlap width and we derive optimized transmission boundary conditions of Robin type. The theoretical results and convergence estimates are verified through some numerical experiments.

LAHCEN LAAYOUNI, MARTIN J. GANDER, S. LOISEL

**Non-Overlapping Optimized Domain Decomposition Methods in Spherical Coordinates**

**Location:** Room 049, **Time:** Friday, 25 July, 9:50

We investigate the performance of non-overlapping domain decomposition methods for solving Poisson equations on the sphere. This equation arises in a global weather model to improve the precipitation forecast over Canada. We consider two different types of non-overlapping algorithms: the Dirichlet-Neumann algorithm and an optimized Schwarz method. We show that both algorithms applied to a simple two subdo-

main decomposition converge in two iterations. This is however not true for a more general subdomain decomposition. Using numerical experiments, we compare the performance of the Dirichlet-Neumann algorithm with the optimal relaxation parameter found for the two subdomain problem, and the optimized Schwarz method with Robin transmission conditions optimized for the two subdomain problem.

CAROLINE JAPHET, YVON MADAY, FRÉDÉRIC NATAF

**A New Cement to Glue Nonconforming Grids with Robin Interface Conditions: The Finite Element Case**

**Location:** Room 049, **Time:** Friday, 25 July, 10:15

We design and analyse a new nonconforming domain decomposition method based on Schwarz type algorithms that allows for an extension to optimized interface conditions on nonconforming grids. Such interface conditions are developed in [2]. When the grids are conforming, the implementation of such interface conditions on the discretized problem has already been considered in

[2].

On the other hand, using nonconforming grids allows for parallel generation of meshes, for local adaptive meshes and fast and independent solvers. The mortar method, first introduced in [1], enables the use of nonconforming grids.

It is also well suited to the use of “Dirichlet-Neumann”

or “Neumann-Neumann” preconditioned conjugate gradient method applied to the Schur complement matrix. But the mortar method can’t be used easily with optimized interface conditions in the framework of Schwarz type methods.

We design and study a nonconforming domain decomposition method which allows for the use of Robin interface conditions for Schwarz type methods. We consider a diffusion equation discretized by a finite element method. The nonconforming domain decomposition method is proved to be well posed, and the iterative solver to converge. The error analysis is performed.

## MS04 Domain Decomposition Methods for Wave Propagation in Unbounded Media

Organized by: XAVIER ANTOINE, FRANK SCHMIDT

The minisymposium covers recent results on theoretical and numerical aspects of wave propagation in unbounded media. Most methods both for the solution of time-dependent and time-harmonic scattering problems rely on a decomposition into an exterior and an interior problem. The minisymposium will focus on the numerical solution of such coupled interior/exterior problems and provide an overview of ideas for the solution of exterior acoustic, electromagnetic, and quantum mechanical problems.

THORSTEN HOHAGE

### New Transparent Boundary Conditions for Coupled Interior/Exterior Wave Propagation Problems

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 16:00

We consider Helmholtz-type equations with inhomogeneous exterior domains, e.g. exterior domains containing wave guides, which arise in the simulation of photonic devices. Usually, Sommerfeld’s radiation condition is not valid for such problems. We present an alternative radiation condition called *pole condition*, which has been suggested by Frank Schmidt. Roughly speaking, it says that the Laplace transform of a radiating solution along a family of rays tending to infinity has a holomorphic extension to the lower half of the complex plane. To justify the validity of this condition, we show that it is equivalent to Sommerfeld’s radiation condition for bounded obstacle scattering problems. Moreover, we show that for scattering problems by rough surfaces and for wave guide problems, the pole condition is also equivalent to the standard radiation conditions used in these fields.

For the numerical solution of scattering problems, the radiation condition has to be replaced by a transparent boundary condition on the artificial boundary of the computational domain. We describe a construction of exact transparent boundary conditions based on the pole condition which does not rely on the explicit knowl-

Then we present numerical results that illustrate the method.

- [1] C. Bernardi, Y. Maday and A. Patera, *A new non-conforming approach to domain decomposition: the mortar element method*, Nonlinear Partial Differential Equations and their Applications, eds H. Brezis and J.L. Lions (Pitman, 1989).
- [2] C. Japhet, F. Nataf and F. Rogier, *The Optimized Order 2 Method. Application to convection-diffusion problems*, Future Generation Computer Systems, 18(1) (2001), pp. 17-30, Elsevier Science.

edge of a fundamental solution or a series representation of the solution. There exists an explicit formula expressing the exterior solution in a stable way in terms of quantities defined in the Laplace domain. This provides an efficient numerical method for the evaluation of the exterior solution and distinguishes the proposed method from other methods, e.g. the Perfectly Matched Layer (PML) method. The total computational cost is typically dominated by the finite element solution of the interior problem.

- [1] T. Hohage, F. Schmidt, L. Zschiedrich: *Solving time-harmonic scattering problems based on the pole condition. I:Theory*. SIAM J. Math. Anal., to appear.
- [2] T. Hohage, F. Schmidt, L. Zschiedrich: *Solving time-harmonic scattering problems based on the pole condition. II:Convergence of the PML method*. SIAM J. Math. Anal., to appear.
- [3] T. Hohage, F. Schmidt, L. Zschiedrich: *A new method for the solution of scattering problems*. In B. Michielsen and F. Decavele (eds) Proceedings of the JEE’02 Symposium, p. 251-256, Toulouse, ON-ERA, 2002

NOLWENN BALIN, A. BENDALI

### Domain Decomposition and Additive Schwarz Techniques in the Solution of a TE Model of the Scattering by an Electrically Deep Cavity

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 16:25

It is well known, by the experts in stealth technology, that an electrically deep cavity significantly contributes to the RCS of the structure in which it is residing. Several difficulties prevent the use of standard methods to deal with this problem:

- asymptotic methods cannot be used to determine

the field scattered by the cavity,

- direct solutions are not possible due to the huge size of the linear system to be solved,
- fast methods, like the multipole method, cannot directly apply because of the very slow conver-



gence of the Krylov-like iterative methods due to the cavity.

Two directions have been explored (for a TE model) to enhance the solution.

A non-overlapping domain decomposition consisted of successive pieces the interfaces of which being a sectional surface of the cavity clearly takes advantage of the geometry of the problem. The electromagnetic field inside each piece can then be represented by equivalent currents. Successive eliminations, each time at the level of only one piece, reduce the cavity effect, at the end of the process, to the determination of the electric and

magnetic equivalent currents at the opening of the cavity.

An additive overlapping Schwarz method, based on partition of unity to decompose the currents, is used to solve the resulting integral equations on the opening of the cavity and the exterior part of the structure. This leads to an iterative method, in which at each step, are solved problems set on the support of each partition of unity function. A high convergence rate has been observed.

This work is a part of a PhD thesis which is done in the frame of a collaboration involving MBDA-F, CERFACS and MIP Laboratory.

XAVIER ANTOINE, H. BARUCQ

### **On the Construction of Approximate Boundary Conditions for Solving the Interior Problem of the Acoustic Scattering Transmission Problem**

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 16:50

This talk is about the construction of generalized impedance boundary conditions at the interface between two three-dimensional homogeneous media for the penetration of a time-harmonic wave into a dissipative obstacle. The exact model is given through a transmission problem that couples the propagations into an absorbing domain and an exterior domain. The

conditions arise from an asymptotic analysis of the interior solution whose propagation is studied by pseudodifferential technics classically involved for the construction of radiation boundary conditions. First and second-order conditions are analyzed and some numerical experiments illustrate their validity domain.

FRANK SCHMIDT, LIN ZSCHIEDRICH

### **Numerical Methods to Realize the Pole Condition Concept**

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 17:15

Sommerfeld's radiation condition is of basic importance for time-harmonic acoustic scattering problems, both for uniqueness considerations as well as for the foundation of numerical methods.

Recently we proposed an alternative radiation condition called pole condition. The pole condition is more general than Sommerfeld's radiation condition and may be applied, in merely the same way, to a number of different problems like time-harmonic and time-dependent scattering problems as well as to eigenproblems on un-

bounded domains. Whereas recent studies on pole condition concentrated on theoretical properties of the condition itself, this talk aims to discuss how to apply the pole condition concept to solve scattering problems on unbounded domains. Here, the decomposition into interior and exterior domain, a semi-discretization of the exterior domain, and the application of the pole condition lead in a natural way to suitable transparent boundary conditions for the interior domain.

MATTHIAS EHRHARDT, ANTON ARNOLD, IVAN SOFRONOV

### **Approximation, Stability and Fast Calculation of non-Local Boundary Conditions for the Schrödinger Equation**

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 17:40

We propose a way to efficiently treat the well-known transparent boundary conditions for the Schrödinger equation. Our approach is based on two ideas: to write out a discrete transparent boundary condition (DTBC) using the Crank-Nicolson finite difference scheme for the governing equation, and to approximate the discrete convolution kernel of DTBC by sum-of-exponentials for

a rapid recursive calculation of the convolution.

We prove stability of the resulting initial-boundary value scheme, give error estimates for the considered approximation of the boundary condition, and illustrate the efficiency of the proposed method on several examples.

## **MS05 Heterogeneous Domain Decomposition with Applications in Multiphysics**

Organized by: RALF KORNHUBER, ALFIO QUARTERONI

Coupled heterogeneous phenomena are not an exception but the rule in advanced numerical simulations of fluid dynamics, microelectronics, hydrodynamics, hemodynamics, electrodynamics or acoustics. Mathematical understanding and efficient numerical solvers become more and more important. The aim of this minisymposium is to bring together scientists working in this field who will both present talks on topics within the theme and contribute to discussion during the minisymposium.

RONALD H.W. HOPPE

### Domain Decomposition Methods in Electrothermomechanical Coupling Problems

Location: Room 005, Time: Tuesday, 22 July, 16:00

The functionality of modern electronic devices and systems often depends on the coupling of various physical phenomena on different scales in both time and space. The mathematical modeling on a macroscopic level typically leads to coupled systems of partial differential equations whose numerical solution requires suitable problem oriented discretizations and efficient iterative solvers such as domain decomposition and multi-level methods.

In this contribution, we consider electrothermomechanical coupling problems as they arise in the modeling and simulation of high power electronic devices. In particular, we are faced with a hierarchy of coupled physical effects in so far as electrical energy is converted to Joule heat causing heat stresses that have an impact on

the mechanical behavior of the devices and may lead to mechanical damage without appropriate cooling mechanisms.

Moreover, there are structural coupling effects due to the sandwich-like construction of the devices featuring multiple layers of specific materials with different thermal and mechanical properties. The latter motivates the application of domain decomposition techniques on nonmatching grids based on individual finite element discretizations of the substructures. We will address in detail the modeling aspects of the hierarchy of coupling phenomena as well as the discretization-related couplings in the numerical simulation of the operating behavior of the devices.

VSEVOLOD NEFEDOV

### Subgridding in Finite-Difference Time-Domain Method

Location: Room 005, Time: Tuesday, 22 July, 16:25

Finite-Difference Time-Domain (FDTD) method is a simple and effective solution method for the Maxwell Equations. It most commonly used on structured uniform grids and for that reason lacks adaptivity.

In this talk we consider various ways to improve FDTD algorithm by employing local refinement algorithms, for instance, Local Defect Correction (LDC). We construct a combination of LDC and FDTD, discuss stability and

accuracy issues and consider an application of LDC and FDTD to a number of problems.

- [1] K.S. Yee, *Numerical solution of initial boundary value problem involving Maxwell's equations in isotropic media*. IEEE Transactions on antennas and propagation, AP-14:302-307, May 1966.
- [2] V. Nefedov, *Subgridding in FDTD*, technical report RANA 02-29, TU Eindhoven, Eindhoven, 2002

FAUSTO SALERI, E. MIGLIO, S. PEROTTO

### A Multiphysics Strategy for Free Surface Flows

Location: Room 005, Time: Tuesday, 22 July, 16:50

Several environmental engineering applications involve free surface flows phenomena. In this context the wide variety of situations leads to consider a large spectrum of space and time scales related to the presence of different physical phenomena [2,6,7].

Various models have been developed in order to cope with the above mentioned problems; these models can be grouped into the following categories (in descending order of complexity): as for the 3D case one can consider either the Free Surface Navier-Stokes or the Hydrostatic Shallow Water equations (see [1]); concerning the 2D situation the Boussinesq, Serre or Shallow Water models can be adopted (see [4], [5]); finally the 1D counterpart of these latter models can be used. Most of these models are well established, both in terms of a sound mathematical formulation and of a robust numerical implementation.

Ideally one should use a full 3D model to capture all the physical features of the problem at hand. However this approach is characterized by a huge computational effort that we aim to reduce by suitably coupling models of different dimensions among the ones mentioned above.

We present a multiphysics strategy in order to take into

account different scales by combining 3D, 2D and 1D models retaining a reasonable computational cost.

- [1] Causin, P.; Miglio, E.; Saleri, F.: *Algebraic factorizations for 3D non-hydrostatic free surface flows*. Comput. Visual. Sci., 5 (2002), no. 2, 85–94,
- [2] Debnath, L.: *Nonlinear Water Waves*, Academic Press, San Diego, 1994.
- [3] Marrocu, M.; Ambrosi, D.: *Mesh adaptation strategies for shallow water flow*. Internat. J. Numer. Methods Fluids 31 (1999), no. 2, 497–512.
- [4] Grasselli, M.; Perotto, S.; Saleri, F.: *Space-time finite elements for Boussinesq equations*. East-West J. Numer. Math., 7 (1999), no. 4, 283–306.
- [5] Perotto, S.; Saleri, F.: *Adaptive finite element methods for Boussinesq equations*. Numer. Methods Partial Differential Equations, 16 (2000), no. 2, 214–236.
- [6] Vreugdenhil, C.B.: *Numerical methods for Shallow Water flows*, Kluwer Academic Press, Dordrecht, 1998.
- [7] Whitham, G.B.: *Linear and Nonlinear Waves*, Wiley, New York, 1974.

FRIEDHELM SCHIEWECK, W.J. LAYTON, I. YOTOV  
**Coupling Fluid Flow with Porous Media Flow**  
**Location:** Room 005, **Time:** Tuesday, 22 July, 17:15

The transport of substances back and forth between surface and ground water is a very serious problem. We study herein the mathematical model of this setting consisting of the Stokes equations in the fluid region coupled with Darcy's equations in the porous medium, coupled across the interface by the Beavers-Joseph-Saffman conditions.

We prove existence and uniqueness of a weak solution. For the approximation of velocity and pressure, we use an arbitrary pair of conforming LBB-stable finite element spaces in the fluid region and any pair of the well-known mixed finite element spaces like RT-spaces or BDM-spaces in the region of the porous medium. The coupling of the normal components of the velocity approximation across the interface can be characterized

easily by means of hanging nodes. For the coupled approximation spaces of both regions, we prove the LBB-condition and optimal interpolation estimates. This leads to an estimate of the discretization error which is of optimal order.

The analysis of our finite element scheme suggests a way to uncouple the solution of the global discrete problem into steps involving only solvers for the porous media and fluid flow subproblems. This is important because there are many codes available which have been optimized for solving these subproblems.

[1] W.J. Layton, F. Schieweck, I. Yotov, *Coupling Fluid Flow with Porous Media Flow*. SIAM J. Numer. Anal., Vol. 40, No. 6, pp. 2195-2218 (2003).

PAOLO ZUNINO  
**Iterative Substructuring Methods for Advection-Diffusion Problems in Heterogeneous Media**  
**Location:** Room 005, **Time:** Tuesday, 22 July, 17:40

This work is devoted to the numerical approximation of a system of advection-diffusion equations set in adjacent domains and coupled with non-standard matching conditions. The specific application of the model at hand is the study of the transfer of chemicals through media of heterogeneous nature, for example a free fluid and a porous medium. This problem has several practical applications, for instance the study of the motion of a pollutant from a basin to the adjacent soil or the study of the transport of chemicals from the blood flow to the wall of the vessels.

Because of the heterogeneity of the media we consider, the concentration of chemicals may feature large variations from one medium to the other. For example, if a pollutant is released into a basin, its concentration in the free fluid is reasonably higher than the one in the surrounding soil. Consequently, the numerical simulation of such problems could be an extremely challenging task. To this aim, we propose a particular choice of the matching conditions between the advection-diffusion equations on each medium, which

allow the discontinuity of the concentration across the interface.

After the description of the model, we focus our attention on its numerical treatment. In particular, since our model couples subproblems in different media, we study an iterative procedure where the solutions provided separately on each subdomain are suitably matched (a so called iterative substructuring method). More precisely, we consider a strategy based on Robin interface conditions for both subdomains. The convergence of this iterative strategy is analyzed at both the continuous and the discrete level, by means of suitable interface operators of Steklov-Poincaré type. Moreover, an algebraic reinterpretation of this technique is provided, leading to the definition of optimal preconditioners for the linear system arising from the discretization of the global problem.

Finally, numerical results are presented, in order to assess the computational efficiency of the numerical methods proposed.

### MS06 Robust Decomposition Methods for Parameter Dependent Problems

Organized by: ULRICH LANGER, SERGEY NEPOMNYASCHIKH

The main topic of this Minisymposium consists of the construction and analysis of robust solvers parameter dependent problems. The following problems will be a subject to consider.

