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MS08 Domain Decomposition on Nonmatching Grids

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Domain decomposition methods on nonmatching grids have attracted a lot of attention during the last couple of years. Originating from discretizations by spectral elements, they have been considered for a wide variety of finite element discretizations and for applications in computational electromagnetics, fluid dynamics, and mechanics. The aim of this minisymposium is to report on recent advances in this area.

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Dietrich Braess

A Cascadic Multigrid Algorithm for Mortar Elements

Location: Lecture Room, **Time:** Monday, 21 July, 16:00

It is typical for domain decomposition methods that auxiliary problems have to be solved on the interfaces/skeleton and that approximate Schur complements are involved. The methods differ by the choice of the approximation.

We discuss a multigrid method where an approximate Schur complement solver is hidden in the smoothing procedure, and it is the advantage of the multigrid method that a very cheap approximation can be used. The robustness of the method is confirmed by the fact that even the cascadic version of the multigrid method is robust and efficient.

[1] D. Braess, P. Deuflhard, and K. Lipnikov: *A cascadic multigrid algorithm for mortar elements.* Computing 69 (2002), 205–225

Yvon Maday, Bernd Flemisch, Francesca Rapetti, Barbara I. Wohlmuth

Coupling Scalar and Vector Potentials on Nonmatching Grids for Eddy Currents in Moving Conductor

Location: Lecture Room, **Time:** Monday, 21 July, 16:25

The $T - \Omega$ formulation of the magnetic field has been introduced in many papers for the approximation of the magnetic quantities modeled by the eddy current equations. This decomposition allows to use a scalar function in the main part of the computational domain, reducing the use of vector quantities to the conducting parts.

We propose to approximate these two quantities on nonmatching grids so as to be able to tackle a problem where the conducting part can move in the global domain. The connection between the two grids is managed with mortar element techniques.

[1] Y. Maday, F. Rapetti, B.I. Wohlmuth, *Coupling between scalar and vector potentials by the mortar element method*, C. R. Acad. Sci. Paris, Ser. I 334 (2002) pp. 933-938.

[2] B. Flemisch, Y. Maday, F. Rapetti, B. Wohlmuth, *Coupling scalar and vector potentials on nonmatching grids for eddy currents in a moving conductor*, Journal of Computational and Applied Mathematics, 2003, to appear.

Christian Wieners, Barbara I. Wohlmuth

Multigrid Analysis for Saddle Point Problems Arising from Mortar Discretizations

Location: Lecture Room, **Time:** Monday, 21 July, 16:50

We present an abstract framework for the analysis of multigrid methods for a saddle point problem, and we discuss the application systems arising from mortar finite element discretizations. In contrast to other approaches, the iterates do not have to be in the positive definite subspace. This is required for the case of nonnested Lagrange multiplier spaces.

We apply the multigrid method with a smoother of Braess-Sarazin type to different mortar settings including dual Lagrange multipliers, linear elasticity and a rotating geometry. Numerical results in 2D and 3D realized in the software system UG demonstrate the flexibility, efficiency and reliability of our multigrid method.

[1] C. Wieners and B. Wohlmuth. *Duality estimates for saddle point problems arising from mortar discretizations.* 2003 (to appear in SIAM Sci. Comp.)

Bishnu Prasad Lamichhane, Barbara I. Wohlmuth

Second Order Lagrange Multiplier Spaces for Mortar Finite Element Discretizations

Location: Lecture Room, **Time:** Monday, 21 July, 17:15

The coupling of different discretization schemes or of nonmatching triangulations can be analyzed within the framework of mortar methods. These nonconforming domain decomposition techniques provide a more flexible approach than standard conforming approaches. They are of special interest for time-dependent problems, diffusion coefficients with jumps, problems with local anisotropies, corner singularities, and when different terms dominate in different regions of the simulation domain. To obtain a stable and optimal discretization scheme for the global problem, the information transfer between the subdomains is of crucial importance.

In the first part, we will concentrate on mortar techniques for quadratic finite elements, in particular, with hexahedral triangulations in 3D. We will discuss different Lagrange multiplier spaces for standard triquadratic finite elements and Serendipity elements. Finally, the numerical results for linear and quadratic mortar finite elements in 2D and 3D will be presented. Different Lagrange multiplier spaces will be compared and some applications will be discussed.

