15th International Conference on Domain Decomposition Methods
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MS04 Domain Decomposition Methods for Wave Propagation in Unbounded Media

Organized by: Xavier Antoine, Frank Schmidt

The minisymposium covers recent results on theoretical and numerical aspects of wave propagation in unbounded media. Most methods both for the solution of time-dependent and time-harmonic scattering problems rely on a decomposition into an exterior and an interior problem. The minisymposium will focus on the numerical solution of such coupled interior/exterior problems and provide an overview of ideas for the solution of exterior acoustic, electromagnetic, and quantum mechanical problems.

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New Transparent Boundary Conditions for Coupled Interior/Exterior Wave Propagation Problems

Location: Lecture Room, **Time:** Tuesday, 22 July, 16:00

We consider Helmholtz-type equations with inhomogeneous exterior domains, e.g. exterior domains containing wave guides, which arise in the simulation of photonic devices. Usually, Sommerfeld's radiation condition is not valid for such problems. We present an alternative radiation condition called *pole condition*, which has been suggested by Frank Schmidt. Roughly speaking, it says that the Laplace transform of a radiating solution along a family of rays tending to infinity has a holomorphic extension to the lower half of the complex plane. To justify the validity of this condition, we show that it is equivalent to Sommerfeld's radiation condition for bounded obstacle scattering problems. Moreover, we show that for scattering problems by rough surfaces and for wave guide problems, the pole condition is also equivalent to the standard radiation conditions used in these fields.

For the numerical solution of scattering problems, the radiation condition has to be replaced by a transparent boundary condition on the artificial boundary of the computational domain. We describe a construction of exact transparent boundary conditions based on the pole condition which does not rely on the explicit knowledge of a fundamental solution or a series representation of the solution. There exists an explicit formula expressing the exterior solution in a stable way in terms of quantities defined in the Laplace domain. This provides an efficient numerical method for the evaluation of the exterior solution and distinguishes the proposed method from other methods, e.g. the Perfectly Matched Layer (PML) method. The total computational cost is typically dominated by the finite element solution of the interior problem.

[1] T. Hohage, F. Schmidt, L. Zschiedrich: *Solving time-harmonic scattering problems based on the pole condition. I:Theory.* SIAM J. Math. Anal., to appear.

[2] T. Hohage, F. Schmidt, L. Zschiedrich: *Solving time-harmonic scattering problems based on the pole condition. II:Convergence of the PML method.* SIAM J. Math. Anal., to appear.

[3] T. Hohage, F. Schmidt, L. Zschiedrich: *A new method for the solution of scattering problems.* In B. Michielsen and F. Decavele (eds) Proceedings of the JEE'02 Symposium, p. 251-256, Toulouse, ONERA, 2002

Nolwenn Balin, A. Bendali

Domain Decomposition and Additive Schwarz Techniques in the Solution of a TE Model of the Scattering by an Electrically Deep Cavity

Location: Lecture Room, **Time:** Tuesday, 22 July, 16:25

It is well known, by the experts in stealth technology, that an electrically deep cavity significantly contributes to the RCS of the structure in which it is residing. Several difficulties prevent the use of standard methods to deal with this problem:

- asymptotic methods cannot be used to determine the field scattered by the cavity,

- direct solutions are not possible due to the huge size of the linear system to be solved,
- fast methods, like the multipol method, cannot directly apply because of the very slow convergence of the Krylov-like iterative methods due to the cavity.

Two directions have been explored (for a TE model) to enhance the solution.

A non-overlapping domain decomposition consisted of successive pieces the interfaces of which being a sectional surface of the cavity clearly takes advantage of the geometry of the problem. The electromagnetic field inside each piece can then be represented by equivalent currents. Successive eliminations, each time at the level of only one piece, reduce the cavity effect, at the end of the process, to the determination of the electric and magnetic equivalent currents at the opening of the cavity.

An additive overlapping Schwarz method, based on partition of unity to decompose the currents, is used to solve the resulting integral equations on the opening of the cavity and the exterior part of the structure. This leads to an iterative method, in which at each step, are solved problems set on the support of each partition of unity function. A high convergence rate has been observed.

This work is a part of a PhD thesis which is done in the frame of a collaboration involving MBDA-F, CERFACS and MIP Laboratory.

Xavier Antoine, H. Barucq

On the Construction of Approximate Boundary Conditions for Solving the Interior Problem of the Acoustic Scattering Transmission Problem
Location: Lecture Room, **Time:** Tuesday, 22 July, 16:50

This talk is about the construction of generalized impedance boundary conditions at the interface between two three-dimensional homogeneous media for the penetration of a time-harmonic wave into a dissipative obstacle. The exact model is given through a transmission problem that couples the propagations into an absorbing domain and an exterior domain. The conditions arise from an asymptotic analysis of the interior solution whose propagation is studied by pseudodifferential technics classically involved for the construction of radiation boundary conditions. First and second-order conditions are analyzed and some numerical experiments illustrate their validity domain.

Frank Schmidt, Lin Zschiedrich

Numerical Methods to Realize the Pole Condition Concept
Location: Lecture Room, **Time:** Tuesday, 22 July, 17:15

Sommerfeld's radiation condition is of basic importance for time-harmonic acoustic scattering problems, both for uniqueness considerations as well as for the foundation of numerical methods.

Recently we proposed an alternative radiation condition called pole condition. The pole condition is more general than Sommerfeld's radiation condition and may be applied, in merely the same way, to a number of different problems like time-harmonic and time-dependent scattering problems as well as to eigenproblems on unbounded domains. Whereas recent studies on pole condition concentrated on theoretical properties of the condition itself, this talk aims to discuss how to apply the pole condition concept to solve scattering problems on unbounded domains. Here, the decomposition into interior and exterior domain, a semi-discretization of the exterior domain, and the application of the pole condition lead in a natural way to suitable transparent boundary conditions for the interior domain.

Matthias Ehrhardt, Anton Arnold, Ivan Sofronov

Approximation, Stability and Fast Calculation of non-Local Boundary Conditions for the Schrödinger Equation
Location: Lecture Room, **Time:** Tuesday, 22 July, 17:40

We propose a way to efficiently treat the well-known transparent boundary conditions for the Schrödinger equation. Our approach is based on two ideas: to write out a discrete transparent boundary condition (DTBC) using the Crank-Nicolson finite difference scheme for the governing equation, and to approximate the discrete convolution kernel of DTBC by sum-of-exponentials for a rapid recursive calculation of the convolution.

We prove stability of the resulting initial-boundary value scheme, give error estimates for the considered approximation of the boundary condition, and illustrate the efficiency of the proposed method on several examples.