- elliptic problems with singularities in the coefficients,
- anisotropic boundary value problems,
- Effective solvers of hp schemes,
- preconditioning operators in weighted Sobolev spaces

SERGEY V. NEPOMNYASCHIKH, E.-J. PARK  
**Preconditioning for Heterogeneous Problems**  
**Location:** Room 049, **Time:** Monday, 21 July, 16:00

Preconditioning for heterogeneous elliptic problems is considered. The basis for the construction of preconditioning operators in this talk is domain decomposition methods. Suggested domain decomposition technique involves the construction of local preconditioning op-

erators and interface preconditioning operators. For the latter preconditioners we need to investigate Schur complements or trace theorems in the corresponding Sobolev spaces.

KARL SCHERER

### Weighted Norm-Equivalences for Preconditioning

**Location:** Room 049, **Time:** Monday, 21 July, 16:25

Let  $\mathcal{V}_j$  be a sequence of hierarchical spaces of piecewise polynomial functions with respect to triangulations  $\mathcal{T}_0 \subset \mathcal{T}_1 \subset \dots \subset \mathcal{T}_J$  with respect to a domain  $\Omega \in \mathbb{R}^2$  and consider the Ritz-Galerkin approximation of an elliptic problem associated with the bilinear form  $a(u, v) := \int_{\Omega} \sum_{i,k} a_{i,k}(D_i u)(D_k v)$ . We replace or approximate it by a discretized bilinear form satisfying the local ellipticity condition ( $\underline{\omega}_i, \bar{\omega}_i > 0$ )

$$\underline{\omega}_i \sum_{i=1}^2 \xi_i^2 \leq \sum_{i,k=1}^2 a_{i,k}(x) \xi_i \xi_k \leq \bar{\omega}_i \sum_{i=1}^2 \xi_i^2, \\ x \in Z_i, \text{ for each triangle } Z_i \in \mathcal{T}_J$$

Under this assumption robust norm equivalences of the

form

$$\beta_0 \left( \|P_0\|_X^2 + \sum_{j=1}^J 4^j \|P_j u - P_{j-1} u\|_X^2 \right) \\ \leq a(u, u) \leq \\ \beta_1 \left( \|P_0\|_X^2 + \sum_{j=1}^J 4^j \|P_j u - P_{j-1} u\|_X^2 \right)$$

for projections  $P_j : \mathcal{V}_j \rightarrow \mathcal{V}_J$  in weighted  $L_2$ -norms are established. "Robust" means that the constants  $\beta_0, \beta_1$  do not depend on the weights, the number  $J$  of levels and  $u \in \mathcal{V}$ . The essential tools are weighted Jackson and Bernstein-inequalities. Applications to preconditioning are indicated.

SVEN BEUCHLER

### A Dirichlet-Dirichlet DD-preconditioner for $p$ -fem

**Location:** Room 049, **Time:** Monday, 21 July, 16:50

In this talk, a uniformly elliptic second order boundary value problem in 2D is discretized by the  $p$ -version of the finite element method. An inexact Dirichlet-Dirichlet domain decomposition preconditioner for the system of linear algebraic equations is investigated. The solver for the problem in the subdomains [1] and a preconditioner for the Schur-complement, [2] are proposed as ingredients for the inexact DD-preconditioner. Finally, several numerical experiments are given.

- [1] S. Beuchler. *Optimal preconditioners for the  $p$ -version of the fem.* Preprint SFB393/03-03, Technische Universität Chemnitz, Sonderforschungsbereich 393, 2003.
- [2] S. Jensen and V.G. Korneev. *On domain decomposition preconditioning in the hierarchical  $p$ -version of the finite element method.* Comput. Methods Appl. Mech. Eng., 150(1-4):215-238, 1997.

ALMEDIN BEĆIROVIC, JOACHIM SCHÖBERL

### Optimal Extension Operators for High Order Tetrahedral Elements

**Location:** Room 049, **Time:** Monday, 21 July, 17:15

The goal of this work is the construction of fast iterative solvers for matrix equations arising from high order finite elements. We focus on cheap block-Jacobi and block-Gauss-Seidel iterations, where the blocks are defined by the shape functions associated to edge-, face- and interior-nodes. Of course, the speed of convergence depends on the choice of the shape functions.

We present new shape functions leading to nearly optimal iteration numbers. The construction is based on

polynomial extension operators. By the help of symbolic computing we could derive cheap recursion formulas for the efficient computation of the shape functions. Numerical results for 2D and 3D problems are presented.

- [1] A. Bećirović and J. Schöberl, *Hierarchical shape functions based on explicit extension operators*, SFB-Report 03xx, Johannes Kepler University Linz, 2003

LI DENG, ICHIRO HAGIWARA

### Folding Process of Thin-Walled Prismatic Columns by Origami Modeling

**Location:** Room 049, **Time:** Monday, 21 July, 17:40

We study folding process of thin-walled prismatic columns by Origami modeling, its applied background is how to crush a empty can and PET bottle efficiently.

We will give its simple mathematical model, and showed some simulations by nonlinear software Ls-dyna.

## MS07 Parallel Finite Element Software

Organized by: PETER BASTIAN, CHRISTIAN WIENERS

The numerical solution of partial differential equations by the finite element methods consists of a number of steps where the efficient resolution of linear systems by domain decomposition methods is an important but not the only part. The flexible and efficient implementation of all aspects of the finite element method on parallel computers has led to increasingly complex software frameworks. In this minisymposium several new developments will be presented.

KRZYSZTOF BANAS

### A Model for Parallel Adaptive Finite Element Software

**Location:** Room 005, **Time:** Wednesday, 23 July, 11:00

The paper presents a conceptual model and details of an implementation for parallel adaptive finite element systems, particularly their computational kernels. The whole methodology is based on domain decomposition while message passing is used as a model of programming. The proposed finite element architecture consist of independent modules, most of them taken from sequential codes. The sequential modules are only slightly modified for parallel execution and two new modules, explicitly aimed at handling parallelism, are added. The new modules comprise domain decomposition manager

and parallel communication library interface.

The presented principles are thought of as a general guidance for the parallelization of sequential finite element codes. An example implementation utilizes 3D prismatic meshes and discontinuous Galerkin approximation. Two numerical examples, the first in which Laplace's equation is approximated using GMRES with multi-grid preconditioning and the second where dynamic adaptivity with load balancing is utilized for simulating linear convection, illustrate capabilities of the approach.

CHRISTOPH PFLAUM

### Parallelization Concepts of the Library EXPDE

**Location:** Room 005, **Time:** Wednesday, 23 July, 11:25

The aim of the library EXPDE is to provide a user friendly interface for the implementation of PDE software. The interface uses a language which is close to the mathematical language. Efficiency is obtained by expression templates. In this talk, we explain the parallelization concept and coarse grid correction of EXPDE. The parallelization properties are described for different concepts as block structured parallelization and parallelization of semi-unstructured grids.

- [1] C. Pflaum, *Expression Templates for Partial Differential Equations*. Comput Visual Sci, Volume 4, Issue 1, pp.1-8, 2001.
- [2] C. Pflaum, *Semi-Unstructured Grids*. Computing, Nummer 2, vol. 67, pp. 141-166, 2001.
- [3] A. Linke, C. Pflaum, B. Bergen, *Scientific Progress in the Par-EXPDE-Project*, Preprint No. 246, Mathematische Institute, Universität Würzburg, 2002.

CHRISTIAN WIENERS

### Distributed Point Objects: A New Concept for Parallel Finite Elements

**Location:** Room 005, **Time:** Wednesday, 23 July, 11:50

We present a new concept for the realization of finite element computations on parallel machines which is based on a dynamic data structure address by points. All geometric objects (cells, faces, edges) are referenced by its midpoint, and all algebraic data structures (vectors and matrices) are tied to the nodal points of the finite elements. Together, they build Distributed Point Objects (DPO), where the parallel distribution is made transparent by processor lists assigned to the points. All objects are stored in hash tables (where the keys are points) so that pointers can be completely avoided.

The purpose of our new model and its prototype implementation is to provide a platform for developing, testing, and improving lean interfaces between specific problem classes and general parallel solver. It represents a compromise between flexibility and compactness of the code and the requirements for an optimal perfor-

mance. So, we avoid constructions which are machine dependent, and we restrict ourselves to a very small set of parallel commands in the message passing interface. Finally, we consider the application of the parallel programming model to a geomechanical porous media problem (cooperation with Ehlers / Ammann, Universität Stuttgart). This demonstrates that demanding 3-d nonlinear and time-dependent engineering applications on unstructured meshes can be parallelized very efficiently within a very small overhead for the parallel implementation.

- [1] C. Wieners, M. Ammann, W. Ehlers. *Distributed Point Objects: A new concept for parallel finite elements applied to a geomechanical problem*. 2003 (submitted to Future Generation Computer Systems)

PETER BASTIAN

### Towards a Unified Framework for Finite Element Computations

**Location:** Room 005, **Time:** Wednesday, 23 July, 12:15

Finite Element implementations range from small and simple model problem applications to large frameworks incorporating many different finite element variants, state-of-the-art solvers, adaptive mesh refinement and parallel computation capability. The large frameworks typically have evolved over many years, are difficult to use and may be inefficient for particular applications with respect to memory and run-time.

In this talk we present an object oriented approach to finite elements that is intended as an open, public domain software platform. Its particular features are: (1) Flexibility (higher order, systems, adaptivity, parallelism),

(2) reusability of old code (E.g. Albert, UG) and (3) efficiency. These features are achieved through the use of static polymorphism.

The talk discusses the design of the code with emphasis on the grid abstraction. Already existing applications include a large scale explicit finite volume scheme and a parallel algebraic multigrid solver. Both codes are used to illustrate the concept.

This is joint work with M. Blatt (Heidelberg), M. Droske (Duisburg), C. Engwer (Heidelberg), R. Klöforn (Freiburg), M. Ohlberger (Freiburg) and M. Rumpf (Duisburg).

### MS08 Domain Decomposition on Nonmatching Grids

Organized by: RONALD H.W. HOPPE, BARBARA WOHLMUTH, YURI KUZNETSOV

Domain decomposition methods on nonmatching grids have attracted a lot of attention during the last couple of years. Originating from discretizations by spectral elements, they have been considered for a wide variety of finite element discretizations and for applications in computational electromagnetics, fluid dynamics, and mechanics. The aim of this minisymposium is to report on recent advances in this area.

DIETRICH BRAESS

#### A Cascadic Multigrid Algorithm for Mortar Elements

**Location:** Lecture Room, **Time:** Monday, 21 July, 16:00

It is typical for domain decomposition methods that auxiliary problems have to be solved on the interfaces/skeleton and that approximate Schur complements are involved. The methods differ by the choice of the approximation.

We discuss a multigrid method where an approximate Schur complement solver is hidden in the smoothing procedure, and it is the advantage of the multigrid

method that a very cheap approximation can be used. The robustness of the method is confirmed by the fact that even the cascadic version of the multigrid method is robust and efficient.

- [1] D. Braess, P. Deuffhard, and K. Lipnikov: *A cascadic multigrid algorithm for mortar elements*. Computing 69 (2002), 205–225

YVON MADAY, BERND FLEMISCH, FRANCESCA RAPETTI, BARBARA I. WOHLMUTH

#### Coupling Scalar and Vector Potentials on Nonmatching Grids for Eddy Currents in Moving Conductor

**Location:** Lecture Room, **Time:** Monday, 21 July, 16:25

The  $T - \Omega$  formulation of the magnetic field has been introduced in many papers for the approximation of the magnetic quantities modeled by the eddy current equations. This decomposition allows to use a scalar function in the main part of the computational domain, reducing the use of vector quantities to the conducting parts.

We propose to approximate these two quantities on nonmatching grids so as to be able to tackle a problem where the conducting part can move in the global domain. The connection between the two grids is managed

with mortar element techniques.

- [1] Y. Maday, F. Rapetti, B.I. Wohlmuth, *Coupling between scalar and vector potentials by the mortar element method*, C. R. Acad. Sci. Paris, Ser. I 334 (2002) pp. 933-938.  
 [2] B. Flemisch, Y. Maday, F. Rapetti, B. Wohlmuth, *Coupling scalar and vector potentials on nonmatching grids for eddy currents in a moving conductor*, Journal of Computational and Applied Mathematics, 2003, to appear.

CHRISTIAN WIENERS, BARBARA I. WOHLMUTH

#### Multigrid Analysis for Saddle Point Problems Arising from Mortar Discretizations

**Location:** Lecture Room, **Time:** Monday, 21 July, 16:50

We present an abstract framework for the analysis of multigrid methods for a saddle point problem, and we discuss the application systems arising from mortar finite element discretizations. In contrast to other approaches, the iterates do not have to be in the positive definite subspace. This is required for the case of non-nested Lagrange multiplier spaces.

We apply the multigrid method with a smoother of

Braess-Sarazin type to different mortar settings including dual Lagrange multipliers, linear elasticity and a rotating geometry. Numerical results in 2D and 3D realized in the software system UG demonstrate the flexibility, efficiency and reliability of our multigrid method.

- [1] C. Wieners and B. Wohlmuth. *Duality estimates for saddle point problems arising from mortar dis-*

cretizations. 2003 (to appear in SIAM Sci. Comp.)

BISHNU PRASAD LAMICHHANE, BARBARA I. WOHLMUTH

**Second Order Lagrange Multiplier Spaces for Mortar Finite Element Discretizations**

**Location:** Lecture Room, **Time:** Monday, 21 July, 17:15

The coupling of different discretization schemes or of nonmatching triangulations can be analyzed within the framework of mortar methods. These nonconforming domain decomposition techniques provide a more flexible approach than standard conforming approaches. They are of special interest for time-dependent problems, diffusion coefficients with jumps, problems with local anisotropies, corner singularities, and when different terms dominate in different regions of the simulation domain. To obtain a stable and optimal discretization scheme for the global problem, the information transfer between the subdomains is of crucial importance.

In the first part, we will concentrate on mortar techniques for quadratic finite elements, in particular, with hexahedral triangulations in 3D. We will discuss different Lagrange multiplier spaces for standard triquadratic finite elements and Serendipity elements. Finally, the numerical results for linear and quadratic mortar finite

elements in 2D and 3D will be presented. Different Lagrange multiplier spaces will be compared and some applications will be discussed.

- [1] C. Bernardi and Y. Maday and A.T. Patera (1993), *Domain decomposition by the mortar element method*, In: Asymptotic and numerical methods for partial differential equations with critical parameters
- [2] B.I. Wohlmuth (2001) *Discretization Methods and Iterative Solvers Based on Domain Decomposition*, Springer Heidelberg, 17
- [3] B.P. Lamichhane and B.I. Wohlmuth (2002), *Higher Order Lagrange Multiplier Spaces For Mortar Finite Element Discretizations*, CALCOLO, 39(4), 219-237
- [4] B.I. Wohlmuth (2001) *A Comparison of Dual Lagrange Multiplier Spaces for Mortar Finite Element Discretizations*, *M<sup>2</sup>AN*, 36, 995-1012

XUEJUN XU, ZHONG-CI SHI

**A Mortar Element Method for a Plate Bending Problem**

**Location:** Lecture Room, **Time:** Monday, 21 July, 17:40

Recently, Marcinkowski [Numer. Math.,2002] presented a mortar element method for some plate elements such as the conforming Hsieh-Clough-Tocher(HCT) and the reduced Hsieh-Clough-Tocher(RHCT) elements. Optimal energy norm error estimate was obtained under the elliptic regularity assumption that the solution is in the space  $H^4(\Omega) \cap H_0^2(\Omega)$ . However, it is known that a realistic regularity assumption for a plate bending problem is  $H^3(\Omega) \cap H_0^2(\Omega)$ .

In this talk, using the techniques developed by Shi for

deriving the error estimates for the nonconforming Morley element we will show that a mortar element method is applicable to the plate bending problem with the above weaker regularity assumption. The optimal energy norm error estimate and  $H^1$  norm estimate will be given, which cannot be derived by Marcinkowski's approach.

An optimal multigrid method for the mortar element will also be presented.

BERND HEINRICH

**Nitsche-Type Mortaring for Elliptic Problems with Singularities and Boundary Layers**

**Location:** Lecture Room, **Time:** Thursday, 24 July, 16:00

The paper deals with the Nitsche-type mortaring for treating weak continuity across non-matching meshes for some non-overlapping domain decomposition. The approach is derived as a generalization of some method of J.A. Nitsche(1971), a survey about recent results and a comparison with the classical mortar method are given. In particular, the method is applied to transmission problems with coefficients being discontinuous at polygonal interfaces as well as to singularly perturbed reaction-diffusion problems. The solutions of

such problems are provided with corner singularities or/and boundary layers. The interface of the domain decomposition may pass corners of the boundary or of the physical interface, or this interface may be aligned with the boundary layer. For such problems and non-matching meshes of triangles, which are anisotropic in the boundary layers, stability as well as error estimates of the finite element schemes are proved. Some numerical examples illustrate the approach and the results.

YURI KUZNETSOV

**Mixed Finite Element Methods for Diffusion Equations on Nonmatching Grids**

**Location:** Lecture Room, **Time:** Thursday, 24 July, 16:25

The algebraic aspects of the mixed and mixed-hybrid finite element discretizations on nonmatching grids for the diffusion equations are discussed. Using the standard condensation technique the underlying algebraic

saddle point problems can be reduced to the systems with symmetric positive definite matrices. A number of preconditioners are proposed and analysed for both saddle point and condensed matrices.

