

MS10 Recent Advances for the Parareal in Time Algorithm

Organized by: Yvon Maday

The parareal in time algorithm has been introduced by Lions Maday and Turinici. It is an iterative procedure based on the alternate use of a coarse and a fine solvers that allows to squeeze the resolution time if parallel architectures are available.

Since its introduction, many developments and applications have been added and the coupling of this idea with standard domain decompositions or control iterations allow to increase the efficiency of the combined iterative solvers. An effort has also been made on the understanding of the effects of the use different schemes for the time resolution. Finally, the definition of the coarse solvers can be relaxed to allow for very fast resolution in this orthogonal direction.

This minisymposium will present 4 contributions illustrating the different features and providing a wide range of applications of this new direction for parallelization.

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Yvon Maday

The Parareal Algorithm: Basics and Combination with Domain Decomposition Iterations

Location: Room 005, **Time:** Tuesday, 22 July, 11:00

In this first talk we introduce the basics of the parareal algorithm in particular we focus on its new presentation that can be understood as a predictor-corrector procedure. The combination of the iterative method in time with other types of iterative techniques in other

directions as the spacial one for domain decomposition techniques or control one for optimal control iterative techniques can then be proposed and results illustrating the potentiality of the algorithm are presented.

Guillaume Bal

On the Analysis and Implementation of the Parareal Algorithm

Location: Room 005, **Time:** Tuesday, 22 July, 11:25

The parareal algorithm allows us to parallelize differential equations in time. It is based on solving successively non-parallel coarse discretizations and parallel fine discretizations, and can replace a discretization of order m by a discretization of order $(k + 1)m$ after k iterations.

We present convergence results for ordinary differential equations and stochastic ordinary differential equations, which shows robustness of the algorithm for a certain

class of stiff equations. We analyze the strategies that should be used to maximize the total speedup or the system efficiency (maximize the time where processors are active).

Finally we present some numerical simulations corresponding to applications where we believe the algorithm may be useful.

Gunnar Andreas Staff

Stability and Convergence of the Parareal Algorithm

Location: Room 005, **Time:** Tuesday, 22 July, 11:50

The point of departure for this talk is the parareal algorithm presented in [1]. This is an algorithm for solving time-dependent differential equations, and is genuinely only of interest in a parallel context. We discuss the stability of the parareal algorithm for autonomous differential equations, and present theoretical and numerical results for the linear diffusion equation and the

viscous Burger's equation. The spatial discretization is here based on pure spectral methods, while various choices for time integration schemes are used. A coarse propagator based on a coarse discretization in both time and space will also be discussed. We conclude by giving recommendations regarding suitable choices of time integration schemes for both the coarse and the fine prop-

agator for various types of partial differential equations.

- [1] Guillaume Bal and Yvon Maday. *A "parareal" time discretization for non-linear PDE's with application to the pricing of an American put*. In Luca

F. Pavarino and Andrea Toselli, editors, Recent Developments in domain Decomposition Methods, volume 23 of Lecture Notes in Computational Science and Engineering, pages 189-202. Springer, 2002

Paul F. Fischer

Investigation of the Parareal Algorithm for Semi-Implicit Incompressible Navier-Stokes Simulations

Location: Room 005, **Time:** Tuesday, 22 July, 12:15

We present preliminary results on the performance of the time-parallel ("Parareal") algorithm in a semi-implicit spectral element code for the solution of the incompressible Navier-Stokes equations.