MS06 Robust Decomposition Methods for Parameter Dependent Problems

Organized by: Ulrich Langer, Sergey Nepomnyaschikh

The main topic of this Minisymposium consists of the construction and analysis of robust solvers parameter dependent problems. The following problems will be a subject to consider.

- elliptic problems with singularities in the coefficients,

- anisotropic boundary value problems,

- Effective solvers of hp schemes,

- preconditioning operators in weighted Sobolev spaces

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Sergey V. Nepomnyaschikh, E.-J. Park

Preconditioning for Heterogeneous Problems
Location: Room 049, **Time:** Monday, 21 July, 16:00

Preconditioning for heterogeneous elliptic problems is considered. The basis for the construction of preconditionig operators in this talk is domain decomposition methods. Suggested domain decomposition technique involves the construction of local preconditioning operators and interface preconditioning operators. For the latter preconditioners we need to investigate Schur complements or trace theorems in the corresponding Sobolev spaces.

Karl Scherer

Weighted Norm-Equivalences for Preconditioning
Location: Room 049, **Time:** Monday, 21 July, 16:25

Let $\mathcal{V}_j$ be a sequence of hierarchical spaces of piecewise polynomial functions with respect to triangulations $\mathcal{T}_0 \subset \mathcal{T}_1 \subset \cdots \subset \mathcal{T}_J$ with respect to a domain $\Omega \in \mathrm{R}^2$ and consider the Ritz-Galerkin approximation of an elliptic problem associated with the bilinear form $a(u,v) := \int_\Omega \sum_{i,k} a_{i,k}(D_i u)(D_k v)$. We replace or approximate it by a discretized bilinear form satisfying the local ellipticity condition ($\underline{\omega}_i, \overline{\omega}_i > 0$)

$$\underline{\omega}_i \sum_{i=1}^2 \xi_i^2 \le \sum_{i,k=1}^2 a_{i,k}(x)\xi_i\xi_k \le \overline{\omega}_i \sum_{i=1}^2 \xi_i^2,$$
$$x \in Z_i, \text{ for each triangle } Z_i \in \mathcal{T}_J$$

Under this assumption robust norm equivalences of the form

$$\beta_0\Big(||P_0||_X^2 + \sum_{j=1}^J 4^j||P_j u - P_{j-1}u||_X^2\Big)$$
$$\le a(u,u) \le$$
$$\beta_1\Big(||P_0||_X^2 + \sum_{j=1}^J 4^j||P_j u - P_{j-1}u||_X^2\Big)$$

for projections $P_j : \mathcal{V}_j \to \mathcal{V}_J$ in weighted L_2- norms are established. "Robust" means that the constants β_0, β_1 do not depend on the weights, the number J of levels and $u \in \mathcal{V}$. The essential tools are weighted Jackson- and Bernstein-inequalities. Applications to preconditioning are indicated.

Sven Beuchler

A Dirichlet-Dirichlet DD-preconditioner for p-fem
Location: Room 049, **Time:** Monday, 21 July, 16:50

In this talk, a uniformly elliptic second order boundary value problem in 2D is discretized by the p-version of the finite element method. An inexact Dirichlet-Dirichlet domain decomposition preconditioner for the system of linear algebraic equations is investigated. The solver for the problem in the subdomains [1] and a preconditioner for the Schur-complement, [2] are proposed as ingredients for the inexact DD-preconditioner. Finally, several numerical experiments are given.

[1] S. Beuchler. *Optimal preconditioners for the p-version of the fem.* Preprint SFB393/03-03, Technische Universität Chemnitz, Sonderforschungsbereich 393, 2003.

[2] S. Jensen and V.G. Korneev. *On domain decomposition preconditioning in the hierarchical p−version of the finite element method.* Comput. Methods Appl. Mech. Eng., 150(1–4):215–238, 1997.

Almedin Becirovic, Joachim Schöberl

Optimal Extension Operators for High Order Tetrahedral Elements
Location: Room 049, **Time:** Monday, 21 July, 17:15

The goal of this work is the construction of fast iterative solvers for matrix equations arising from high order finite elements. We focus on cheap block-Jacobi and block-Gauss-Seidel iterations, where the blocks are defined by the shape functions associated to edge-, face- and interior-nodes. Of course, the speed of convergence depends on the choice of the shape functions.

We present new shape functions leading to nearly optimal iteration numbers. The construction is based on polynomial extension operators. By the help of symbolic computing we could derive cheap recursion formulas for the efficient computation of the shape functions. Numerical results for 2D and 3D problems are presented.

[1] A. Bećirović and J. Schöberl, *Hierarchical shape functions based on explicit extension operators*, SFB-Report 03xx, Johannes Kepler University Linz, 2003

Li Deng, Ichiro Hagiwara

Folding Process of Thin-Walled Prismatic Columns by Origami Modeling
Location: Room 049, **Time:** Monday, 21 July, 17:40

We study folding process of thin-walled prismatic columns by Origami modeling, its applied background is how to crush a empty can and PET bottle efficiently. We will give its simple mathematical model, and showed some simulations by nonlinear software Ls-dyna.