15th International Conference on Domain Decomposition Methods
July 21-25, 2003, Berlin, Germany

MS01 Collaborating Subdomains for Multi-Scale Multi-Physics Modelling

Organized by: Peter Chow, Choi-Hong Lai

Many physical problems such as solidification process, aeroacoustics, groundwater simulation, and electronic packaging involve the localisation of the physics and their transient variability across the geometric domain. While it is impossible to address all of the problems mentioned, it is important to understand various options of coupling subdomains in relation to the multi-physics problems. In particular it is important to address how subdomains may be collaborated in achieving a fast global solution to the original problem based on distributed computing.
The minisymposium will discuss the collaboration between subdomains in electronic packaging which is a multi-scale multi-physics problem. The collaboration between subdomains is done using classical non-overlapping subdomains and the corresponding acceleration techniques. The main aim is to reduce total engineer's modelling time in an engineering process. Other topics involved in this minisymposium include coupling of various physical properties in many fluid dynamics involving nonlinear Navier-Stokes solvers. A list of tentative speakers and areas of the talks is given below.

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Xiao-Chuan Cai, Feng-Nan Hwang

ASPIN for Incompressible Navier-Stokes Equations

Location: Room 005, **Time:** Thursday, 24 July, 16:00

A nonlinear additive Schwarz preconditioned inexact Newton method (ASPIN) was introduced recently by Cai and Keyes for solving large sparse nonlinear systems of equations obtained from the discretization of nonlinear partial differential equations.

In this talk, we discuss some recent development of ASPIN for solving the steady state incompressible Navier-Stokes equations in the velocity-pressure formulation. The sparse nonlinear system is obtained by using a $Q_1 - Q_1$ Galerkin least squares finite element discretization on two dimensional unstructured meshes. The key idea of ASPIN is that we find the solution of the original system $F(u) = 0$ by solving a nonlinearly preconditioned system $G(u) = 0$ that has the same solution as the original system, but with more balanced nonlinearities. The nonlinear preconditioners are based on the solution of the Navier-Stokes equations defined on the overlapping subdomains with some proper boundary conditions. In this talk we present some numerical results obtained on parallel computers for two challenging CFD benchmark problems: a driven cavity flow problem and a back facing step problem.

We compare our approach with some inexact Newton method with different choices of forcing terms. The numerical results show that ASPIN is more robust than the traditional inexact Newton method for high Reynolds number flows as well as for large number of processors.

Tobias Knopp, Gert Lube, Ralf Gritzki, Markus Roesler

Iterative Substructuring Methods for Indoor Air Flow Simulation

Location: Room 005, **Time:** Thursday, 24 July, 16:25

The numerical simulation of turbulent indoor air flows is performed using iterative substructuring methods. We consider the k/ϵ model based on the nonstationary, incompressible Navier-Stokes problem [4]. The resolution of turbulent layers in the near-wall region is avoided using an improved wall function concept [3, 2].

For quasi-stationary flows we apply the semidiscretization in time with the simplest discontinuous Galerkin method. After proper linearization in each time step, the iterative process requires the fast solution of linearized Navier-Stokes problems and of advection-diffusion-reaction problems. These subproblems are discretized using stabilized FEM together with a shock-capturing technique for the transport equations for the temperature and turbulence quantities k and ϵ, cf. [1, 5]. For the linearized problems we apply an iterative substructuring technique which couples the subdomain problems via Robin-type transmission conditions. Proper chosen parameter functions of these transmission conditions allow a considerable acceleration of this well-posed convergent method [6]. The applicability of the approach is shown for some benchmark problems, including comparison with experimental data [7], and realistic ventilation problems [3, 2].

[1] Codina, R., Soto, O.: *Finite element implementations of two-equation and algebraic stress turbulence models for steady incompressible flow*, Intern. J. Numer. Meths. Fluids 90 (1999) 3, 309-334

[2] Gritzki, R.: *Determination of the efficiency of window ventilation using numerical simulation methods* (in German), Ph.D. Thesis, TU Dresden, 2001

[3] Knopp, T.: *Finite element simulation of buoyancy-driven turbulent flows*, Ph.D. Thesis, University of Göttingen, 2003

[4] Knopp, T., Lube, G., Gritzki, R., Rösler, R..: *Iterative substructuring techniques for incompressible nonisothermal flows and its application to to indoor air flow simulation*, Intern. J. Numer. Meths. Fluids 40 (2002), 1527-1538

[5] Knopp, T., Lube, G., Rapin, G.: *Stabilized finite element methods with shock-capturing for advection-diffusion problems*, Comput. Meths. Appl. Mech. Engrg. 191 (2002) 27-28, 2997-3013

[6] Lube, G., Müller, L., Otto, F.C.: *A non-overlapping domain decomposition method for stabilized finite element approximations of the Oseen equations*, J. Comp. Appl. Math. 132 (2001) 2, 211-236

[7] Tiam, Y., Karayiannis, T.: *Low turbulence natural convection in an air filled square cavity, Part I: The thermal and fluid flow field.* Int. J. Heat Mass Transfer 43 (2000), 849-866.