[1] C. Bernardi and Y. Maday and A.T. Patera (1993), *Domain decomposition by the mortar element method*, In: Asymptotic and numerical methods for partial differential equations with critical parameters

[2] B.I. Wohlmuth (2001) *Discretization Methods and Iterative Solvers Based on Domain Decomposition*, Springer Heidelberg, 17

[3] B.P. Lamichhane and B.I. Wohlmuth (2002), *Higher Order Lagrange Multiplier Spaces For Mortar Finite Element Discretizations*, CALCOLO, 39(4), 219-237

[4] B.I. Wohlmuth (2001) *A Comparison of Dual Lagrange Multiplier Spaces for Mortar Finite Element Discretizations*, M^2AN, 36, 995-1012

Xuejun Xu, Zhong-Ci Shi

A Mortar Element Method for a Plate Bending Problem

Location: Lecture Room, **Time:** Monday, 21 July, 17:40

Recently, Marcinkowski [Numer. Math.,2002] presented a mortar element method for some plate elements such as the conforming Hsieh-Clough-Tocher(HCT) and the reduced Hsieh-Clough-Tocher(RHCT) elements. Optimal energy norm error estimate was obtained under the elliptic regularity assumption that the solution is in the space $H^4(\Omega) \cap H_0^2(\Omega)$. However, it is known that a realistic regularity assumption for a plate bending problem is $H^3(\Omega) \cap H_0^2(\Omega)$.

In this talk, using the techniques developed by Shi for deriving the error estimates for the nonconforming Morley element we will show that a mortar element method is applicable to the plate bending problem with the above weaker regularity assumption. The optimal energy norm error estimate and H^1 norm estimate will be given, which cannot be derived by Marcinkowski's approach.

An optimal multigrid method for the mortar element will also be presented.

Bernd Heinrich

Nitsche-Type Mortaring for Elliptic Problems with Singularities and Boundary Layers

Location: Lecture Room, **Time:** Thursday, 24 July, 16:00

The paper deals with the Nitsche-type mortaring for treating weak continuity across non-matching meshes for some non-overlapping domain decomposition. The approach is derived as a generalization of some method of J.A. Nitsche(1971), a survey about recent results and a comparison with the classical mortar method are given. In particular, the method is applied to transmission problems with coefficients being discontinuous at polygonal interfaces as well as to singularly perturbed reaction-diffusion problems. The solutions of such problems are provided with corner singularities or/and boundary layers. The interface of the domain decomposition may pass corners of the boundary or of the physical interface, or this interface may be aligned with the boundary layer. For such problems and nonmatching meshes of triangles, which are anisotropic in the boundary layers, stability as well as error estimates of the finite element schemes are proved. Some numerical examples illustrate the approach and the results.

Yuri Kuznetsov

Mixed Finite Element Methods for Diffusion Equations on Nonmatching Grids

Location: Lecture Room, **Time:** Thursday, 24 July, 16:25

The algebraic aspects of the mixed and mixed-hybrid finite element discretizations on nonmatching grids for the diffusion equations are discussed. Using the standard condensation technique the underlying algebraic saddle point problems can be reduced to the systems with symmetric positive definite matrices.A number of preconditioners are proposed and analysed for both saddle point and condensed matrices.

Talal Rahman, Ronald H.W. Hoppe, Xuejun Xu

Additive Schwarz Method for the Mortar Crouzeix-Raviart Element

Location: Lecture Room, **Time:** Thursday, 24 July, 16:50

We present an additive Schwarz method for the mortar Crouzeix-Raviart finite element for solving systems of algebraic equations resulting from the approximation of second order elliptic boundary value problems with discontinuous coefficients.

The finite element approximation is done by independently triangulating each subregion (subdomain) with the P1 non-conforming or the Crouzeix-Raviart element, and then by using the mortar technique (originally introduced by Bernardi et al. for conforming elements, recently extended to the CR element by Marcinkowski) to describe the discrete problem.

It is well known that large jumps in the coefficients cause trouble for most iterative methods. There already exists some Schwarz methods which are insensitive to such jumps, for instance, for the standard P1 conforming element and the mortar P1 conforming element. The aim of this talk is to extend their ideas to using the mortar CR element, hoping that the new algorithm, due to special construction, will have several advantages (algorithmic) over the old ones.