TALAL RAHMAN, RONALD H.W. HOPPE, XUEJUN XU

### **Additive Schwarz Method for the Mortar Crouzeix-Raviart Element**

**Location:** Lecture Room, **Time:** Thursday, 24 July, 16:50

We present an additive Schwarz method for the mortar Crouzeix-Raviart finite element for solving systems of algebraic equations resulting from the approximation of second order elliptic boundary value problems with discontinuous coefficients.

The finite element approximation is done by independently triangulating each subregion (subdomain) with the P1 non-conforming or the Crouzeix-Raviart element, and then by using the mortar technique (originally introduced by Bernardi et al. for conforming elements, recently extended to the CR element by

Marcinkowski) to describe the discrete problem.

It is well known that large jumps in the coefficients cause trouble for most iterative methods. There already exists some Schwarz methods which are insensitive to such jumps, for instance, for the standard P1 conforming element and the mortar P1 conforming element. The aim of this talk is to extend their ideas to using the mortar CR element, hoping that the new algorithm, due to special construction, will have several advantages (algorithmic) over the old ones.

ANDREAS GANTNER, MAX DRYJA, OLOF B. WIDLUND, BARBARA I. WOHLMUTH

### **Multilevel Additive Schwarz Preconditioner for Nonconforming Mortar Finite Element Methods**

**Location:** Lecture Room, **Time:** Thursday, 24 July, 17:15

Mortar elements form a family of special non-overlapping domain decomposition methods which allows the coupling of different triangulations across subdomain boundaries. We discuss and analyze a multilevel preconditioner for mortar finite elements on nonmatching triangulations. The analysis is carried out within

the abstract framework of additive Schwarz methods. Numerical results show a performance of our preconditioner as predicted by the theory. Our condition number estimate depends quadratically on the number of refinement levels

PADMANABHAN SESHAIYER

### **Non-Conforming Finite Element Methods for Nonmatching Grids Tuned to Parallel Implementation**

**Location:** Lecture Room, **Time:** Thursday, 24 July, 17:40

Engineering applications often require finite element analysis to be carried out over complex domains. Often such analysis, particularly the labor-intensive meshing phase, may be accomplished by dividing the global domain into several local sub-domains. These sub-domains may be constructed separately by different analysts and the global domain can then be constructed by piecing together these individually modeled sub-domains. However, during the assembly, it is often too cumbersome, or even infeasible, to coordinate the meshes over separate subdomains. The mortar finite element method ([1, 2, 3]) is a non-conforming domain decomposition technique which helps to accomplish such a modeling task. An added advantage of this approach is that mesh refinement can be imposed selectively on those components where it is required. Moreover, different variational problems in different subdomains can be combined.

In this talk, the hp-version of the non-conforming technique developed in [3, 4, 8, 9, 10] will be discussed where the stability and convergence were shown to be “uniform” in terms of “both” the polynomial degree “and” the mesh refinement used, without assuming quasiuniformity for the meshes. The numerical analysis of the mortar finite element method relies on the coercivity of

the bilinear form and an evaluation of approximation and consistency error terms ([5, 6, 7]). Our numerical results show optimality for the resulting non-conforming method for various h, p and hp discretizations, including the case of exponential hp convergence over geometric meshes. We will also present numerical results for the Lagrange multiplier and also discuss extensions to three-dimensions.

- [1] C. Bernardi, Y. Maday, and A.T. Patera, *Domain decomposition by the mortar element method*, in Asymptotic and Numerical Methods for PDEs with Critical Parameters. H.G. Kaper and M. Garbey, (eds.), NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci, 384, Kluwer, 269-286(1993).
- [2] F. Ben Belgacem, *The mortar finite element method with Lagrange Multipliers*, Numerische Mathematik, 84, 173-197, (1999).
- [3] P. Seshaiyer and M. Suri, *Uniform hp convergence results for the mortar finite element method*, Mathematics of Computation, 69, 521-546 (2000).
- [4] P. Seshaiyer and M. Suri, *hp submeshing via non-conforming finite element methods*, Computer Methods in Applied Mechanics and Engineering, 189, 1011-1030 (2000).



- [5] F. Ben Belgacem, P. Seshaiyer and M. Suri, *Optimal convergence rates of hp mortar finite element methods for second-order elliptic problems*, RAIRO Mathematical Modeling and Numerical Analysis, 34, 591-608 (2000).
- [6] P. Seshaiyer and M. Suri, *Convergence results for non-conforming hp methods: The mortar finite element method*, Contemporary Mathematics, 218, 453-459 (1998).
- [7] P. Seshaiyer, *Stability and convergence of non-conforming hp finite element methods*, Computers and Mathematics with Applications, Accepted and to appear, 2001.
- [8] F. Ben Belgacem, L.K. Chilton and P. Seshaiyer, *The hp-mortar finite element method for the mixed elasticity and stokes problems*, Computers and Mathematics with Applications, Accepted and to appear, 2001.
- [9] F. Ben Belgacem, L.K. Chilton and P. Seshaiyer, *Non-conforming hp finite element methods for Stokes Problem*, Lecture Notes in Computer Science, Springer, 23, 133-146 (2001).
- [10] L.K. Chilton and P. Seshaiyer, *The hp mortar domain decomposition method for problems in fluid mechanics*, International Journal of Numerical Methods in Fluids, 40, 1561-1570 (2002).

## MS09 FETI and Neumann-Neumann Domain Decomposition Methods

Organized by: AXEL KLAWONN, KENDALL PIERSON, OLOF B. WIDLUND

The FETI and Neumann-Neumann families of iterative substructuring methods are among the best known and most severely tested domain decomposition methods for elliptic partial differential equations. The minisymposium will cover new theoretical and computational developments with applications in fluid and structural mechanics and acoustic scattering.

CHARBEL FARHAT, JING LI

### An Iterative Domain Decomposition Method for the Solution of a Class of Indefinite Problems in Computational Structural Dynamics

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 11:00

The FETI-DP domain decomposition method (DDM) is extended to address the iterative solution of a class of indefinite problems of the form  $(\mathbf{A} - \sigma\mathbf{M})\mathbf{x} = \mathbf{b}$ , where  $\mathbf{A}$  and  $\mathbf{M}$  are two symmetric positive semi-definite matrices arising from the finite element discretization of elastodynamic problems, and  $\sigma$  is a positive number. A key component of this extension is a new coarse problem based on the free-space solutions of Navier's homogeneous displacement equations of motion. These solutions are waves, and therefore the resulting DDM is

reminiscent of the FETI-H method. For this reason, it is named here the FETI-DPH method. For a given  $\sigma$ , this method is numerically shown to be scalable with respect to all of the global problem size, subdomain problem size, and number of subdomains. Its intrinsic performance for various ranges of  $\sigma$  is illustrated with the solution on an Origin 3800 parallel processor as well as a beowulf cluster of several large-scale structural dynamics and vibration problems.

OLOF B. WIDLUND, AXEL KLAWONN, OLIVER RHEINBACH

### Selecting Primal Constraints for FETI-DP Algorithms for Linear Elasticity

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 11:25

The FETI algorithms form one of three families of domain decomposition methods that have been implemented and severely tested on the very largest existing parallel computer systems; the other two are the Balancing Neumann-Neumann methods and the Overlapping Schwarz methods with at least two levels.

For the classical Balancing Neumann-Neumann algorithm, the global space has a dimension of six times the number of floating substructures; it also helps to include six degrees of freedom for the substructures that are constrained by boundary conditions. This count is a direct reflection of the fact that there are six linearly independent rigid body modes; we will only discuss problems in three dimensions. The same count is also typical for the coarse space of a standard Overlapping Schwarz

methods which include a second coarse level.

In this contribution, the much more subtle issue of selecting a small but efficient set of primal constraints for Dual-Primal FETI methods will be discussed. The goal is, if possible, to select a smaller or otherwise cheaper coarse problem than those of the main competitors while also guaranteeing a rapid convergence of the FETI-DP algorithm even in the presence of large discontinuities in the Lamé parameters. The possibility of using auxiliary computations involving a few substructures at a time in the selection of effective primal constraints will also be discussed.

This research is part of a project conducted jointly with Axel Klawonn and Oliver Rheinbach of the University of Essen, Germany.

RADEK KUCERA, JAROSLAV HASLINGER, ZDENEK DOSTAL

### The FETI Based Domain Decomposition Method for Solving 3D-Multibody Contact Problems with Coulomb Friction

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 11:50

The contribution deals with the numerical solution of contact problems with Coulomb friction for 3D bodies. First we introduce auxiliary problems with given friction defining a mapping  $\Phi$  which associates with a given slip bound the normal contact stress in the equilibrium state. Solutions to contact problems with Coulomb friction are defined as fixed points of  $\Phi$  and are computed by using the method of successive approximations. The mathematical model of contact problems with given friction leads to a variational inequality of the second kind. Its discretization is based on the so called mixed variational formulation in terms of displacements and stresses on the contact boundary. Therefore the mixed finite element method is used with a piecewise linear ap-

proximation of displacements and a piecewise constant approximation of stresses. This discretization reduces the size of the resulting algebraic problem providing a coarse grid for the approximation of contact stresses. In contrast to 2D case, constraints imposed on tangential components of contact stresses are quadratic. After a linear approximation of these constraints, we are able to use our fast algorithm based on the augmented Lagrangian to the solution of quadratic programming problems with simple bounds and equality constraints. The final version of the algorithm for contact problems with Coulomb friction is a variant of the FETI based domain decomposition method.

HYEAHYUN KIM, CHANG-OCK LEE

### A FETI-DP Method for the Stokes Problems on Nonmatching Grids

**Location:** Lecture Room, **Time:** Tuesday, 22 July, 12:15

In this talk, we consider a FETI-DP method for the Stokes problems on nonmatching grids in 2D. FETI-DP method is a domain decomposition method that uses Lagrange multipliers to match the solutions continuously across the subdomain boundaries in the sense of dual-primal variables. We use the  $P_1(h) - P_0(2h)$  inf-sup stable finite elements solving for the Stokes problems and the mortar matching condition on the velocity functions as the constraints for the FETI-DP formulation. Moreover, to satisfy the compatibility condition of local Stokes problems, redundant constraints are introduced. The Lagrange multipliers corresponding to the redun-

dant constraints are treated as primal variables in the FETI-DP formulation. We propose a preconditioner for the FETI-DP operator, which is derived from the dual norm on the Lagrange multiplier space. The dual norm is obtained from the duality pairing between the Lagrange multiplier space and the velocity function space restricted on the slave sides. Then, we show that the condition number of the preconditioned FETI-DP operator is bounded by  $C \max_{i=1, \dots, N} \{(1 + \log(H_i/h_i))^2\}$ , where  $H_i$  and  $h_i$  are the sizes of the subdomains and meshes for each subdomain, respectively, and  $C$  is a constant independent of  $H_i$ 's and  $h_i$ 's.

DAN STEFANICA

### Parallel FETI Algorithms for Mortars

**Location:** Lecture Room, **Time:** Wednesday, 23 July, 11:00

We propose a new version of the FETI algorithm which preserves the parallelization properties of the classical FETI algorithms when applied to mortar discretizations. This new version is based on generalized coupling conditions across the interface replacing the mortar conditions.

We present numerical results showing that the new FETI algorithm has the same scalability properties as the classical FETI method. We compare the numerical

performance of the algorithm proposed here with that of the FETI and FETI-DP methods for mortars. We also discuss implementation details and storage requirements for the new algorithm, both for general geometrically nonconforming situations and for the case when mortar conditions are required only on a small part of the interface, while continuity is required elsewhere.

We conclude by discussing extensions of our algorithm to problems containing Linear Multipoint Constraints.

OLIVER RHEINBACH, AXEL KLAWONN, OLOF B. WIDLUND

### Some Computational Results for Dual-Primal FETI Methods for Three Dimensional Elliptic Problems

**Location:** Lecture Room, **Time:** Wednesday, 23 July, 11:25

Iterative substructuring methods with Lagrange multipliers for elliptic problems are considered. The algorithms belong to the family of dual-primal FETI methods which were introduced for linear elasticity problems in the plane by Farhat, Lesoinne, Le Tallec, Pierson, and Rixen and were later extended to three dimensional elasticity problems by Farhat, Lesoinne, and Pierson.

In a recent paper, the family of algorithms for scalar diffusion problems was extended to three dimensions and was successfully analyzed by Klawonn, Widlund, and Dryja. It has been shown that the condition number

of these dual-primal FETI algorithms can be bounded polylogarithmically as a function of the dimension of the individual subregion problems and that the bounds are otherwise independent of the number of subdomains, the mesh size, and jumps in the diffusion coefficients.

In this talk, numerical results for some of these algorithms are presented and their relation to the theoretical bounds is investigated.

The presented results are joint work with Axel Klawonn, Essen, Germany and Olof Widlund, New York, USA.

**An Hybrid Domain Decomposition Method****Location:** Lecture Room, **Time:** Wednesday, 23 July, 11:50

In the field of non overlapping domain decomposition methods, two classical choices are the primal and the dual Schur complement approaches. The first leads to the Balanced Domain Decomposition [1,2] and the second to the Finite Element Tearing and Interconnecting method [3]. Once preconditioned with their most efficient preconditioner, these methods are highly similar: from a mechanical point of view they are based on the same concepts [4], from a mathematical point of view they reach the same theoretical performance levels [5], from a numerical point of view little improvements [6] make them equally efficient, from a computational point of view usually the same solver is used (projected conjugate gradient), from an implementation point of view only few programming tricks differ from one to another. We here propose a non overlapping domain decomposition method which provides a general frame which includes traditional primal and dual Schur complement approaches. This so called “hybrid” domain decomposition is based on a partition of the interface degrees of freedom into two sets. DOFs belonging to the first set are processed as primal variables whereas DOFs belonging to the second set are processed as dual variables (note that unlike FETIDP [7] primal DOFs are not suppressed from the resolution process) ; the condensation of the global problem onto the interface requires the computation of a hybrid Schur complement. The resulting interface problem is the sum of local contributions, it is solved with a Krylov iterative solver (GMRes), the preconditioner we propose is composed by a scaled sum of inverses of local contributions. Handling zero energy modes leads to two coarse grid problems (one for the operator, one for the preconditioner) which are managed using projectors ; a third optional coarse problem can also be added.

This new approach has several interests. From a numerical point of view, it leads to performance results comparable to the classical approaches. From an implementation point of view, it proves that primal and dual approaches can be joined into a unique code. From

a mechanical point of view, it enables new treatments of multifield problems such as the one involving displacement and pressure fields arising from the study of porous media or incompressible materials. From a mathematical point of view, it offers a framework which may enable to unify theoretical performance results of primal and dual approaches. Last, the open framework provided enables to define more than two sets of DOFs, one can imagine to add a third set of DOFs processed as “direct” variables (like in FETIDP) and a fourth set of DOFs processed as mixed variables (Robin conditions), which may lead to a method including most of the non-overlapping domain decomposition approaches. Numerical assessments will be presented on various structural mechanic problems and multiphysics problems.

- [1] J. Mandel, *Balancing domain decomposition*, Comm. Appl. Num. Meth. Engrg. 9 (1993) 233-241
- [2] P. Le Tallec, *Domain decomposition methods in computational mechanics*, Computational Mechanics Advances 1 (2) (1994) 121-220, north-Holland.
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**A Family of FETI-Derived Preconditioners for the Primal Substructuring Method: Application to Multiple Right-Hand Side Problems and Implicit Dynamic Analysis****Location:** Lecture Room, **Time:** Wednesday, 23 July, 12:15

Over the last decades, DDM have emerged as a powerful computational tool for the static or dynamic analysis of several problems of Computational Mechanics. The last decade, in particular, witnessed the introduction of two major families of DDM, namely the FETI [1] and the Balancing Domain Decomposition (BDD) [2] families of methods. Since their introduction, the two methods evolved fast and were successfully applied to static or dynamic analysis of several problems in Computational Mechanics.

Recently, two new families of DDM, namely a family of FETI-derived primal preconditioners (also called a primal class of FETI methods) and a family of two-level

primal substructuring methods were introduced [3,4]. Theoretical aspects and the computational performance of these new methods and the standard FETI and BDD methods were compared in a general study [4]. In that study, it was found that while the introduced two-level primal substructuring methods are equivalent to the BDD methods, the new primal class of FETI methods is superior to the standard FETI methods when applied to heterogeneous problems and/or plate and shell problems.

The first comparison [3,4] of the standard and novel DDM was focused on the basic problem of single right-hand side static structural analysis, while the present

work extends this comparison to multiple right-hand side problems and Implicit Dynamics. Furthermore, the discussed methods are equipped with a new technique that accelerates their convergence, while a new method particularly tailored for dynamic problems is proposed. In general, the incorporation of the proposed methods in the family of high-performance DDM brings new possibilities for the efficient analysis of multiple right-hand side and dynamic problems.

