MS05 Heterogeneous Domain Decomposition with Applications in Multiphysics

Organized by: Ralf Kornhuber, Alfio Quarteroni

Coupled heterogeneous phenomena are not an exception but the rule in advanced numerical simulations of fluid dynamics, microelectronics, hydrodynamics, hemodynamics, electrodynamics or acoustics. Mathematical understanding and efficient numerical solvers become more and more important. The aim of this minisymposium is to bring together scientists working in this field who will both present talks on topics within the theme and contribute to discussion during the minisymposium.

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Ronald H.W. Hoppe

Domain Decomposition Methods in Electrothermomechanical Coupling Problems

Location: Room 005, **Time:** Tuesday, 22 July, 16:00

The functionality of modern electronic devices and systems often depends on the coupling of various physical phenomena on different scales in both time and space. The mathematical modeling on a macroscopic level typically leads to coupled systems of partial differential equations whose numerical solution requires suitable problem oriented discretizations and efficient iterative solvers such as domain decomposition and multilevel methods.

In this contribution, we consider electrothermomechanical coupling problems as they arise in the modeling and simulation of high power electronic devices. In particular, we are faced with a hierarchy of coupled physical effects in so far as electrical energy is converted to Joule heat causing heat stresses that have an impact on the mechanical behavior of the devices and may lead to mechanical damage without appropriate cooling mechanisms.

Moreover, there are structural coupling effects due to the sandwich-like construction of the devices featuring multiple layers of specific materials with different thermal and mechanical properties. The latter motivates the application of domain decomposition techniques on nonmatching grids based on individual finite element discretizations of the substructures. We will address in detail the modeling aspects of the hierarchy of coupling phenomena as well as the discretization-related couplings in the numerical simulation of the operating behavior of the devices.

Vsevolod Nefedov

Subgridding in Finite-Difference Time-Domain Method

Location: Room 005, **Time:** Tuesday, 22 July, 16:25

Finite-Difference Time-Domain (FDTD) method is a simple and effective solution method for the Maxwell Equations. It most commonly used on structured uniform grids and for that reason lacks adaptivity.

In this talk we consider various ways to improve FDTD algorithm by employing local refinement algorithms, for instance, Local Defect Correction (LDC). We construct a combination of LDC and FDTD, discuss stability and accuracy issues and consider an application of LDC and FDTD to a number of problems.

[1] K.S. Yee, *Numerical solution of initial boundary value problem involving Maxwell's equations in isotropic media.* IEEE Transactions on antennas and propagation, AP-14:302-307, May 1966.

[2] V. Nefedov, *Subgridding in FDTD*, technical report RANA 02-29, TU Eindhoven, Eindhoven, 2002

Fausto Saleri, E. Miglio, S. Perotto

A Multiphysics Strategy for Free Surface Flows

Location: Room 005, **Time:** Tuesday, 22 July, 16:50

Several environmental engineering applications involve free surface flows phenomena. In this context the wide variety of situations leads to consider a large spectrum of space and time scales related to the presence of different physical phenomena [2,6,7].

Various models have been developed in order to cope with the above mentioned problems; these models can be grouped into the following categories (in descending order of complexity): as for the 3D case one can consider either the Free Surface Navier-Stokes or the Hydrostatic Shallow Water equations (see [1]); concerning the 2D situation the Boussinesq, Serre or Shallow Water models can be adopted (see [4], [5]); finally the 1D counterpart of these latter models can be used. Most of these models are well established, both in terms of a sound mathematical formulation and of a robust numerical implementation.

Ideally one should use a full 3D model to capture all the physical features of the problem at hand. However this approach is characterized by a huge computational effort that we aim to reduce by suitably coupling models of different dimensions among the ones mentioned above.

We present a multiphysics strategy in order to take into account different scales by combining 3D, 2D and 1D models retaining a reasonable computational cost.

[1] Causin, P.; Miglio, E.; Saleri, F.: *Algebraic factorizations for 3D non-hydrostatic free surface flows.* Comput. Visual. Sci., 5 (2002), no. 2, 85–94,

[2] Debnath, L.: *Nonlinear Water Waves*, Academic Press, San Diego, 1994.