Andreas Gantner, Max Dryja, Olof B. Widlund, Barbara I. Wohlmuth

Multilevel Additive Schwarz Preconditioner for Nonconforming Mortar Finite Element Methods

Location: Lecture Room, **Time:** Thursday, 24 July, 17:15

Mortar elements form a family of special nonoverlapping domain decomposition methods which allows the coupling of different triangulations across subdomain boundaries. We discuss and analyze a multilevel preconditioner for mortar finite elements on nonmatching triangulations. The analysis is carried out within the abstract framework of additive Schwarz methods. Numerical results show a performance of our preconditioner as predicted by the theory. Our condition number estimate depends quadratically on the number of refinement levels

Padmanabhan Seshaiyer

Non-Conforming Finite Element Methods for Nonmatching Grids Tuned to Parallel Implementation

Location: Lecture Room, **Time:** Thursday, 24 July, 17:40

Engineering applications often require finite element analysis to be carried out over complex domains. Often such analysis, particularly the labor-intensive meshing phase, may be accomplished by dividing the global domain into several local sub-domains. These subdomains may be constructed separately by different analysts and the global domain can then be constructed by piecing together these individually modeled subdomains. However, during the assembly, it is often too cumbersome, or even infeasible, to coordinate the meshes over separate subdomains. The mortar finite element method ([1, 2, 3]) is a non-conforming domain decomposition technique which helps to accomplish such a modeling task. An added advantage of this approach is that mesh refinement can be imposed selectively on those components where it is required. Moreover, different variational problems in different subdomains can be combined.

In this talk, the hp-version of the non-conforming technique developed in [3, 4, 8, 9, 10] will be discussed where the stability and convergence were shown to be "uniform" in terms of "both" the polynomial degree "and" the mesh refinement used, without assuming quasiuniformity for the meshes. The numerical analysis of the mortar finite element method relies on the coercivity of the bilinear form and an evaluation of approximation and consistency error terms ([5, 6, 7]). Our numerical results show optimality for the resulting non-conforming method for various h, p and hp discretizations, including the case of exponential hp convergence over geometric meshes. We will also present numerical results for the Lagrange multiplier and also discuss extensions to three-dimensions.

[1] C. Bernardi, Y. Maday, and A.T. Patera, *Domain decomposition by the mortar element method*, in Asymptotic and Numerical Methods for PDEs with Critical Parameters. H.G. Kaper and M. Garbey, (eds.), NATO Adv. Sci. Inst. Ser. C Math. Phs. Sci, 384, Kluwer, 269-286(1993).

[2] F. Ben Belgacem, *The mortar finite element method with Lagrange Multipliers*, Numerische Mathematik, 84, 173–197, (1999).

[3] P. Seshaiyer and M. Suri, *Uniform hp convergence results for the mortar finite element method*, Mathematics of Computation, 69, 521-546 (2000).

[4] P. Seshaiyer and M. Suri, *hp submeshing via non-conforming finite element methods*, Computer Methods in Applied Mechanics and Engineering, 189, 1011-1030 (2000).

[5] F. Ben Belgacem, P. Seshaiyer and M. Suri, *Optimal convergence rates of hp mortar finite element methods for second-order elliptic problems*, RAIRO Mathematical Modeling and Numerical Analysis, 34, 591-608 (2000).

[6] P. Seshaiyer and M. Suri, *Convergence results for non-conforming hp methods: The mortar finite element method*, Contemporaty Mathematics, 218, 453-459 (1998).

[7] P. Seshaiyer, *Stability and convergence of non-conforming hp finite element methods*, Computers and Mathematics with Applications, Accepted and to appear, 2001.

[8] F. Ben Belgacem, L.K. Chilton and P. Seshaiyer, *The hp-mortar finite element method for the mixed elasticity and stokes problems*, Computers and Mathematics with Applications, Accepted and to appear, 2001.

[9] F. Ben Belgacem, L.K. Chilton and P. Seshaiyer, *Non-conforming hp finite element methods for Stokes Problem*, Lecture Notes in Computer Science, Springer, 23, 133-146 (2001).

[10] L.K. Chilton and P. Seshaiyer, *The hp mortar domain decomposition method for problems in fluid mechanics*, International Journal of Numerical Methods in Fluids, 40, 1561-1570 (2002).