MS02 Discretization Techniques and Algorithms for Multibody Contact Problems

Organized by: Barbara Wohlmuth, Taoufik Sassi

Domain decomposition techniques on non-matching grids provide a flexible and efficient tool for the numerical approximation of non linear multibody contact problems with friction.Generalized mortar methods based on Lagrange multipliers can be used to discretize contact problems. The resulting variational inequalities are defined on a discrete convex set. The non-penetration condition is realized in terms of suitable weak integral conditions. Here, a priori error estimates will be considered as well as iterative solvers based on domain decomposition techniques. Different concepts as monotone multigrid methods, non linear Dirichlet–Neumann algorithms and FETI techniques will be discussed and analyzed.

Barbara Wohlmuth (Organizer)
Universität Stuttgart
Fachbereich Mathematik
Pfaffenwaldring 57
70569 Stuttgart
GERMANY
mailto:wohlmuth@mathematik.uni-stuttgart.de

Taoufik Sassi (Organizer)
INSA of Lyon
MAPLY-INSA of Lyon, bat.. L. de Vinci
20, avenue Albert Einstein
69621 Villeurbanne
FRANCE
mailto:sassi@insa-lyon.fr

Zdenek Dostal, D. Horák

Optimal Penalty and Scalable FETI Based Algorithms for Numerical Solution of Variational Inequalities

Location: Room 005, **Time:** Monday, 21 July, 16:00

We shall first briefly review the FETI methodology proposed by C. Farhat and F.-X. Roux adapted for solution of variational inequalities. Using the classical results by Mandel and Tezaur related to the scalability of FETI for solution of linear elliptic boundary value problems, we shall show how to exploit their results to reduce the discretized variational inequality to the bound and equality constrained quadratic problem with the condition number of the Hessian independent on the discretization parameter h. Then we shall show that such problems may be solved efficiently by recently proposed algorithms for solution of quadratic programming problems with the rate of convergence in terms of the spectral condition of the Hessian of the cost function. In particular, it follows that if we impose the equality constraints by the penalty method, we can solve our penalized problem to the prescribed relative precision in a number of iterations that does not depend on h. Since we managed to prove that a prescribed bound on the relative feasibility error of the solution may be achieved with the value of the penalty parameter that does not depend on the discretization parameter h, we conclude that these results may be used to develop scalable algorithms for numerical solution of elliptic variational inequalities. We give results of numerical experiments with parallel solution of both coercive and semicoercive model problems discretized by up to more than eight million of nodal variables to demonstrate numerically optimality of the penalty and scalability of the algorithm presented.

Taoufik Sassi, Jaroslav Haslinger

A Mixed Finite Element Approximation of 3D Contact Problems with Given Friction: Approximation and the Numerical Realization

Location: Room 005, **Time:** Monday, 21 July, 16:25

This contribution deals with a mixed variational formulation of 3D contact problems with the simplest model involving friction. This formulation is based on a dualization of the set of admissible displacements and the regularization of the nondifferentiable term. Displacements are approximated by piecewise linear elements while the respective dual variables by piecewise constant functions on a dual partition of the contact zone. The rate of convergence is established provided that the solution is smooth enough. The numerical realization of such problems will be discussed and results of several model examples will be shown.

Rolf Krause, O. Sander

Fast Solving of Contact Problems on Complicated Geometries

Location: Room 005, **Time:** Monday, 21 July, 16:50

The efficient solution of contact problems involving friction in three space dimension is a non-trivial task. The mathematical modeling of the non-penetration condition and of the frictional effects at the interface gives rise to a non-linear and non-differentiable energy functional which has to be minimized. In our talk, we present a non-linear monotone multigrid method, by means of which one-sided frictional contact problems can be solved with optimal complexity. For the case of two sided contact problems, the information transfer at the interface is realized in terms of Mortar methods. Our method uses a combination of dual Basis functions for the Lagrange Multiplier space and a suitable transformation of the arising discrete system based on work by Wohlmuth and Krause. Thus the proposed method is also applicable in case of non-matching triangulations at the interface.

We also apply our monotone method to problems on complicated geometries. To this end, we combine the multigrid method with boundary parametrizations, by means of which the possibly complex geometry of the computational domain can be successively regained starting from a simple and coarse initial mesh. We also show how these parametrization techniques can be applied in order to realize the information transfer between two bodies coming into contact by means of Mortar methods in the case of curvilinear boundaries. Numerical results in three space dimensions illustrate the efficiency and accuracy of our method. In particular, we consider the case of curvilinear contact boundaries in three space dimensions.

Taoufik Sassi, Laurent Baillet, Guy Bayada, Jalila Sabil

Domain Decomposition Algorithms for Contact Problems

Location: Room 005, **Time:** Monday, 21 July, 17:15

We propose and we prove the convergence of a Neumann-Dirichlet and a Neumann-Neumann algorithms in order to approximate a Signorini problem between two elastic bodies. The Neumann-Neumann algorithm is a paralell one, in which we have to solve a Dirichlet problem then a Neumann one simultaneously on each domain. Primary interest of these algorithms is to retain the natural interface between the two bodies as a numerical interface for the domain decomposition.

Panayot Vassilevski

Monotone Element Agglomeration AMG$_e$ for Contact Problems

Location: Room 005, **Time:** Monday, 21 July, 17:40

In this talk we will present an application of the recently developed sparse matrix element topology [1] to create agglomerated elements coarsened away from a (contact) boundary. Based on thus constructed hierarchy of agglomerated elements one then uses the AMGe in the form proposed in [2] to construct coarse finite element spaces to be used in multilevel schemes. A main feature is that the contact boundary is present (not coarsened) on all coarse levels, hence the constraints are straightforward to satisfy. We consider two multilevel schemes – a classical FAS and a subspace minimization scheme. Both, when combined with monotone smoothers, such as the projected Gauss–Seidel, lead to overall monotone multilevel algorithms. Some numerical illustration will be presented as well.

Major part of the talk is based on a joint work with Ana Iontcheva [3].

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[1] P.S. Vassilevski. *Sparse matrix element topology with application to AMG and preconditioning*, Numer. Linear Algebra Appl., **9**: 429–444 (2002).

[2] Jones, J.E. and P.S. Vassilevski. *AMGe based on element agglomeration*, SIAM J. Sci. Comput, **23**: 109–133 (2001).

[3] Iontcheva, A.H. and P.S. Vassilevski. *Monotone multigrid methods based on agglomeration AMG$_e$ coarsening away from the contact boundary for the Signorini's problem.* preprint (2003).