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[3] Y. Fragakis, M. Papadrakakis, *A Family of New FETI Methods for Structural Mechanics*, Proceedings of the Fifth World Congress on Computational Mechanics (WCCM-V), Vienna, Austria, July 7-12, 2002, <http://wccm.tuwien.ac.at>

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MAX DRYJA, OLOF B. WIDLUND

### **A Dual-Primal FETI Method with Face Constraints for Mortar Discretization of Elliptic Problems**

**Location:** Lecture Room, **Time:** Friday, 25 July, 9:00

A discretization of elliptic problems on nonmatching triangulations is considered. The discrete problem is formulated using the finite element mortar method. For solving this problem a Dual-Primal FETI method, with

face constraints only, is designed and analyzed and it is proved that its rate of convergence is almost optimal. The algorithm is well suited for parallel computations

WLODEK PROSKUROWSKI, MAX DRYJA

### **A FETI-DP Method for the Mortar Discretization of Elliptic Problems with Discontinuous Coefficients**

**Location:** Lecture Room, **Time:** Friday, 25 July, 9:25

We consider an elliptic problem with discontinuous coefficients. The problem is discretized on non-matching triangulation determined by the jump of coefficients. The resulting discrete problem is obtained employing a mortar technique.

For this discrete problem we design and analyze a FETI-DP method. We extend the results obtained in [1] for problems with continuous coefficients on many subregions and in [2] for problems with discontinuous coefficients on two subdomains.

We prove that the convergence rate of the method is

almost optimal and independent of the jump of the discontinuity. Theoretical results are confirmed by numerical experiments.

[1] M. Dryja and O. Widlund, *A generalized FETI-DP method for the mortar discretization of elliptic problems*, in I. Herrera et al. (eds.), Fourteenth Conference on Domain Decomposition Methods, 2003.

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ANDREA TOSELLI, XAVIER VASSEUR

### **FETI and Neumann-Neumann Preconditioners for hp Finite Element Approximations on Anisotropic Meshes: Algorithms and Theory**

**Location:** Lecture Room, **Time:** Friday, 25 July, 9:50

We consider Neumann-Neumann and a class of FETI methods for hp finite element approximations of scalar elliptic problems on geometrically refined boundary layer meshes in two and three dimensions. These are meshes that are highly anisotropic: the aspect ratio typically grows exponentially with the polynomial degree.

The condition number of our preconditioners is independent of arbitrarily large aspect ratios of the mesh and of potentially large jumps of the coefficients.

In addition, it only grows polylogarithmically with the polynomial degree, as in the case of p approximations on shape-regular meshes.

XAVIER VASSEUR, ANDREA TOSELLI

### **FETI and Neumann-Neumann Preconditioners for hp Finite Element Approximations on Anisotropic Meshes: Numerical Validation**

**Location:** Lecture Room, **Time:** Friday, 25 July, 10:15

We investigate numerically certain Balancing Neumann-Neumann and one-level FETI domain decomposition methods for the solution of algebraic linear systems arising from  $hp$  finite element approximations of scalar elliptic problems on geometrically refined boundary layer meshes. These are meshes that are highly anisotropic where the aspect ratio grows exponentially with the polynomial degree. Various two- and three-dimensional applications will be presented. The numerical results are found to be in good agreement with the theoretic

cal bounds for the condition numbers of the preconditioned operators derived in Andrea Toselli's talk ("FETI and Neumann-Neumann preconditioners for hp finite element approximations on anisotropic meshes: Algorithms and theory"). They confirm that the condition numbers are independent of the aspect ratio of the mesh and of potentially large jumps of the coefficients. In addition, they only grow polylogarithmically with the polynomial degree, as in the case of  $p$  approximations on shape-regular meshes.

## MS10 Recent Advances for the Parareal in Time Algorithm

Organized by: YVON MADAY

The parareal in time algorithm has been introduced by Lions Maday and Turinici. It is an iterative procedure based on the alternate use of a coarse and a fine solvers that allows to squeeze the resolution time if parallel architectures are available.

Since its introduction, many developments and applications have been added and the coupling of this idea with standard domain decompositions or control iterations allow to increase the efficiency of the combined iterative solvers. An effort has also been made on the understanding of the effects of the use different schemes for the time resolution. Finally, the definition of the coarse solvers can be relaxed to allow for very fast resolution in this orthogonal direction.

This minisymposium will present 4 contributions illustrating the different features and providing a wide range of applications of this new direction for parallelization.

YVON MADAY

### The Parareal Algorithm: Basics and Combination with Domain Decomposition Iterations

**Location:** Room 005, **Time:** Tuesday, 22 July, 11:00

In this first talk we introduce the basics of the parareal algorithm in particular we focus on its new presentation that can be understood as a predictor-corrector procedure. The combination of the iterative method in time with other types of iterative techniques in other

directions as the spacial one for domain decomposition techniques or control one for optimal control iterative techniques can then be proposed and results illustrating the potentiality of the algorithm are presented.

GUILLAUME BAL

### On the Analysis and Implementation of the Parareal Algorithm

**Location:** Room 005, **Time:** Tuesday, 22 July, 11:25

The parareal algorithm allows us to parallelize differential equations in time. It is based on solving successively non-parallel coarse discretizations and parallel fine discretizations, and can replace a discretization of order  $m$  by a discretization of order  $(k+1)m$  after  $k$  iterations. We present convergence results for ordinary differential equations and stochastic ordinary differential equations, which shows robustness of the algorithm for a certain

class of stiff equations. We analyze the strategies that should be used to maximize the total speedup or the system efficiency (maximize the time where processors are active).

Finally we present some numerical simulations corresponding to applications where we believe the algorithm may be useful.

GUNNAR ANDREAS STAFF

### Stability and Convergence of the Parareal Algorithm

**Location:** Room 005, **Time:** Tuesday, 22 July, 11:50

The point of departure for this talk is the parareal algorithm presented in [1]. This is an algorithm for solving time-dependent differential equations, and is genuinely only of interest in a parallel context. We discuss the stability of the parareal algorithm for autonomous differential equations, and present theoretical and numerical results for the linear diffusion equation and the viscous Burger's equation. The spatial discretization is here based on pure spectral methods, while various choices for time integration schemes are used. A coarse

propagator based on a coarse discretization in both time and space will also be discussed. We conclude by giving recommendations regarding suitable choices of time integration schemes for both the coarse and the fine propagator for various types of partial differential equations.

- [1] Guillaume Bal and Yvon Maday. *A "parareal" time discretization for non-linear PDE's with application to the pricing of an American put*. In Luca F. Pavarino and Andrea Toselli, editors, Recent Developments in domain Decomposition Methods, vol-

PAUL F. FISCHER

**Investigation of the Parareal Algorithm for Semi-Implicit Incompressible Navier-Stokes Simulations**

**Location:** Room 005, **Time:** Tuesday, 22 July, 12:15

We present preliminary results on the performance of the time-parallel (“Parareal”) algorithm in a semi-implicit spectral element code for the solution of the incompressible Navier-Stokes equations.

**MS11 Space Decomposition and Subspace Correction Methods for Linear and Nonlinear Problems**

Organized by: XUE-CHENG TAI, JINCHAO XU

Domain decomposition and multigrid methods can be interpreted as space decomposition and subspace correction methods. There are different ways to extend these methods to different nonlinear problems – physical problems with nonlinear differential operators and with constraints over the solutions. The purpose of this minisymposium is to bring people in this research direction to present new results and discuss new progresses.

LORI BADEA

**On a Multilevel Schwarz Method for the Constraint Minimization of non-Quadratic Functionals**

**Location:** Room 049, **Time:** Thursday, 24 July, 16:00

The literature on the domain decomposition methods is very large. We can see, for instance, the papers in the proceedings of the annual conferences on domain decomposition methods starting in 1988 with [5]. Naturally, the most of papers dealing with these methods are dedicated to the linear elliptic problems. Also, for the variational inequalities, the convergence proofs refer in general to the inequalities coming from the minimization of quadratic functionals. To our knowledge, even if sometimes the authors make some remarks in their papers on the nonlinear cases, very few papers really deal with the application of these methods to nonlinear problems. We can cite in this direction the papers written by Tai and Espedal [12], Tai and Xu [13] for nonlinear equations, Hoffmann and Zhou [6], Lui [8], Zeng and Zhou [14] for inequalities having nonlinear source terms, and Badea [3], [2] for the minimization of non-quadratic functionals.

The multilevel or multigrid methods can be viewed as domain decomposition methods and we can cite the results obtained by Kornhuber [7], Mandel [9], [10], Smith, Bjørstad and Gropp [11], and Badea, Tai and Wang [4]. Evidently, this list is not exhaustive and it can be completed by many other papers.

In [1], the convergence of a Schwarz method for variational inequalities coming from the minimization of a quadratic functional has been proved. This method has been extended to the one and two-level methods in [4]. Also, its convergence for the constraint minimization of the non-quadratic convex functionals is proved in [2]. The goal of this paper is the extension of the results in [2] to the study of the multilevel methods. The error estimates we give show the same dependence of the error on the mesh and overlapping parameters as in the linear cases. Numerical examples are given to illustrate the convergence of the methods.

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*problems*, SIAM J. Numer. Anal., 28 (1991), pp. 179-204.

[2] L. BADEA, Convergence rate of a multiplicative Schwarz method for strongly nonlinear inequalities, in *Analysis and optimization of differential systems*, V. Barbu, I. Lasiecka, D. Tiba and C. Varsan Eds., Kluwer Academic Publishers, Boston/Dordrecht/London, ISBN 1-4020-7439-5, 2003.

[3] L. BADEA, *Domain decomposition method for strongly nonlinear inequalities*, Preprint series of the Institute of Mathematics of the Romanian Academy, nr. 6, 2002.

[4] L. BADEA, X.-C. TAI AND J. WANG, *Convergence rate analysis of a multiplicative Schwarz method for variational inequalities*, SIAM J. Numer. Anal., accepted for publication, 2002.

[5] R. GLOWINSKI, G.H. GOLUB, G.A. MEURANT, Eds. 1998, *First Int. Symp. on Domain Decomposition Methods*, SIAM, Philadelphia.

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[7] R. KORNHUBER, *Monotone multigrid methods for elliptic variational inequalities I*, Numer. Math. 69 (1994), pp. 167-184.

[8] S.-H. LUI, *On monotone Schwarz alternating methods for nonlinear elliptic Pdes*, Modél. Math. Anal. Num., ESIAM:M2AN, vol. 35, no. 1, 2001, pp. 1-15.

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[10] J. MANDEL, *Hybrid domain decomposition with unstructured subdomains*, *Proceedings of the 6th International Symposium on Domain Decomposition*

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- [11] F.S. BARRY, P.E. BJØRSTAD AND W. GROPP, *Domain decomposition: Parallel Multilevel Methods for Elliptic Differential Equations*, Cambridge University Press, 1996.
- [12] X.-C. TAI AND M. ESPEDAL, *Rate of convergence of some space decomposition methods for linear and nonlinear problems* SIAM J. Numer. Anal., vol. 35, no. 4 (1998), pp. 1558-1570.
- [13] X.-C. TAI AND J. XU, *Global and uniform convergence of subspace correction methods for some convex optimization problems*, Math. of Comp., electronically published on May 11, 2001.
- [14] J. ZENG AND S. ZHOU, *Schwarz algorithm for the solution of variational inequalities with nonlinear source terms*, Appl. Math. Comput., 97, 1998, pp.23-35.

RALF KORNUBER, SUSANNA GEBAUER, HARRY YSERENTANT  
**Hierarchical Decomposition of Domains with Fractures**  
**Location:** Room 049, **Time:** Thursday, 24 July, 16:25

We consider a diffusion problem with strongly varying diffusion coefficients. More precisely, the diffusion  $k_0$  in a subdomain consisting of a network of fractures is much larger than it is in the remaining porous matrix. The network of fractures consists of long thin rectangles with width  $\varepsilon > 0$ . Such kind of problems occur not only in hydrology (where the above notions come from) but also has other applications in engineering sciences or medicine. The main difficulty in constructing fast solvers is that the permeability  $k_0$  may become arbitrary large while the width  $\varepsilon$  may become arbitrary small.

In order to reduce the degrees of freedom, we use an anisotropic quadrilateral partition of the network of fractures while the porous matrix is partitioned in the usual way by isotropic triangles. For the resulting discrete problem we present an hierarchical domain decomposition algorithm with appropriate multigrid solvers for the subproblems. It turns out that under reasonable assumptions on the mesh parameters our algorithm converges robustly for arbitrary large permeability  $k_0$  and vanishing width  $\varepsilon$ . These theoretical findings are illustrated by numerical computations.

XUE-CHENG TAI  
**Nonlinear Positive Intepolation Operators for Analysis with Multilevel Grids**  
**Location:** Room 049, **Time:** Thursday, 24 July, 16:50

In order to analyse the convergence of multigrid and domain decomposition methods for obstacle type of problems, we need to use some special interpolation operators. The following properties are typically needed:

1. It should have the standard approximation errors.
2. It should preserve positive functions.
3. It should be monotone, i.e. the interpolation for a given functions on a coarser mesh should not less than the interpolation on a finer mesh.
4. The interpolation operator should be "stable" in some suitable norm.

We shall construct a nonlinear interpolation operator which satisfies all the above properties and show how can we use it for the analysis for multigrid methods for obstacle type of problems.

- [1] Nochetto, R.H. and Wahlbin, Lars B.: *Positivity preserving finite element approximation*. Math. Comp. 71 (2002), no. 240, 1405–1419.
- [2] Lori Badea, Xue-Cheng Tai and Junping Wang; *Convergence rate analysis of a multiplicative Schwarz method for variational inequalities*. To appear in: SIAM J. Numer. Anal..
- [3] Xue-Cheng Tai: *Rate of convergence for some constraint decomposition methods for nonlinear variational inequalities*, Numer. Math., 2003, 93:755-786.
- [4] X.-C. Tai, B. Heimsund and J. Xu: *Rate of Convergence for Parallel Subspace Correction Methods for nonlinear variational inequalities*. "Thirteenth international domain decomposition conference", 2001, pp. 127–138, CIMNE.

JINCHAO XU, LONG CHEN, PENGTAO SUN  
**Anisotropic grid adaptation and multigrid methods**  
**Location:** Room 049, **Time:** Thursday, 24 July, 17:15

Some recent studies are reported in this talk on using multilevel subspace correction techniques in grid adaptation. Results presented include gradient and Hessian recovery schemes by using averaging and smoothing

(as in multigrid), interpolation error estimates for both isotropic and anisotropic grids and multilevel techniques for global grid moving and local grid refining/moving. Some applications will also be given.

IZASKUN GARRIDO, MAGNE S. ESPEDAL, G. E. FLADMARK  
**A Convergent Algorithm for Time Parallelization Applied to Reservoir Simulation**  
**Location:** Room 049, **Time:** Thursday, 24 July, 17:40

Parallel methods are usually applied to the space domain because the sequential nature of time is considered to be a handicap for the development of competitive algorithms. However, this sequential nature can also play to our advantage by ensuring convergence within a given number of iterations. This new algorithm for time parallelization is derived from the classical Alternating Schwarz method and acts as a predictor corrector improving both speed and accuracy with respect to the sequential solvers. It reduces significantly the numer-

ical effort for the computation of the molar masses in our complex reservoir simulator **Athene**, which will be illustrated with several numerical examples.

- [1] G. Bal, Y. Maday. *A parareal time discretization for non-linear PDE's with application to the pricing of an American put*
- [2] Jacques-Louis Lions, Y. Maday, Gabriel Turinici. *Résolution d'EDP par un schéma en temps pararéel*. In C. R. Acad. Sci. Paris , **332**, No. 1, pages 1-6. 2001

## MS12 Trefftz-Methods

Organized by: ISMAEL HERRERA, ROBERT YATES

Trefftz originally proposed his method in 1926 [1], but it has been in the last decades, especially since Jirousek and his collaborators originated hybrid-Trefftz (HT) finite element (FE) model [2-4] that it has become a highly efficient computational tool for the solution of difficult boundary value problems. More recently, Herrera interpreted Trefftz method as a domain decomposition method (DDM) and proposed a unified theory of DDMs based on it [5-7]. Trefftz methods, when seen from this perspective, are classified into two broad categories: direct (or Trefftz-Jirousek) and indirect methods. This latter approach is frequently called Trefftz-Herrera method because was originated by Herrera and his collaborators [8,9]. The unified theory expands considerably Trefftz method scope and yields new avenues of thought for numerical methods of partial differential equations. In particular, it has produced an elegant and very general formulation of DDMs [10], together with broad families of numerical approximations such as Localized Adjoint Methods (LAM) [11,12].

Trefftz methods have received important contributions for their development from many different fields of application [13]; among them: potential problems, plane elasticity, plate bending (thin, thick, post-buckling), heat conduction, advective diffusive transport. In spite of their obvious connections, the community that studies DDMs and that which studies Trefftz method have remained apart and, to a large extent, unaware of each other. Clearly, putting them in contact is a very worthwhile endeavor since an exchange of experiences would be very profitable for both of them. The purpose of this minisymposium is to present and discuss recent progress of Trefftz method, and its implications in different fields of application, when it is interpreted as a domain decomposition method. Key representatives of the communities mentioned above will make such presentations and discussions.