Edward Swim, Padmanabhan Seshaiyer

Fluid-Structure Interaction with Nonconforming Finite Elements

Location: Room 005, **Time:** Thursday, 24 July, 16:50

We investigate the interaction between a viscous incompressible fluid and a structure whose deformation defines the interface between the two. Direct numerical solution of the highly nonlinear equations governing even the most simplified models of fluid-structure interaction requires that both the flow field and the domain shape be determined as part of the solution since neither is known a priori. To accomplish this, previous algorithms have decoupled the solid and fluid mechanics, solving for each separately and converging iteratively to a solution which satisfies both.

In this talk, we describe a non-conforming hp finite element method which solves the problem simultaneously on each subdomain. Mortar finite elements are used to construct approximate solutions of the corresponding partial differential equations on the fluid and structure domains as well as the flexible boundary between them. The local approximation within each subdomain is designed using stable hp finite elements, where both mesh refinement and degree enhancement are combined to increase accuracy.

Choi-Hong Lai, Jianwen Cao

Some Effctive Techniques of Nonlinear Solvers for Black-Oil Modelling

Location: Room 005, **Time:** Thursday, 24 July, 17:15

In this paper, we discuss how to improve the performance of solving the set of nonlinear equations originated from a fully implicit scheme for multiphase flow in pourous media on Beuwolf PCs cluser.

A hybrid nonlinear method is suggested which combines global convergent inexact newton method and rank-2 correction BFGS quasi-newton algorithm.

In order to improve the computational efficiency of the nonlinear solver, parallel strategy is adopted based on distributed memory HPC by means of using DDM method,and each newtonian iteration step uses the so-called inner-outer nested algorithm with a proper preconditioner. As one of the most important part, the preconditioner uses multipurpose oblique projection correction strategy which based on iterative algorithms to solve a small linear system of $(P^TAP)z = r$ involved. This preconditioner involves several preconditioning such as AMG,Relaxed ILU,upscaling,DDM, Constaint Residual Precondition etc.

Numerical results display the performace and scalability on 4 - 128 CPUs Beuwolf Linux Cluster.

Martijn Anthonissen

Local Defect Correction Techniques Applied to a Combustion Problem

Location: Room 005, **Time:** Friday, 25 July, 9:00

We consider a discretization method for elliptic boundary value problems introduced by Hackbusch. In this technique, called the *local defect correction* (LDC) method, the discretization on the *composite grid* is based on a combination of standard discretizations on several uniform grids with different grid sizes that cover different parts of the domain. At least one grid, the coarse grid, should cover the entire domain, and its grid size should be chosen in agreement with the relatively smooth behavior of the solution outside the high activity areas. Apart from this *global coarse grid*, one or several *local fine grids* are used which are also uniform. Each of the local grids covers only a (small) part of the domain and contains a high activity region. The grid sizes of the local grids are chosen in agreement with the behavior of the continuous solution in that part of the domain.

The LDC method is an iterative process: a basic global discretization is improved by local discretizations defined in subdomains. The update of the coarse grid solution is achieved by adding a defect correction term to the right hand side of the coarse grid problem.

We extend the LDC algorithm by successively adding adaptivity, multilevel refinement, domain decomposition and regridding. The final result will be a technique for discretizing and solving boundary value problems on a composite grid found by adaptive grid refinement given a code for solving a boundary value problem on a tensor-product grid in a rectangular domain.

We apply our proposed adaptive multilevel LDC algorithm to a Bunsen flame problem.

[1] M.J.H. Anthonissen. *Local defect correction techniques: analysis and application to combustion.* PhD thesis, Eindhoven University of Technology, Eindhoven, 2001.

[2] M.J.H. Anthonissen, R.M.M. Mattheij, and J.H.M. ten Thije Boonkkamp. *Convergence results for the local defect correction method as an iterative process.* Technical Report RANA 00-14, Eindhoven University of Technology, Eindhoven, June 2000. Accepted for publication in *Numerische Mathematik*.

[3] M.J.H. Anthonissen, B. van 't Hof, and A.A. Reusken. *A finite volume scheme for solving elliptic boundary value problems on composite grids.* Computing, 61:285–305, 1998.