[3] Marrocu, M.; Ambrosi, D.: *Mesh adaptation strategies for shallow water flow.* Internat. J. Numer. Methods Fluids 31 (1999), no. 2, 497–512.

[4] Grasselli, M.; Perotto, S.; Saleri, F.: *Space-time finite elements for Boussinesq equations.* East-West J. Numer. Math., 7 (1999), no. 4, 283–306.

[5] Perotto, S.; Saleri, F.: *Adaptive finite element methods for Boussinesq equations.* Numer. Methods Partial Differential Equations, 16 (2000), no. 2, 214–236.

[6] Vreugdenhil, C.B.: *Numerical methods for Shallow-Water flows*, Kluwer Academic Press, Dordrecht, 1998.

[7] Whitham, G.B.: *Linear and Nonlinear Waves*, Wiley, New York, 1974.

Friedhelm Schieweck, W.J. Layton, I. Yotov

Coupling Fluid Flow with Porous Media Flow

Location: Room 005, **Time:** Tuesday, 22 July, 17:15

The transport of substances back and forth between surface and ground water is a very serious problem. We study herein the mathematical model of this setting consisting of the Stokes equations in the fluid region coupled with Darcy's equations in the porous medium, coupled across the interface by the Beavers-Joseph-Saffman conditions.

We prove existence and uniqueness of a weak solution. For the approximation of velocity and pressure, we use an arbitrary pair of conforming LBB-stable finite element spaces in the fluid region and any pair of the well-known mixed finite element spaces like RT-spaces or BDM-spaces in the region of the porous medium. The coupling of the normal components of the velocity approximation across the interface can be characterized easily by means of hanging nodes. For the coupled approximation spaces of both regions, we prove the LBB-condition and optimal interpolation estimates. This leads to an estimate of the discretization error which is of optimal order.

The analysis of our finite element scheme suggests a way to uncouple the solution of the global discrete problem into steps involving only solvers for the porous media and fluid flow subproblems. This is important because there are many codes available which have been optimized for solving these subproblems.

[1] W.J. Layton, F. Schieweck, I. Yotov, *Coupling Fluid Flow with Porous Media Flow.* SIAM J. Numer. Anal., Vol. 40, No. 6, pp. 2195-2218 (2003).

Paolo Zunino

Iterative Substructuring Methods for Advection-Diffusion Problems in Heterogeneous Media

Location: Room 005, **Time:** Tuesday, 22 July, 17:40

This work is devoted to the numerical approximation of a system of advection-diffusion equations set in adjacent domains and coupled with non-standard matching conditions. The specific application of the model at hand is the study of the transfer of chemicals through media of heterogeneous nature, for example a free fluid and a porous medium. This problem has several practical applications, for instance the study of the motion of a pollutant from a basin to the adjacent soil or the study of the transport of chemicals from the blood flow to the wall of the vessels.

Because of the heterogeneity of the media we consider, the concentration of chemicals may feature large variations from one medium to the other. For example, if a pollutant is released into a basin, its concentration in the free fluid is reasonably higher than the one in the surrounding soil. Consequently, the numerical simulation of such problems could be an extremely challenging task. To this aim, we propose a particular choice of the matching conditions between the advection-diffusion equations on each medium, which allow the discontinuity of the concentration across the interface.

After the description of the model, we focus our attention on its numerical treatment. In particular, since our model couples subproblems in different media, we study an iterative procedure where the solutions provided separately on each subdomain are suitably matched (a so called iterative substructuring method). More precisely, we consider a strategy based on Robin interface conditions for both subdomains. The convergence of this iterative strategy is analyzed at both the continuous and the discrete level, by means of suitable interface operators of Steklov-Poincaré type. Moreover, an algebraic reinterpretation of this technique is provided, leading to the definition of optimal preconditioners for the linear system arising from the discretization of the global problem.

Finally, numerical results are presented, in order to assess the computational efficiency of the numerical methods proposed.