- [1] Trefftz E. *Ein Gegenstück zum Ritzschen Verfahren*. In Proceedings 2nd International Congress of Applied mechanics, Zurich, 131-137, 1926
- [2] Jirousek J. & Leon N. *A powerful finite element for plate bending*. Comp.Meth.Appl. Mech. Eng., 12, 77-96, 1977
- [3] Jirousek J. *Basis for development of large finite elements locally satisfying all field equations*. Comp. Meth. Appl. Mech. Eng., 14, 65-92, 1978
- [4] Jirousek J. & Zielinski A. P. *Survey of Trefftz-type element formulations*. Compu. & Struc., 63, 225-242, 1997
- [5] Herrera, I. *Trefftz-Herrera Domain Decomposition*. Special Volume on Trefftz Method: 70 Years Anniversary; Advances in Engineering Software, 24, pp 43-56,1995
- [6] Herrera, I. *On Jirousek Method and its Generalizations*. Computer Assisted Mechanics in Engineering Sciences. Special Issue 8 pp. 325-342, 2001
- [7] Herrera, I. *Trefftz Method: A General Theory*. Numerical Methods for Partial Differential Equations, 16(6) pp. 561-580, November 2000.
- [8] Herrera, I., Yates, R. and Daz M. *General Theory of Domain Decomposition: Indirect Methods*. Numerical Methods for Partial Differential Equations, 18 (3), pp.296-322, 2002
- [9] Herrera, I. *The indirect Approach to Domain Decomposition*. 14th International Conference on Domain Decomposition Methods, Cocoyoc, Mor., Mex., 2002. Invited Plenary Lecture. www.ddm.org
- [10] Herrera, I., *A Unified Theory of Domain Decomposition Methods*. 14th International Conference on Domain Decomposition Methods, Cocoyoc, Mor., Mex., 2002. www.ddm.org



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- [13] Qin, Q.H. *The Trefftz Finite and Boundary Element Method*. WIT Press, Southampton, 2000

ISMAEL HERRERA, ROBERT YATES

**A New and More General Version of the Hybrid-Trefftz Finite Element Model, Derived by Application of th-Domain Decomposition**

**Location:** Room 049, **Time:** Tuesday, 22 July, 11:00

In recent years the hybrid-Trefftz finite element (hT-FE) model, which originated in the work by Jirousek and his collaborators [1,2], and makes use of an independently defined auxiliary inter-element frame, has been considerably improved. It has indeed become a highly efficient computational tool for the solution of difficult boundary value problems [3-5]. In parallel and to a large extent independently, a general and elegant theory of Domain Decomposition Methods (DDM) has been developed by Herrera and his coworkers [6-8], which has already produced very significant numerical results [8-10]. There is a general formulation of DDM, which subsumes and generalizes the standard approaches. In particular, it is the natural theoretical framework for Trefftz methods, as has been recognized by some of the most conspicuous researchers of this area [11-13]. To clarify further this point, it is important to spell out in greater detail than has been done so far, the relation between Herrera's theory and the procedures studied by researchers working in standard approaches to Trefftz method (Trefftz-Jirousek approach). As a contribution to this end, in this paper the hybrid-Trefftz finite element model is derived in considerable detail, from Herrera's theory of DDM. In addition, by so doing, almost automatically, the hT-FE model is generalized to non-symmetric systems (actually, to any linear differential equation, or system of such equations, independently of its type) and for boundary value problems with prescribed jumps.

- [1] Jirousek J. and Leon N., *Comp. Meth. Appl. Mech. Eng.*, 12, 77-96, 1977
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ROBERT YATES, ISMAEL HERRERA

**Trefftz-Herrera Method: Highly Accurate Numerical Algorithms for Parabolic Equations**

**Location:** Room 049, **Time:** Tuesday, 22 July, 11:25

Trefftz-Herrera method, or indirect method, has attracted much interest recently due to its generality and flexibility in deriving solutions on the internal boundary of the domain decomposition. For the case of elliptic equations, of both second and higher order, the method has been shown to give excellent results. In this paper, for the first time, the method has been suc-

cessfully applied to parabolic equations and a family of numerical procedures which yield solutions to any desired accuracy is presented. A particular member of this family is the well-known Crank-Nicolson second order procedure -thus, the family of numerical algorithms here presented may be thought as a generalization of the Crank-Nicolson procedure-.

ERNESTO RUBIO-ACOSTA, ISMAEL HERRERA, ROBERT YATES

**Parallel Implementation of Indirect Collocation Methods**

**Location:** Room 049, **Time:** Tuesday, 22 July, 11:50

Implementing collocation methods by parallel processing, presents complications that must be overcome in order to profit from the advantages of these resources [1,2]. However, when a TH-Discretization approach is used, a very novel and effective manner of incorporating parallel processors in the computational models has recently been introduced. These new procedures are also based on TH-Domain Decomposition. In particular, a novel, simple and systematic manner of producing the Schur complement and the local matrices, which completely reduces the global equation, is here reported. A discussion of the implications of these new procedures, as well as comparisons with other approaches, will also be presented.

MARTIN DIAZ-VIERA, ISMAEL HERRERA

**Trefftz-Herrera Domain Decomposition Method for Biharmonic Equation**

**Location:** Room 049, **Time:** Tuesday, 22 July, 12:15

Trefftz-Herrera Domain Decomposition Method, that were introduced in previous papers [1-3], are formulated and applied to the biharmonic equation in two dimensions. The basic unifying concept of that theory consists on interpreting domain decomposition methods as procedures for obtaining information about the sought solution at the 'internal boundary' ( $\Sigma$ , which separates the subdomains from each other), sufficient for defining well-posed problems in each one of the subdomains (to be referred as 'local problems'). The distinguishing feature of the Trefftz-Herrera Method is the use of specialized test functions which have the property of yielding any desired information on  $\Sigma$ . The guidelines for the construction of such weighting functions are supplied by a special kind of Green's formulas (Green-Herrera formulas), formulated in Sobolev spaces of discontinuous functions, which permit analyzing the information on  $\Sigma$ ,

[1] Herrera, I., Guarnaccia, J. y Pinder, G.F. *Domain Decomposition Method for Collocation Procedures*, Computational Methods in Water Resources X, Vol. 1, Eds. A. Peters, et. al., Kluwer Academic Publishers, Heidelberg, pp. 273-280, Julio, 1994. (Invited talk).

[2] Guarnaccia, J., Herrera, I. y Pinder, G. *Solution of Flow and Transport Problems by a Combination of Collocation and Domain Decomposition Procedures*, Computational Methods in Water Resources X, Vol. 1, Eds. A. Peters, et.al., Kluwer Academic Publishers, International Conference on Computational Methods in Water Resources, Heidelberg, pp. 265-272, Julio, 1994.

contained in approximate solutions. Some preliminary numerical experiments about efficiency and convergence of this procedure are presented.

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### 3 Abstracts of Contributed Talks

AZIZ AZIMI, VAHID ESFAHANIAN, SIAMAK KAZEMZADEH HANNANI

#### Geometrical Discretization of the Computational Domain for Computations of Axisymmetric Supersonic Flows

**Location:** Room 049, **Time:** Monday, 21 July, 12:20

The construction of single structured grids for complex geometries is not always possible and it can be a high time consuming part in Computation Fluid Dynamics (CFD). The governing equations for computations of supersonic flows are nonlinear and, in general, do not admit an analytical solution. Thus, numerical techniques are indispensable for obtaining the full-scale solution of these equations. In many cases, computational difficulty stems from the inherently complex geometry and boundary conditions of the problem, which excludes the use of high-accuracy global methods.

Therefore, both multi-zone and multi-block methods for complex geometries can be very important tools. These techniques allow us to use various numerical schemes and governing equations in each region of the computational domain and also, multiple regions of structured grid can be joined together to form the optimum grid for the simulation of flow over a complete body. Thus, these abilities increase both the efficiency of the numerical schemes and the accuracy of the results.

By the use of composite-region grids, the computational domain is subdivided into several subdomains bounded by four curves (in two dimensional). Within each sub-region, the grid is generated separately and also the numerical solutions of governing equations can be implemented in all rectangular computational subdomains separately. This allows the solution of large problems, requiring many mesh points, by keeping only the information needed to solve the governing equations in one region in the computer RAM while storing the information of remaining regions in the hard disk. In the multi-block method, the blocks can be had complete communication of flow information across their connecting interfaces, While in multi-zone method, transferring the information of flow variables is done only in streamwise direction.

In practical supersonic aerodynamic calculations, the flow contains various regimes, flow separation regions and strong interaction between inviscid and viscous layers. Thus, the Full compressible Navier-Stokes (FNS) equations or the Reduced Navier-Stokes (RNS) equations such as the Thin-Layer Navier-Stokes (TLNS) and the Parabolized Navier-Stokes (PNS) equations have to be used in these computations. The numerical solution of the TLNS and PNS equations requires less computer memory and calculation time than the FNS equations. Therefore, in order to predict supersonic flow-fields around/in complex configurations, one can choose an appropriate algorithm which uses the TLNS, the PNS equations or the combination of the TLNS and PNS equations along with multi-zone or multi-block schemes to reduce the complexity of grid generation, the computational efforts and required storage.

In this talk, the computations of axisymmetric steady compressible flow which performed over/in complex ge-

ometries to determine the aerodynamic characteristics are presented. The present work is to show the ability of the multi-zone and multi-block methods to simulate the external or interaction of internal-external compressible flow. The numerical scheme used to solve the TLNS and PNS equations in the generalized coordinate system is an efficient, implicit, finite-difference factored algorithm of the Beam and Warming. To limit the generation of wiggles and overshoots near shock waves due to inherent behavior of central differencing method, nonlinear artificial dissipation terms (combination of second and fourth order terms) are added to the numerical method. The present results including surface pressure and temperature are compared to other numerical results and experimental data.

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ISABELLE BOURSIER, A. BOURGEAT, DAMIEN TROMEUR-DERVOUT

### **Modelling of an Underground Waste Disposal Site by Upscaling and Simulation with Domain Decomposition Method**

**Location:** Room 055, **Time:** Thursday, 24 July, 11:00

The global behavior of an underground waste disposal is studied in order to have an accurate upscaled model suitable for the computations involved in safety assessment process. We will decompose the solution to obtain more regular problems. Thanks to this regularity, we can use spectral methods to solve them. So we can get an accurate simulation of the solutions. Moreover, we will apply a multiple domain decomposition technique, here the Aitken Schwarz method, to solve these regular problems. Since spectral methods will give accurate mappings on interfaces, we can expect to be able to apply Aitken acceleration of the Schwarz method and obtain a good convergence.

We need to simulate the auxiliary variables. Their behavior is represented by a diffusive problem on a domain admitting an hole and some periodical conditions. The unit is represented by the flux through the hole. The simulation of these variables needs to be precise since they represent the oscillations at the start of the leak and will influence the rest of the simulation. In order to get an accurate simulation of these variables, we will use spectral methods. Moreover a multiple decomposition technique with mapping techniques will

be realized for spectral methods to solve the auxiliary problems. Where the assumption of the repository periodical structure in the horizontal direction exists, a Fourier discretization is used. In the vertical direction, we use a Chebychev interpolation. Otherwise the problem will be treated only by Chebychev interpolation. We could compare this situation with a simulation in the domain without any hole but with an irregular second member ([4]).

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DAN-GABRIEL CALUGARU, DAMIEN TROMEUR-DERVOUT

### **A non-Overlapping Domain Decomposition Method to Solve Flow in Discontinuous Porous Media**

**Location:** Room 055, **Time:** Thursday, 24 July, 11:20

In realistic hydrological or hydrogeochemical problems, the flow has to be solved in a domain presenting several superposed aquifers separated by layers with low permeability. More complex configurations, taking into account a fault and eventually a landslide along the fault, are also important in some applications as seismic research [1] or oil recovery [2]. In such configurations, each layer can be supposed a homogeneous porous medium and we have a piecewise constant permeability.

To solve the flow problem, a natural non overlapping domain decomposition is considered, taking into account the geological layers and the Dirichlet-Neuman algorithm of Schwarz type is used. However, it is known that this method is non always convergent, the distribution of Dirichlet and Neumann conditions being very important. A theoretical study of the convergence can be given in 1D case, but in 2D or 3D cases it becomes very difficult. On the other hand, the inter-changing of interface conditions being generally difficult to implement, the classical Dirichlet-Neumann algorithm seems not an appropriate decomposition method for such a problem. However, a simple idea can transform this non attractive algorithm in a powerful and simple tool to solve the flow problem: it is the using of its linearity property.

The method described in this communication is based by Aitken acceleration of the iterative solutions obtained by Dirichlet-Neuman algorithm and restricted to the interfaces. The idea of Aitken acceleration has been already used in [3] for the classical additive Schwarz algorithm. We place ourselves this study in the framework of finite elements method and for above configurations, we investigate how one can use the Aitken acceleration for the Dirichlet-Neuman algorithm. Some numerical experiments are presented and the obtained results are compared with the results obtained using other methods (as the method presented in [2] which uses some Robin conditions at the interfaces).

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DIMOS CHARMPIS, MANOLIS PAPADRAKAKIS

### Generation of Subdomains and Subdomain Clusters for Domain Decomposition Methods

**Location:** Lecture Room, **Time:** Thursday, 24 July, 11:20

The practical use of dual-type and balancing Domain Decomposition Methods (DDMs) on parallel computing environments with independent numbers of subdomains and processors constitutes today a basic feature of these solution approaches. The dominant objective during the mesh partitioning task is to produce subdomains with specific geometric characteristics, in order to improve the conditioning of the interface problem. The number of generated subdomains is in general larger than the number of available processors, since the computational performance of dual-type and balancing DDMs is improved both in terms of overall execution time and storage requirements by using mesh partitions with increased numbers of subdomains. Thus, an additional computational task has to be performed, in order to produce subsets of the global problem for the available processors by appropriately organizing the subdomains, generated during the mesh partitioning task, into subdomain clusters.

The performance of dual-type and balancing DDMs is affected by the quality of the produced subdomain clusters. The aim of the additional task for the generation of the clusters is to yield balanced subsets of the domain decomposition problem with minimum communication requirements during the iterative solution procedure. The simple techniques used up to now for this purpose do not always address the issue of communication minimization effectively. This deficiency is overcome with the heuristic approach presented in this work, according to which the task of subdomain cluster generation is handled as a graph partitioning optimization problem: the mesh partitioning output is translated into weighted

graph data (vertices represent subdomains and edges are associated with subdomain connectivity), which are used as input to standard graph partitioning software, in order to organize the subdomains into balanced subdomain clusters with minimum edgecut. Performance results of the presented algorithm for subdomain cluster generation are obtained with a network-distributed FETI implementation executed on ethernet-networked PCs using the Linux operating system and the message passing software PVM.

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YURONG CHEN

### An Accelerated Block-Parallel Newton Method via Overlapped Partitioning

**Location:** Room 049, **Time:** Monday, 21 July, 11:40

In this paper an overlapped block-parallel Newton method is presented for solving large nonlinear systems. The graph partitioning algorithms are first used to partition the Jacobian into weakly coupled overlapping blocks. Then the simplified Newton iteration is directly performed with the diagonal blocks and the overlapping solutions are assembled in a weighted average way at each iteration. In the implementation, an accelerated technique is also proposed to reduce the number of iterations. The conditions under which the algorithm is locally and semilocally convergent are studied, and the numerical results of solving power flow equations with the algorithm in parallel are presented as well.

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JOHN COMPTON, CHRISTOPHER CLOUSE

### **Domain Decomposition and Load Balancing in the AMTRAN Neutron Transport Code**

**Location:** Lecture Room, **Time:** Thursday, 24 July, 11:40

Effective spatial domain decomposition for discrete ordinate (Sn) neutron transport calculations has been critical for exploiting massively parallel architectures typified by the ASCI White computer at Lawrence Livermore National Laboratory. A combination of geometrical and computational constraints has posed a unique

challenge as problems have been scaled up to several thousand processors. Carefully scripted decomposition and corresponding execution algorithms have been developed to handle a range of geometrical and hardware configurations.

JEAN-MICHEL CROS

### **Rigid Body Modes within the Framework of Domain Decomposition Methods**

**Location:** Room 055, **Time:** Monday, 21 July, 11:40

Semi-definite problems arise frequently, especially for aeronautic applications. Within the framework of domain decomposition methods, the difficulty is to compute the rigid body modes such structures. In this paper, we consider domain decomposition methods whose auxiliary problem uses corner modes. The non singularity of each subdomain can be guaranteed simply by

making sure that every subdomain has enough corner nodes. But, the singularity of the global problem induces the singularity of the coarse matrix. Then, we expose how the null space for the global problem can be expressed as a linear arrangement of the corner modes. Numerical results are presented for static and modal analysis

DAOUD S. DAOUD

### **Explicit Implicit Non Overlapping Domain Decomposition Method with Splitting up method for Multi Dimensional Parabolic Problem**

**Location:** Room 049, **Time:** Thursday, 24 July, 11:20

The explicit implicit domain decomposition methods are a non iterative types of methods for non overlapping domain decomposition. In comparison with the classical Schwarz algorithm for parabolic problem the former methods are computationally and communicationally more efficient for each time step but due to the use of the explicit step for the interface prediction the methods suffer from the accuracy of the usual explicit

scheme. In this article a specific type of splitting up method for the dependent variables is initially considered to solve the two or three dimensional parabolic problem over non overlapped subdomains defined for each spatial variable. This type of splitting up will provide a flexibility to start by any spatial variable to solve over the subdomains, and also a flexibility to choose different mesh spacing for each spatial variable. We also

presented the parallel explicit splitting up algorithm to define (predict the interface boundary conditions with respect to each spatial variable and for each nonoverlapping subdomains. The use of the presented explicit scheme will provide an interface boundary conditions to have a square or cubic nonoverlapping subdomains rather than stripes along the specific spatial variable for interface line as in the other techniques. The parallel second order splitting up algorithm is then considered to solve the sub problems defined over each subdomain, the correction step will then be considered for the predicted interface nodal points using the most recent solution values over the subdomains. Finally several model problems are considered to test the efficiency of the presented algorithm.