[4] B.A.V. Bennett and M.D. Smooke. *Local rectangular refinement with application to axisymmetric laminar flames.* Combust. Theory Modelling, 2:221–258, 1998.

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[6] P.J.J. Ferket and A.A. Reusken. *A finite difference discretization method on composite grids.* Computing, 56:343–369, 1996.

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Marc Garbey, Omar Dia, Y. Jobic

Heterogeneous Domain Decomposition for Boundary Layer Problems

Location: Room 005, **Time:** Friday, 25 July, 9:25

In this paper, we propose an heterogeneous domain decomposition solver for Navier-Stokes flow in pipe. The domain of computation is decomposed into a regular domain with Cartesian grids and several boundary layer domains that fit the boundaries with local orthogonal meshes. The domain decomposition is motivated by the physic and/or singular perturbation analysis for large high Reynolds numbers. We have then very different type of meshes between the regular domain and the boundary layer domain. The numerical efficiency of the domain decomposition is the consequences of few factors such as:

1. each sub-domain can use a fast solver that takes full advantage of either the stretching of the mesh in one space direction for boundary layer domains, or the regular data structure with Cartesian grids used for the main part of the flow.
2. simplicity of the implementation, grid generation, and memory allocation due to the use of the additive Schwarz method for the iteration process between overlapping non matching grids.
3. fast convergence of the domain decomposition algorithm thanks to the use of an acceleration procedure to speed up the convergence of the Schwarz method.

We will discuss the numerical accuracy and robustness of this heterogeneous domain decomposition technique.

Arthur E. P. Veldman, E.G.M. Coenen

Interaction Laws in Viscous-Inviscid Coupling

Location: Room 005, **Time:** Friday, 25 July, 9:50

A classical example of domain decomposition is found in aerodynamic boundary-layer theory. About one century ago Prandtl proposed to split an aerodynamic flow field at high Reynolds number into a thin viscous boundary layer and an inviscid external-flow region. The two regions are typically coupled by exchanging information on pressure and boundary-layer displacement effects (in DD terms: Dirichlet-Neuman coupling).

Following Prandtl, the boundary-layer equations are solved with pressure prescribed from the inviscid-flow solution. However, as discussed already at length by Goldstein in 1948, this concept fails as soon as flow separation occurs. Only in the late 60-ies and 70-ies the first understanding came up on why this failure occurs. Subsequently, this has led to alternative approaches, such as the semi-inverse method and the quasi-simultaneous method, that do not suffer from Goldstein's singularity.

In the presented research, the quasi-simultaneous method has been analysed and brought back to its essentials. The result is a method that is very close to Prandtl's original concept – any existing Prandtl boundary-layer code can be modified with modest effort into our approach – yet it has no problem to cope with flow separation. The ideas behind the method are easily generalized to other instances of domain decomposition.

Results on convergence behaviour and numerical robustness of the method will be shown for two- and three-dimensional flow past indented plates and airfoils.

Peter Chow, Choi-Hong Lai

Electronic Packaging and Reduction in Modelling Time Using Domain Decomposition
Location: Room 005, **Time:** Friday, 25 July, 10:15

'Small' is exquisite and cool to the consumers of electronic products, but it throws out enormous technical challenges that need to be overcome by designers and engineers. Elements such as health and safety compliance, power and heat management, and usability are commonly top on the list of issues. The primary technical challenges [chow] are: 1) High density of components leads to an increase of model complexity that needs to address not just the processes of thermal cooling, electromagnetic radiation and solid structure but the interaction and interference between the processes, i.e. Multi-Physics. 2) Creating highly intricate and highly detailed geometry and mesh models with parts of dissimilar scales, such as the entire electronic components in a laptop computer and exposure analysis of electronic devices on an entire human body and tissues. 3) Size of computational demands million plus cells/elements models are common in industrial simulations and models with tens of millions of elements are appearing, but still the resolution is not enough in applications such as electromagnetic and thermal cooling designs. 4) The market demands and speed to market adds to the ever-increasing pressures of shortening the modelling time and number of design cycles.

Whilst domain decomposition has been successfully applied to areas such as parallel solvers and preconditioners, coupling of different numerical methods and physical models, it has not been considered for the entire simulation process chain in order to achieve a comprehensive reduction in modelling time. An early concept of using domain decomposition methods in the reduction of modelling time can be found in [1]. This paper gives a rigorous approach of the concept and uses the framework of defect equations in coupling [2]. Algorithms developed in this paper concentrate on problems with geometrical multi-scale at the macroscopic mathematical models. Numerical experiments, including monophase and multi-phase, are examined with efficiency of the algorithm being linked to the overall modelling time.

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