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MARCO DISCACCIATI

### Domain Decomposition Methods for Coupling Stokes and Darcy Equations

**Location:** Room 055, **Time:** Thursday, 24 July, 11:40

This presentation concerns the study of mathematical and numerical models for simulating incompressible fluid flows through heterogeneous media. In particular, we consider the case of free fluids which can filtrate through a porous medium occupying a domain neighbouring the free fluid itself. This topic has many important applications, among which we recall the hydrological environmental ones and mass transfer in biomechanics.

In this talk we outline the mathematical and numerical analysis of a coupled Stokes/Darcy problem. In particular, by adopting the Beavers and Joseph interface conditions, we will assess the well-posedness of the global problem, and we will introduce a suitable Galerkin finite element approximation.

Then, we will focus our attention on effective iterative substructuring methods, which allow to solve the global problem through the independent solution of both Stokes and Darcy problem in each subdomain. Through the analysis of suitable Steklov-Poincaré interface operators, we can characterize an optimal preconditioner to solve the discrete algebraic problem, which can be applied in the framework of Krylov type methods. The effectiveness of the computational methods we have introduced, will be shown on some test cases.

tioner to solve the discrete algebraic problem, which can be applied in the framework of Krylov type methods. The effectiveness of the computational methods we have introduced, will be shown on some test cases.

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MARC GARBEY, W. SHYY

### Multilevel Solutions, Least Square Extrapolation and a Posteriori Error Estimate

**Location:** Room 049, **Time:** Monday, 21 July, 11:00

A posteriori error estimators are fundamental tools used to provide confidence in the numerical computation of PDEs. Unfortunately, the main theories of a posteriori estimators have been developed largely in the finite element framework, first for linear elliptic operators and second for non-linear PDEs in the absence of disparate length scales such as boundary layers.

On the other hand, there is a strong tradition in using grid refinement combined with Richardson extrapolation to produce CFD solutions with improved accuracy and, therefore, a posteriori error estimates on coarse grid solutions. But in practice, the effective order of a numerical method often depends on space location and is not accurately satisfied on different levels of grids used in the extrapolation formula. The Richardson extrapolation method then becomes unreliable. We propose a more robust and numerically efficient method that automatically finds the order of a method as the solution of a least square minimization problem on the residual. We introduce a multi-level least square extrapolation method that post-processes several grid solutions to recover accuracy and provide a posteriori error esti-

mate. This method is not restricted to uniform refined grid solutions, nor strictly embedded grid levels. Our least square extrapolation method is a post-processing of data produced by existing PDE codes, that is easy to implement and does not require detailed knowledge of the PDE code itself. This work is a continuation of papers presented at DD13 and DD14 — see references.

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MATTHIAS HEINKENSCHLOSS, HOANG NGUYEN

### **Domain Decomposition Preconditioners for the Optimization of Distributed Parameter Systems**

**Location:** Room 005, **Time:** Thursday, 24 July, 11:00

Optimization of distributed systems in the context of parameter estimation, optimal control, or optimal design plays an important role in science and engineering. These optimization problems pose several computational difficulties arising, among other things, from the large number of variables and the conditioning of subproblems, which often depends on the choice of a penalty parameter.

In this talk we present and analyze a Neumann-Neumann DD preconditioner for the solution of elliptic linear quadratic optimal control problems. The Neumann-Neumann DD preconditioner is applied to the optimality system. Theoretical results and numerical

tests show that the dependence of the quality of this preconditioner on mesh size and subdomain size is comparable to its counterpart applied to elliptic equations only.

Numerical tests indicate that, unlike other DD preconditioners for optimality systems, the proposed Neumann-Neumann DD preconditioner is rather insensitive to the choice of the penalty parameter, which makes this preconditioner attractive as a subproblem solver in interior-point methods. Theoretical and numerical results as well as some comparisons of the Neumann-Neumann DD preconditioner with other DD preconditioners for optimality systems are presented.

BERNHARD HIENTZSCH

### **Domain Decomposition Preconditioners for Spectral Nedelec Elements in Two and Three Dimensions**

**Location:** Room 005, **Time:** Thursday, 24 July, 12:00

In previous work, we proposed overlapping Schwarz preconditioners for the model problem

$$\alpha(\mathbf{u}, \mathbf{v}) + \beta(\mathbf{curl} \mathbf{u}, \mathbf{curl} \mathbf{v}) = \mathbf{f}(\mathbf{u})$$

discretized by high-order spectral versions of Nédélec elements and presented a theoretical analysis and experiments in two dimensions.

Our implementation included fast matrix-vector multiplies and fast solvers using the block sum of tensor-product structure of the discretization. This allows the implementation of spectral methods and the use of elements of very high order (up to 50-100 in each direction).

We have now used these building blocks to implement iterative substructuring preconditioners for the two-dimensional case and have extended the fast matrix-vector multiply and the fast solvers to the three-dimensional case, and used it to implement domain decomposition preconditioners in the three-dimensional case. We will present numerical experiments for these methods in two and three dimensions.

We have also extended and simplified our analysis of the overlapping methods and will present some theoretical and numerical results for the analysis of iterative substructuring methods.



FRANK HUELSEMANN, ULRICH RUEDE

### Optimised Data Structures for Efficient Large Scale Parallel Computations

**Location:** Lecture Room, **Time:** Thursday, 24 July, 12:00

Scientific computations strive to be efficient. Yet the standard definition of parallel efficiency does not take the performance of the scalar program into account. Fully efficient parallel programs combine high performance computations on the individual processing elements with high parallel efficiency. The optimised data structures presented in this talk are one step towards such efficient parallel programs.

Current computer architectures perform operations on regular data structures typically several times faster than operations on irregular or unstructured ones. This observation is driving the design of the *hierarchical hybrid grid framework*, which first generates and then ex-

ploits regular substructures on globally unstructured grids. The approach is based on the assumption that the problem geometry can be represented by a much coarser grid than the one needed for an accurate resolution of the solution. The necessary refinement is carried out in such a way as to generate regular subregions in the global grid. The data structures for storing unknowns and operators on the refined grid are designed to exploit the regularity of the subregions. Furthermore, by introducing ghost cells even in the sequential case they are well suited for parallel computations, as will be shown for overlapping Schwarz techniques.

MOURAD ISMAIL, SILVIA BERTOLUZZA, BERTRAND MAURY

### The Fat Boundary Method: Convergence and Error Analysis

**Location:** Room 049, **Time:** Thursday, 24 July, 11:00

The *Fat Boundary Method* (FBM), introduced by B. Maury, is a fictitious domain type method for solving partial differential equations in a domain with holes  $Wu\bar{B}$  – where  $B$  is a collection of smooth open subsets (the holes) – that consists in splitting the initial problem into two parts to be coupled via Schwarz type iterations: the solution, with a fictitious domain approach, of a problem set in the whole domain  $W$ , for which fast solvers can be used, and the solution of a collection of independent problems defined on narrow strips around

the connected components of  $B$ , that can be performed fully in parallel.

We will analyze the FBM in the framework of a finite element discretization and prove convergence of the Schwarz iterations and an almost optimal error estimate for the resulting discrete solution.

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BORIS N. KHOROMSKIJ, W. HACKBUSCH, R. KRIEMANN

### Direct Schur Complement Method by Hierarchical Matrix Techniques

**Location:** Room 005, **Time:** Monday, 21 July, 11:00

A class of hierarchical matrices ( $\mathcal{H}$ -matrices) allows the data-sparse approximation to integral and more general nonlocal operators (say, the elliptic Green function and Poincaré-Steklov operators) with almost linear complexity. We consider the  $\mathcal{H}$ -matrix-based approximation to the Schur complement on the interface [HKhKr1:03] corresponding to the finite element discretisation of an elliptic operator  $\mathcal{L}$  with piecewise constant coefficients in  $\mathbb{R}^2$ . As with the standard Schur complement domain decomposition methods, we split the elliptic inverse  $\mathcal{L}^{-1}$  as a sum of local inverses associated with subdomains (this can be implemented in parallel), and the corresponding Poincaré-Steklov operator on the interface. We focus on the data-sparse approximation to the Poincaré-Steklov operator and its inverse. Using the hierarchical formats based on weakened admissibility criteria (cf. [HKhKr1:02]) we elaborate the *approximate Schur complement inverse* in an explicit form that is proved to have a linear-logarithmic cost  $O(N_\Gamma \log^q N_\Gamma)$ , where  $N_\Gamma$  is the number of degrees of freedom on the interface. In the case of piecewise constant coefficients, the local Schur complements are

approximated by the explicit BEM representations. We also prove the asymptotically optimal error estimate in the case of piecewise linear finite elements. In the case of variable coefficients, our method manifests a linear-logarithmic complexity in the discrete problem size  $N_\Omega$ . Numerical examples confirm the almost linear cost of our direct Schur complement method. In particular, for the discrete Laplacian on  $255 \times 255$  and  $511 \times 511$  grids with  $6 \times 6$  decomposition we have  $N_\Gamma = 2525$  and  $N_\Gamma = 5085$ , respectively. The elapsed CPU times to compute the explicit Schur complement inverses on the interface with the relative error  $1.1e - 03$  are  $29.0sec$  and  $74.0sec$ , correspondingly (SUN 6800).

- [1] W. Hackbusch, B.N. Khoromskij and R. Kriemann. *Hierarchical Matrices Based on Weak Admissibility Criterion*. Preprint MPI MIS 2, Leipzig, 2003; Computing (submitted).
- [2] W. Hackbusch, B.N. Khoromskij and R. Kriemann. *Direct Schur Complement Method by Hierarchical Matrix Techniques*. Preprint MPI MIS, Leipzig, 2003 (in progress).

PIOTR KRZYZANOWSKI

### Domain Decomposition for Discontinuous Galerkin Method with Application to Stokes Flow

Location: Room 055, Time: Thursday, 24 July, 12:20

A domain decomposition based solver for a Discontinuous Galerkin approximation of Stokes equations will be presented and discussed.

DEEPAK KULKARNI, DANIEL TORTORELLI

### Domain Decomposition Based Two-Level Newton Scheme for non-Linear Problems

Location: Room 049, Time: Monday, 21 July, 12:00

There has been a vast amount of research done in the field of domain decomposition[1], most of which deals with linear problems. When solving non-linear problems the equations are linearized, and then the domain decomposition approaches are used to solve the ensuing linear problem. There are often applications however, where non-linear effects are restricted to small regions. The above mentioned schemes are not efficient in handling such localized non-linearities. In this paper we develop a domain decomposition scheme for non-linear problems based on a two-level Newton approach. The scheme consists of a lower level Newton scheme to solve the sub-domain problems. As such, if a sub-domain problem behaves linearly, we need not assemble and factorize its stiffness matrix repeatedly. An upper level Newton scheme is used to solve the interface problem. This latter problem is obtained by enforcing the necessary kinematic and flux interfacial conditions. In our  $C^0$  formulation we prescribe essential boundary conditions on the interface to satisfy the kinematic interfacial conditions. In this way we avoid the complications associated with floating sub-domains in lagrange multiplier based domain decomposition schemes. The interfacial flux condition is satisfied by balancing the reactive nodal fluxes, which result from the prescribed essential boundary conditions on the interface nodes. A similar approach has been used previously for coupling

multi-physics problems[2, 5]. The developed approach has been implemented in parallel using MPI. The decomposition of the domain into several sub-domains is done by METIS<sup>®</sup> a graph partitioning routine. All sub-domain solves are performed in parallel, and we employ a sparse direct solver to factorize the sub-domain matrices. The sensitivities needed to assemble the interface problem involve a forward and back solve using the already factored sub-domain matrices. To solve the interface problem we use the iterative solvers present in PETSc<sup>®</sup>. The probing method is used to precondition the iterative solver. The formation and solution of the preconditioner are also done in parallel. Thus every stage of the proposed approach has been parallelized and shows good scalability.

- [1] D.E. Keyes, T.F. Chan, G. Meurant, J.S. Scroggs, and R.G. Voigt (eds), *Domain Decomposition Methods for Partial Differential Equations*, SIAM J. Sci. Statistic. Comput., 13, 967-993, 1992.
- [2] N.R. Aluru and J. White, *A Multilevel Newton Method for Mixed-Energy Domain simulation of MEMS*, Journal of Microelectromechanical systems, 1999, vol.8,no.3,pp.299-308
- [3] Jin Young Kim, et. al., *Improved multilevel Newton solvers for fully-coupled multi-physics problems*, International Journal for Numerical Methods in Engineering, submitted for review.

SABINE LE BORNE

### Hierarchical Matrices for Convection-Dominated Problems

Location: Room 005, Time: Monday, 21 July, 11:20

Hierarchical matrices ( $\mathcal{H}$ -matrices) provide a technique for the sparse approximation of large, fully populated matrices. This technique has been shown to be applicable to stiffness matrices arising in boundary element method (BEM) and finite element method (FEM) applications. In the latter case, it is the inverse stiffness matrix that is fully populated and approximated by an  $\mathcal{H}$ -matrix.

For elliptic operators with  $L^\infty$ -coefficients, it has been shown that the standard partitioning algorithm in connection with the standard admissibility condition lead to hierarchical matrices that approximate the (inverse) of the stiffness matrix with an error of the same order as the discretization order while having nearly optimal storage complexity.

In this talk, we will briefly review the standard con-

struction of  $\mathcal{H}$ -matrices. We will then demonstrate the shortcomings of applying the standard partitioning and admissibility condition to the singularly perturbed convection-diffusion equation. We will construct a modified (hierarchical) partitioning of the index and block index sets together with a modified admissibility condition, both depending on the (constant) convection direction and the grid parameter  $h$ . An important observation is that the  $\mathcal{H}$ -matrix approximant benefits from the convection direction aligning with the underlying grid. Numerical results will illustrate the effect of the proposed changes.

- [1] S. Le Borne,  *$\mathcal{H}$ -matrices for convection-diffusion problems with constant convection*, to appear in Computing (2003)

HONGWEI LI, JIACHANG SUN

**Robin Transmission Conditions for Overlapping Additive Schwarz Method Applied to Elliptic Problems**

**Location:** Room 049, **Time:** Thursday, 24 July, 12:00

We consider overlapping additive Schwarz method with Robin conditions as the transmission conditions (interior boundary conditions). The main difficulty left in this field is how to select the parameters for Robin conditions – these parameters have strong effect on the convergence rate of ASM. In this paper, we proposed a very simple way to determine the optimal (nearly optimal) parameters for general linear elliptic problems. The parameters can be determined in advance without any calculations. The key idea different from other authors (M.J. Gander, F.Nataf etc.) is that we give up to look for the real optimal, instead, we just try to find the nearly optimal which perform as better as the optimal.

- [1] WEI PAI TANG, *Generalized Schwarz splittings*, SIAM J. Sci. Stat. Comp., 13(2):573-595, 1992.
- [2] M.J. GANDER, L. HALPERN AND F. NATAF, *Optimized Schwarz Methods*, Proceedings of the 12th International Conference on Domain Decomposition, pp. 15-27, 2000.
- [3] B. ENGQUIST, H.K. ZHAO, *Absorbing Boundary Conditions for Domain Decomposition*, Applied Numerical Mathematics, Vol. 27, 1998, pp341-365.

- [4] P.L. LIONS, *On the Schwarz alternating method. I*, In R. Glowinski, G.H. Golub, G.A. Meurant and J. Periaux, editors, "First International Symposium on Domain Decomposition Methods for Partial Differential Equations", SIAM Publications, 1988, 1-42
- [5] P.L. LIONS, *On the Schwarz alternating method. II*, In T.F. Chan, R. Glowinski, J. Periaux, and O.B. Widlund, editors, "Domain Decomposition Methods", SIAM Publications, 1989, 47-70
- [6] P.L. LIONS, *On the Schwarz alternating method III: A variant for non-overlapping subdomains*, *Proceedings of Third International Symposium on Domain Decomposition Methods for Partial Differential Equations*. T.F. Chan, R. Glowinski, J. P eriaux, and O. Widlund, eds., Philadelphia, PA, 1990, SIAM., pp. 202-223.
- [7] F. NATAF, *Interface Connections in Domain Decomposition Methods*, NATO Advanced Study Institute, Modern Methods in Scientific Computing and Applications, Universit e Montral, Juillet 2001, vol.75 in the NATO Science Ser.II.

LESZEK MARCINKOWSKI, XIAO-CHUAN CAI

**Parallel Performance of a Two-Level ASPIN Algorithm**

**Location:** Room 049, **Time:** Monday, 21 July, 11:20

The one-level nonlinear additive preconditioned inexact Newton method (ASPIN1) is a powerful parallel iterative method for solving system of sparse nonlinear equations and the method proved itself to be more robust than inexact Newton algorithms for many problems. ASPIN1 is nonlinearly scalable with respect to both the mesh size and the number of processors, however, it is not linearly scalable with respect to the num-

ber of processors. To improve the linear scalability a two-level version was introduced recently (ASPIN2). In this talk, we present some new numerical results for solving some incompressible flow problems on very fine meshes using the two-level ASPIN algorithm. The focus of the discussion will be on the case of many subdomains.

V ERONIQUE MARTIN

**Domain Decomposition Methods for Viscous Shallow Water Equation**

**Location:** Room 055, **Time:** Thursday, 24 July, 12:00

Classical Schwarz methods have been first applied to stationary scalar equations and for overlapping subdomains (Dirichlet transmission conditions were used at the interface). Then have been introduced more efficient methods (with Robin type transmission conditions) in the sense that overlap is no longer necessary and that these conditions can be chosen such that the convergence rate is minimized .

Some recent works have been proposed to solve time dependent equations, formulating algorithms directly for the original problem without first discretizing in time,

contrary to the classical approach.

In this talk we apply this method to the viscous Shallow Water equation. We first write the optimal transmission conditions which coincide with the transparent boundary conditions. But due to their non local nature, we have to approximate them locally. We propose to choose these approximations using physical properties and by minimization of the convergence rate. We propose numerical results which illustrate the efficiency of the method.

L. ANGELA MIHAI

**A Two-Grid Alternate Strip-Based Domain Decomposition Strategy in Two-Dimensions**

**Location:** Room 049, **Time:** Thursday, 24 July, 12:20

The *Two-Grid Alternate Strip-Based Domain Decomposition Strategy* designs efficient preconditioners with good parallelism properties, for the discrete systems which arise from the finite element approximation of symmetric elliptic boundary value problems in two dimensional Euclidean spaces. This new approach offers an alternative methodology which draws upon the strengths of both overlapping and non-overlapping domain decomposition techniques. The key ingredients are the alternate strip-based-solvers introduced in [1], which generate algorithms in two-stages and allow the use of efficient subdomain preconditioners such as a two-grid  $V$  or  $W$  cycle. Each strip is a union of non-overlapping subdomains and the global interface between subdomains is partitioned as a union of edges between strips (which include also all the vertex points) and edges between subdomains which belong to the same strip (inside strips, the interface edges do not contain their end points). We emphasise that a novel fea-

ture of our approach is that at each stage the direction of the strips changes and with it, the coupling between vertex points and edges. The new strategy extends in a straightforward manner to the three dimensional case.

The presentation is based on the Ph.D. research of L.A.M. under the supervision of Dr. A.W. Craig. Part of this work has been submitted for publication, as [CM1], or is in preparation for submission, as [2] and [3].

- [1] A. W. Craig and L.A. Mihai, *A class of alternate strip-based-solvers for elliptic PDE's*, submitted.
- [2] A.W. Craig and L.A. Mihai, *A class of alternate strip-based iterative substructuring algorithms for elliptic PDE's in two dimensions*, in preparation.
- [3] A.W. Craig and L.A. Mihai, *A class of alternate strip-based iterative substructuring algorithms for elliptic PDE's in three dimensions*, in preparation.

REINHARD NABBen, C. VUIK

### A Comparison of Deflation and Coarse Grid Correction Applied to Porous Media Flow

**Location:** Room 005, **Time:** Monday, 21 July, 11:40

We compare various preconditioners for the numerical solution of partial differential equations. We compare a coarse grid correction preconditioner used in domain decomposition methods with a so-called deflation preconditioner.

We prove that the effective condition number of the deflated preconditioned system is always, i.e. for all deflation vectors and all restrictions and prolongations,

below the condition number of the system preconditioned by the coarse grid correction. This implies that the Conjugate Gradient method applied to the deflated preconditioned system converges always faster than the Conjugate Gradient method applied to the system preconditioned by the coarse grid correction.

Numerical results for porous media flows emphasize the theoretical results.

FABIENNE OUDIN, J. BARANGER, MARC GARBEY

### Acceleration of the Schwarz Method for Elliptic Problems

**Location:** Room 005, **Time:** Monday, 21 July, 12:20

The Schwarz algorithm has two important qualities that make its use in Computational Fluid Dynamic applications very attractive. First, the method is very easy to implement. A popular practice in CFD is to start from an existing code, partition the domain of computation into overlapping sub-domains, run the CFD solvers in parallel between sub-domain and exchanges information on artificial boundaries between neighbor's sub-domains at the end of each iteration cycle in order to match the solution or some fluxes. This practice avoids a complete re-writing of the CFD code, but convergence of the algorithm might be (very) slow.

The second important quality of the Schwarz algorithm is that (memory) scalability of the iteration step in a parallel implementation is implicitly given thanks to the fact that communications are always between neighbor's sub-domains. But as pointed out in Keyes - Domain Decomposition Proceeding of 98-, the numerical algorithm does not scale at all, because the number of iterates to reach convergence increases as the number of sub-domains increases.

In this paper, we present a family of domain decomposition based on Aitken like acceleration of the Schwarz method seen as an iterative procedure with linear rate

of convergence. This paper is a generalization of our method first introduced in the 12th international conference on domain decomposition [2000] that was restricted to Cartesian grids. The general idea is to construct an approximation of the eigenvectors of the trace transfer operator that have the dominant eigenvalues and accelerate these components after few Schwarz iterates. We consider here examples with the finite volume approximation on general quadrangle meshes of Faille [1992], finite differences with non matching grids and Finite element discretisation.

- [1] J. Baranger, M. Garbey and F. Oudin-Dardun, *On Aitken Like Acceleration of Schwarz Domain Decomposition Method Using Generalized Fourier*, to appear in the proceeding of DD14.
- [2] N. Barberou, M. Garbey, M. Hess, M. Resch, T. Rossi, J. Toivanen and D. Tromeur Dervout, *On the Efficient Meta-computing of linear and nonlinear elliptic problems*, to appear in the Journal of Distributed Parallel Computing: special issue on grid computing.
- [3] M. Garbey and D. Tromeur Dervout: *On some Aitken like acceleration of the Schwarz Method*, Int.

LUCA F. PAVARINO, PIERO COLLI FRANZONE

**A Parallel Solver for Reaction-Diffusion Systems in Computational Electrophysiology**

**Location:** Lecture Room, **Time:** Monday, 21 July, 11:00

In this work, a parallel solver for numerical simulations in computational electrophysiology in three dimensions is introduced and studied. The solver is based on the anisotropic Bidomain cardiac model, consisting of a system of degenerate parabolic reaction-diffusion equations describing the intra and extracellular potentials of the myocardial tissue.

This model includes intramural fiber rotation and anisotropic conductivity coefficients, that can be fully orthotropic or axially symmetric around the fiber direction. In case of equal anisotropy ratio, this system reduces to the simpler anisotropic Monodomain model, consisting of one reaction-diffusion equation describing the transmembrane potential only. These cardiac mod-

els are coupled with a membrane model for the ionic currents, consisting of a system of ordinary differential equations that can vary from the simple FitzHugh-Nagumo (FHN) model to the more complex Luo-Rudy phase I (LR1) model.

The solver employs structured isoparametric  $Q_1$  finite elements in space and a semi-implicit adaptive method in time. Parallelization and portability are based on the PETSc parallel library.

Large-scale computations with up to  $O(10^7)$  unknowns have been run on parallel computers, simulating excitation and repolarization phenomena in three dimensional domains.

MICOL PENNACCHIO

**The Mortar Finite Element Method in Computational Electrophysiology**

**Location:** Lecture Room, **Time:** Monday, 21 July, 11:40

The excitation process in the myocardium can be described by the evolution of the transmembrane and extracellular potentials, which are solutions of a reaction diffusion (R-D) system. This system is still unpractical for realistic simulations due to its computational cost.

A non-conforming non-overlapping domain decomposition method is proposed to overcome the computational costs caused by the narrow cardiac excitation wavefront. By appropriate estimation of the transmembrane potential, the part of the domain requiring a finer grid, typically the regions closed to the cardiac wavefront, is determined. Then, this part is cut out and a new finer grid can be constructed allowing a more accurate computation near the wavefront. Finally, using the mortar finite

element method, the matching of different discretizations on adjacent subdomains can be weakly enforced.

The numerical technique is first applied to the elliptic bidomain equation of the extracellular potential and then to the R-D system starting with the case of "equal anisotropy ratio" that allows to work only with a parabolic equation.

The performance of the numerical technique is investigated showing its efficiency if compared with the classical conforming FEM.

- [1] M. Pennacchio, *The mortar finite element method for the cardiac "bidomain" model of extracellular potential*, to appear on J. Sci. Comp.

JACQUES PERIAUX, H.Q. CHEN, R. GLOWINSKI, JACQUES PERIAUX, J. TOIVANEN

**Domain Embedding/Controllability Methods for the Conjugate Gradient Solution of Wave Propagation Problems: Application to Shape Optimization**

**Location:** Room 005, **Time:** Thursday, 24 July, 11:20

The main goal of this paper is to discuss the numerical simulation of propagation phenomena for time harmonic electromagnetic waves by methods combining controllability and fictitious domain technologies.

The method relies on :

1. distributed Lagrangian multipliers, which allow the propagation to be simulated on an obstacle free computational region using regular finite element meshes essentially independent of the geometry of the obstacle;
2. a controllability formulation which leads to conjugate gradient algorithms with good convergence properties to time-periodic solutions.

This novel methodology has been validated by the solutions of test cases associated to non trivial geometries,

possibly non-convex. The numerical experiments show that the new method performs as well as the method discussed in [ref.1] where obstacle fitted meshes were used.

We shall conclude this presentation by describing the results of a shape optimization problem consisting in the RCS minimization of aerodynamic obstacles where the above simulator is coupled with Evolutionary Algorithms [ref. 2].

- [1] M.O. Bristeau, R. Glowinski, J. Periaux, *Controllability methods for the computation of time periodic solutions; applications to scattering*, Journal of Computational Physics, 147, 1998, pp: 265-292.

- [2] R.A.E. Makinen, J. Periaux and J. Toivanen, *Multi-disciplinary shape optimization in aerodynamics and*

JALI PIESKA, E. LAITINEN,, A. LAPIN

**Predictor-Corrector Methods for Solving Continuous Casting Problem**

**Location:** Room 005, **Time:** Thursday, 24 July, 11:40

In this paper we present numerical approach to solve the continuous casting problem. The main tool is to use IPEC<sup>1</sup> method and DDM similar to [lapi] with multi-level domain decomposition. The general idea of these kind of algorithms is first to solve the problem in artificial boundaries (predictor step). After the solution at the boundaries is known then it can be used as Dirichlet type boundary condition and the noncoupled subdomain problems can be solved parallel. The last step of these methods is to correct the solution at the artificial boundaries (corrector step). The numerical results show the linear speedup of the IPEC-method.

The continuous casting problem can be stated as follows. Let  $\Omega$  be an open bounded domain in  $\mathbb{R}^2$  with boundary  $\Gamma$ ,  $T > 0$ ,  $Q \equiv \Omega \times ]0, T[$  and  $\Sigma = \Gamma \times ]0, T[$ . The domain  $\Omega$  is occupied by thermodynamically homo-

geneous and isotropic metal. We denote by  $H(x, t)$  the enthalpy related to unit mass and by  $u(x, t)$  the temperature for  $(x, t) \in Q$ . The continuous casting problem leads to following boundary-value problem: find  $u = u(x, t)$  such that

$$\begin{aligned} \frac{\partial H(u)}{\partial t} + v \frac{\partial H(u)}{\partial x_2} - \Delta u &= 0 && \text{for } x \in \Omega, t > 0 \\ u = z(x_1, t) > 0 &&& \text{for } x \in \Gamma_1, t > 0 \\ \partial u / \partial n + au = g, \quad a \geq 0, g \geq 0 &&& \text{for } x \in \Gamma_2, t > 0 \\ u = u_0(x) > 0 &&& \text{for } x \in \Omega, t = 0. \end{aligned}$$

- [1] A.V Lapin and J. Pieskä, *On the parallel domain decomposition algorithms for time-dependent problems*, Lobachevskii Journal of Mathematics, Vol. 10, p. 27-44, 2002. <http://ljm.ksu.ru/vol10/lpp.htm>

TALAL RAHMAN, XUEJUN XU, S.H. LUI

**An Additive Schwarz Method for the Morley Element Approximation of a Non-Linear Biharmonic Equation**

**Location:** Room 055, **Time:** Monday, 21 July, 12:00

We consider the well-known Morley non-conforming finite element approximation of a nonlinear biharmonic equation which is related to the well-known two-dimensional Navier-Stokes equations.

In this talk, we discuss a two-level additive Schwarz

method for the discrete nonlinear algebraic system. For sufficiently small Reynolds number, the method is optimal, i.e., the convergence rate is independent of the mesh size and the number of subdomains. Numerical results will be presented.

VLADIMIR RYCHKOV, S.P. KOPYSSOV, I.V. KRASNOPYOROV, A.K. NOVIKOV

**Parallel Distributed Object-Oriented Model of Domain Decomposition**

**Location:** Lecture Room, **Time:** Thursday, 24 July, 12:20

Domain decomposition is one of the efficient methods to solve the problems on complex geometries and large meshes with adaptive refinement. It reduces the computational costs to find the solution within given accuracy. In this case it is necessary to combine computational and geometrical models and map the algorithms to hardware software platforms. In present work it is suggested to use object-oriented approach and parallel distributed middleware. The object-oriented model of domain decomposition and adaptive finite element method is developed. All objects of the model are sorted into four groups:

- Modeling classes used to create and describe analytical model of the problem (nodes, elements, domains, boundary conditions, etc.).
- Numerical classes used to carry out the numerical operations and store the data (matrices, vectors, system of equations, graphs, etc.).
- Analytical classes used to perform the analysis of the problem (solution algorithms, integration

schemes, boundary condition imposition, equation ordering methods, etc.)

- Domain Decomposition classes used in the analysis of the problem by domain decomposition (subdomains, subdomain equation solver, etc.), they are based on the objects of the first groups.

Inheritance, polymorphism and encapsulation enable to make experiments with new domain decomposition algorithms using different methods of domain partitioning, storing and solving of systems of equations, boundary conditioning, and, as to adaptive solution, various mesh refinements, error estimations, refinement criteria. At present substructuring, frontal, element-by-element and some other methods are implemented and tested. Different parallel distributed implementations of objects were examined on MPI and CORBA. As a result, parallel distributed component technique are suggested to develop object-oriented software for domain decomposition. It is based on CORBA, component model CCM, asynchronous method invocation AMI and encapsula-

<sup>1</sup>Implicit predictor-explicit corrector

tion of MPI applications.

- [1] V.N. Rychkov, I.V. Krasnopyorov, S.P. Kopyssov, *Parallel distributed object-oriented computing system for finite element analysis*, Mathematical Modeling, 9, pp. 81-86 (2002).
- [2] A.K. Novikov, S.P. Kopyssov, *Parallel element-by-element conjugate gradient method with decreased communications costs*, The International Summer School "Iterative Methods and Matrix Computations" Editor Gene H. Golub, Lev A. Krukier, 2-9 June 2002, Rostov-on-Don, Russia, pp. 450-454 (2002).
- [3] S.P. Kopyssov, M. Yu. Alies, A.K. Novikov, S.L. Us-tuzhanin, *Domain decomposition for Elastic Problem Solving with Dynamic Refinement Mesh Model*, 2nd Russian Conference on High Performance Computing and Their Application. Moscow State University, Moscow, Russia, pp. 119-122 (2000).
- [4] S.P. Kopyssov, A.K. Novikov, *Parallel algorithms of adaptive refinement and partitioning of unstructured grids*, Mathematical modeling, 9, pp. 91-96 (2002).
- [5] S.P. Kopyssov, A.K. Novikov, *Parallel Adaptive Mesh Refinement with Load Balancing for Finite Element Method*, Lecture Notes in Computer Science, 2127, p.266-273 (2001).
- [6] V.N. Rychkov, I.V. Krasnopyorov, S.P. Kopyssov, *Middleware for high performance computing*, Numerical methods and programming, 2, pp. 117-132 (2001).

DONGWOO SHEEN, JIM DOUGLAS JR., C. PARK, JUAN E. SANTOS

### **$P_1$ Nonconforming Finite Element Method on Quadrilaterals and its Domain Decomposition Procedure**

**Location:** Room 055, **Time:** Monday, 21 July, 11:20

Recently a new  $P_1$  nonconforming finite element space on quadrilaterals has been proposed in [1] for second-order elliptic problems. This new nonconforming finite element space has only 3 degrees of freedom on quadrilaterals, and thus the global degrees of freedom for this space will be about half the usual nonconforming finite element space on quadrilaterals.

This notion is easily generalized to three dimension so that only 4 degrees of freedom will be needed for hexa-

hedrons.

After introducing this element and discussing several computational issues briefly, we will discuss some ongoing research results on domain decomposition procedures using this new element.

- [1] C. Park and D. Sheen,  *$P_1$ -nonconforming quadrilateral finite element methods for second-order elliptic problems* SIAM J. Numer. Anal., 41, pp. 624-640, 2003.

DAN STEFANICA

### **Lower Bounds for Overlapping and Nonoverlapping Domain Decomposition Preconditioners for Mortar Element Methods**

**Location:** Room 055, **Time:** Monday, 21 July, 11:00

We establish lower bounds for the condition numbers of two domain decomposition methods for elliptic problem discretized by geometrically nonconforming mortar finite elements:

- two-level overlapping additive Schwarz algorithms with unstructured coarse spaces; and

- iterative substructuring algorithms.

The lower bounds coincide, up to constants, with the upper bounds established elsewhere in the literature. The optimality of the condition number estimates is thus established.

DANIEL B. SZYLD

### **Algebraic Analysis of Schwarz Methods for Singular Systems**

**Location:** Room 005, **Time:** Monday, 21 July, 12:00

Recently, an algebraic formulation of additive and multiplicative Schwarz method was developed. In this paper we use this algebraic formulation to prove new convergence results for these methods when applied to singular systems of linear equations. In particular, we apply these to the numerical solutions of Markov chains.

- [1] M. Benzi, A. Frommer, R. Nabben, and D.B. Szyld. *Algebraic theory of multiplicative Schwarz methods*. Numerische Mathematik, 89:605-639, 2001.
- [2] A. Frommer and D.B. Szyld. *Weighted max norms, splittings, and overlapping additive Schwarz iterations*. Numerische Mathematik, 83:259-278, 1999.
- [3] R. Nabben. *Comparisons between additive and multiplicative Schwarz iterations in domain decomposition methods*. Numerische Mathematik, on line, 18 February 2003, to appear in print.
- [4] I. Marek and D.B. Szyld. *Algebraic Schwarz Methods for the Numerical Solution of Markov Chains*. Research Report, Temple University, March 2003.

DAISUKE TAGAMI, HIROSHI KANAYAMA, SHIN-ICHIRO SUGIMOTO

### Numerical Computations of 3-D Eddy Current Problems by Iterative Domain Decomposition Method

**Location:** Lecture Room, **Time:** Monday, 21 July, 12:20

We have considered computations of 3-D eddy current problems by an iterative domain decomposition method, where the magnetic vector potential is only used as an unknown function. While this method may be able to analyze larger scale problems, it takes much CPU time of those computations. In this paper, the change of its formulation is considered as a strategy for reducing the CPU time, that is, both the magnetic vector potential and the electric scalar potential are considered as unknown functions. Although the number of the degrees of freedom increases, it is known that the convergence property is improved and the CPU time is shortened in the case of the conventional finite element method. Numerical results show that such improvement

occurs also in the case of the iterative domain decomposition method: the CPU time is reduced by half.

- [1] Kanayama, H., Shioya, R., Tagami, D., and Matsumoto, S., *3-D Eddy current computation for a transformer tank*, COMPEL, Vol.21, pp.554–562, 2002
- [2] Glowinski, R., Dinh, Q.V., and Periaux, J., *Domain decomposition methods for nonlinear problems in fluid dynamics*, Compt. Meths. Appl. Mech. Engrg., Vol.40, pp.27–109, 1983
- [3] Yagawa, G., and Shioya, R., *Parallel finite elements on a massively parallel computer with domain decomposition*, Compt. Sys. Engrg., Vol.4, pp.495–503, 1993

STEFAN TUREK

### Adaptivity Concepts and Load Balancing Strategies for a Generalized Parallel Multigrid/Domain Decomposition Solver

**Location:** Lecture Room, **Time:** Thursday, 24 July, 11:00

While processor technology is still dramatically advancing and promises further enormous improvements in ‘processing data’ for the next decade, much lower advances in ‘moving data’ are expected such that the efficiency of many sequential and parallel simulation tools for partial differential equations (PDE’s) are restricted by the cost for memory access. We demonstrate how ‘data locality’ and ‘pipelining’ can achieve a

significant percentage of the available huge computing power on single processors, and we describe corresponding hardware-oriented concepts for the parallel treatment of ‘adaptive grid refinement’ and ‘adaptive error control’ in combination with special multigrid/domain decomposition schemes. We present their numerical and computational characteristics and discuss corresponding dynamic a posteriori load balancing strategies.

SAULE URAZBAEVA

### System of Queuing Research with the Decomposition Technique and its Application to the Analysis of the Fiber Optic Transmission Network with the DQDB Protocol

**Location:** Room 005, **Time:** Thursday, 24 July, 12:20

Domain decomposition methods for queuing system are one of the fundamental lines of investigation in modern computer science computer mathematics. The mathematical transmission network model with the DQDB protocol has been studied in this paper. We analysed only two models: priority queuing system and cyclic service system.

The research of cyclic system of queuing presents considerable difficulties that is why we have done the decomposition of cyclic system of queuing with N incom-

ing calls stream to the N single line systems of queuing. The course-of-value function of probability distribution of number of the messages on an subscriber servers is given by.

This cyclic system of queuing has been studied by the simulation technique. The results of theoretical studies and of simulation technique show coincidence that is why we can conclude that the domain of applicability of decomposition technique for the given cyclic queuing system analysis is determined only by the correlation.

RODRIGO WEBER DOS SANTOS, G. PLANK, S. BAUER, E.J. VIGMOND

### Preconditioning Techniques for the Bidomain Equations

**Location:** Lecture Room, **Time:** Monday, 21 July, 12:00

In this work we discuss parallel preconditioning techniques for the bidomain equations. The bidomain model has been widely used for the simulation of electrical activity in cardiac tissue. The equations arise from electrostatic formulations describing the potentials in the intracellular and extracellular tissue domains which are

coupled through a non-linear model describing the current flow through the cell membrane.

We approached the non-linear system of partial differential equations with an operator splitting technique. Our numerical algorithm is based on a three step scheme which involves the solution of a parabolic equation, an



elliptic equation and a non-linear system of ordinary differential equations at each time step. We focused on the solution of the linear system associated with the elliptic part of the bidomain model, since it dominates computation, with the preconditioned conjugate gradient method. We compared different parallel preconditioning techniques, such as Jacobi, block incomplete LU,

Gauss-Seidel, additive Schwarz and different multigrid methods. The implementation is based on the Petsc library and we report results for a 16 node HP cluster (each node consists of two McKinley 900 MHz CPU's with 2 GB RAM).

The results suggest the multigrid preconditioner is the best option for the bidomain equations.



## 4 Abstracts of Posters

XIAO-CHUAN CAI, MARIA MURILLO

### **A Restricted Additive Schwarz Method for the Bidomain Model of Cardiac Excitation**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

In this presentation we discuss a fully implicit parallel Newton-Krylov-Schwarz method (NKS) for solving the bidomain equations describing the electrical excitation process of the heart. NKS has been used successfully for many nonlinear problems, but this is the first attempt to use this method for the bidomain model which consists of a system of time dependent partial differential equations of mixed type. Our experiments on parallel computers show that the method is scalable and robust with respect to many of the parameters in the bidomain model. In the outer layer of the algorithm, we use a nonlinearly implicit backward Euler method to discretize the time derivative, and the resulting systems of

large sparse nonlinear equations are solved using an inexact Newton method. The Jacobian system required to solve in each Newton iteration is solved with a GMRES method preconditioned by a new component-wise restricted additive Schwarz preconditioner. The efficiency and robustness of the overall method depend heavily on what preconditioner we use. By comparing several preconditioners, we found our new restricted additive Schwarz method offers the best performance. Our parallel software is developed using the PETSc package of Argonne National Laboratory. Numerical results obtained on an IBM SP will be reported.

XIAO-CHUAN CAI, XUE YUE, FENG-NAN HWANG, ROBIN SHANDAS

### **Simulations of Branching Blood fluids on Parallel Computers**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

In this paper we present a parallel nonlinearly implicit algorithm for the numerical solution of the unsteady incompressible Navier-Stokes equations on unstructured meshes. We focus on the simulations of some branching biofluid dynamics problems, which require some efficient and robust solver technologies in order to handle the high nonlinearity and the complex geometry. Parallel processing is also a must because of the large

number of mesh points needed to accurately discretize the system of differential equations. We study a parallel Newton-Krylov-Schwarz based implicit method for solving the nonlinear algebraic systems arising from a Q2-Q1 finite element discretization of the unsteady Navier-Stokes equations. We show numerically that the method and the PETSc based software are robust and scalable for the complex and unsteady flow problem.

XIAO-CHUAN CAI, ERNESTO PRUDENCIO

### **Domain Decomposition Methods for A PDE Constrained Optimization Problem**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

Optimization problems constrained by nonlinear equality partial differential equations (PDE) have been the focus of intense research in scientific computation lately. The state-of-the-art methods for the parallel numerical solution of such problems involve sequential quadratic programming (SQP), with either reduced or full space approaches.

In this presentation we propose a class of parallel full space SQP Lagrange-Newton-Krylov-Schwarz (LNKSz) algorithms. In LNKSz, a Lagrangian functional is formed and differentiated to obtain an optimality system of nonlinear equations. Inexact Newton's method with line search is then applied and at each Newton's

iteration the Karush-Kuhn-Tucker (KKT) system is solved with a Krylov subspace method preconditioned with an overlapping additive Schwarz method.

We apply LNKSz to some boundary control problems of steady-state flows of viscous incompressible fluids described by Navier-Stokes equations in the velocity-vorticity formulation. We propose the application of LNKSz to flow control problems as a natural extension to the successful application of NKSz to flow simulations. We report the results of a PETSc based implementation of LNKSz for different combinations of Reynolds numbers, grid sizes and number of processors.

SUNGMIN CHO, SERGEY V. NEPOMNYASCHIKH, E.-J. PARK

### **Domain Decomposition Preconditioning for Elliptic Problems with Jumps in the Coefficients**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

The main focus of this work is a numerical analysis of a domain decomposition method for finite element approximations of elliptic problems with jumps in the coefficients. The theorems on traces of functions from Sobolev spaces are very important in studying boundary value problems of partial differential equations. These

theorems are commonly used for a priori estimates of the stability with respect to boundary conditions, and also play very important role in constructing and investigating effective domain decomposition methods. Using these results, the robust domain decomposition methods are considered without using extension operators of

functions from boundaries of subdomains to inside of subdomains.

LI DENG, TAKAO TAYOSHI, TAKASHI KAKO, ICHIRO HAGIWARA

### **Perturbation Analysis and its Approximation by FEM for Coupled Systems Between Structure and Acoustic Field**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

In the present study, we formulate the coupled vibration problem between a structure and an acoustic field in a mathematically rigorous fashion. A typical example of the structure is a car body which can be modeled by a cluster of thin plates. This problem leads to a non-standard eigenvalue problem in some function space.

Furthermore, to clarify the meaning of the coupling strength, we introduce a coupling strength parameter  $T$  as a multiplier applied to the non-diagonal coupling

terms, and a natural interpretation of this parameter is given. We represent an eigen-pair for the coupled system by a perturbation series with respect to  $T$ , which enables us to express the eigen-pair for the coupled case by those for the decoupled case. It is proved that the series consists only of the even order term of  $T$ . We also give its approximation by FEM. We confirm the adequacy of this perturbation analysis and its FEM approximation through some numerical examples.

ABUL K.M. FAHIMUDDIN, MARKUS KROSCHKE, RAINER NIEKAMP, JOACHIM K. AXMANN, HERMANN G. MATTHIES

### **PLATON - An Environment for Coupling Optimisation and Simulation Codes**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

PLATON (Parallel Adaptive Techniques for Optimisation) is a component based distributed software architecture used for coupling optimization and simulation software. The broader spectrum of the usage of PLATON encompasses various facets of general coupling problems. The focus of this poster is on the generic description used in PLATON to describe the necessary

software components and couple them together. In this context, we introduce our Communication Template Library (CTL), a communication middle-ware based on C++ templates. It is worth to mention here that PLATON uses our CTL to provide standard, easy and abstract mechanism for defining interfaces between Distributed Components.

DAVID HORAK, ZDENEK DOSTAL

### **Scalability of FETI Based Algorithms for Variational Inequalities**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

The point of this poster is to review experimental results related to scalability of FETI based domain decomposition augmented Lagrangian algorithm suggested by Dostál, Friedlander, Santos, and Gomes and dual penalty for numerical solution of discretized variational inequalities. Both scalabilities are demonstrated by numerical experiments with parallel solution of a model problem discretized by up to more than 8.5 million of nodal variables and solved in 20 minutes using PETSc and having such optimality, that the number of iterations are bounded independently of the discretization parameter. For more see [1] ... [3].

[1] Z. Dostál, F.A.M. Gomes, and S.A. Santos: *Solu-*

*tion of contact problems by FETI domain decomposition with natural coarse space projection*, Computer Meth. in Appl. Mech and Engineering 190, 13-14 (2000) 1611-1627.

[2] Z. Dostál and D. Horák: *Scalability and FETI based algorithm for large discretized variational inequalities*, submitted to Math. and Comput. in Simulation.

[3] Z. Dostál and D. Horák: *Numerical and parallel scalability of FETI algorithm for variational inequalities: Numerical experiments*, Transactions of VŠB-Technical University of Ostrava, Computer Science and Mathematics Series I,1 2001, 63-76.

DAVID KEYES, FELLOW PRINCIPAL INVESTIGATORS TOPS PROJECT

### **Optimization of PDE-constrained Systems in the Terascale Optimal PDE Simulations Project**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

In support of known and anticipated application requirements for parameter identification, design optimization, optimal control, and data assimilation in complex systems, the Terascale Optimal PDE Simulations project is creating optimization packages that leverage

and integrate its scalable solvers.

This poster, the third of three for presentation at DD-15, discusses two of TOPS' initial forays into PDE-constrained optimization, as well as two unconstrained problems in chemistry.

DAVID KEYES, FELLOW PRINCIPAL INVESTIGATORS TOPS PROJECT

### **Scalable Solvers in the Terascale Optimal PDE Simulations Project**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

In support of magnetically confined fusion, supernovae astrophysics, engine combustion, and other simulations, the Terascale Optimal PDE Simulations project is creating a new generation of publicly available software with new interoperability features for PDE field problems. The packages include PETSc (Argonne National

Lab), Hypre (Lawrence Livermore National Lab), and SUNDIALS (Lawrence Livermore National Lab).

This poster, the second of three for presentation at DD-15, discusses some of TOPS' initial collaborative efforts to upgrade the solvers in extent community physics codes.

DAVID KEYES, FELLOW PRINCIPAL INVESTIGATORS TOPS PROJECT

**Terascale Optimal PDE Solvers: Project Overview**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

The Terascale Optimal PDE Simulations (TOPS) project of the U.S. Department of Energy Scientific Discovery through Advanced Computing (SciDAC) initiative is developing a toolkit of open source solvers for PDEs that arise in many DOE mission areas. These algorithms — primarily domain-decomposed multilevel

methods — aim to reduce solver bottlenecks by magnitudes at the terascale, with goals of usability, robustness, algorithmic efficiency, and the highest performance consistent with these.

This poster, the first of three for presentation at DD-15, outlines the scope and philosophy of the TOPS project.

JIRI STARY, R. BLAHETA, O. JAKL, K. KRECMEK

**Parallel Iterative Solvers in Geomechanics**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

The paper describes parallel iterative solvers for the solution of large-scale linear systems arising from the finite element analysis of elasticity problems in geomechanics. The solvers are based on the conjugate gradient method and space decomposition techniques, primarily on the displacement decomposition and domain decomposition, which are used for both the parallelization of

the solvers and construction of effective preconditioners. The algorithms were implemented firstly with the support of a message passing library only and secondly with the aid of the PETSc library. The resulting program codes were compared at a solution of one purely academic benchmark on shared-memory and distributed-memory machines.

BARBARA STECKEL, HANNO BAEHR

**MpCCI: A Tool for the Simulation of Coupled Problems, eg. Using Domain Decomposition**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

Many industrial applications are characterized by the interaction of different engineering disciplines, like fluid-structure interaction. For the individual disciplines sophisticated and validated solvers exist. MpCCI (Mesh-based parallel Code Coupling Interface) enables industrial users as well as code owners to combine different simulation tools.

Thereby new solvers for the solution of multidisciplinary problems are created. MpCCI is independent of the solver, used within the single code, and independent of the coupling algorithm. The user has to specify and to

implement the coupling strategy.

MpCCI supports the user by providing subroutines for all necessary data transfer including interpolation. A coupling algorithm can be a domain decomposition method applied to the special situation of a coupled problem.

The poster will give an overview over MpCCI. The combination of MpCCI and domain decomposition will be demonstrated for the Farhat-Lesoinne coupling algorithm.

OLDRICH VLACH, JAROSLAV HASLINGER

**Signorini Problem with a Solution Dependent Coefficient of Friction (Model with Given Friction): Approximation and the Numerical Realization**

**Location:** Room 046, **Time:** Thursday, 24 July, 17:40

Contact problems with given friction and coefficient of friction depending on their solutions are studied. We prove the existence of at least one solution eventually its uniqueness under additional assumptions on the coefficient of friction. The method of successive approximations combined with the dual formulation of each iterative step is used for the numerical realization. Numerical results of model examples are shown.

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[2] J. Haslinger, P.D. Panagiotopoulos: *The reciprocal variational approach to the Signorini problem with friction. Approximation results*, Proc of the Royal Society of Edinburgh, 98A, 365-383, 1984

- [3] J. Haslinger, Z. Dostál, R. Kučera: *On a splitting type algorithm for the numerical realization of contact problems with Coulomb friction*, Comput. Methods Appl. Mech. Engrg. , 191(2002), pp. 2261-2881

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