15th International Conference on Domain Decomposition Methods
July 21-25, 2003, Berlin, Germany

MS11 Space Decomposition and Subspace Correction Methods for Linear and Nonlinear Problems

Organized by: Xue-Cheng Tai, Jinchao Xu

Domain decomposition and multigrid methods can be interpreted as space decomposition and subspace correction methods. There are different ways to extend these methods to different nonlinear problems – physical problems with nonlinear differential operators and with constraints over the solutions. The purpose of this minisymposium is to bring people in this research direction to present new results and discuss new progresses.

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Lori Badea

On a Multilevel Schwarz Method for the Constraint Minimization of non-Quadratic Functionals

Location: Room 049, **Time:** Thursday, 24 July, 16:00

The literature on the domain decomposition methods is very large. We can see, for instance, the papers in the proceedings of the annual conferences on domain decomposition methods starting in 1988 with [5]. Naturally, the most of papers dealing with these methods are dedicated to the linear elliptic problems. Also, for the variational inequalities, the convergence proofs refer in general to the inequalities coming from the minimization of quadratic functionals. To our knowledge, even if sometimes the authors make some remarks in their papers on the nonlinear cases, very few papers really deal with the application of these methods to nonlinear problems. We can cite in this direction the papers written by Tai and Espedal [12], Tai and Xu [13] for nonlinear equations, Hoffmann and Zhou [6], Lui [8], Zeng and Zhou [14] for inequalities having nonlinear source terms, and Badea [3], [2] for the minimization of non-quadratic functionals.

The multilevel or multigrid methods can be viewed as domain decomposition methods and we can cite the results obtained by Kornhuber [7], Mandel [9], [10], Smith, Bjørstad and Gropp [11], and Badea, Tai and Wang [4]. Evidently, this list is not exhaustive and it can be completed by many other papers.

In [1], the convergence of a Schwarz method for variational inequalities coming from the minimization of a quadratic functional has been proved. This method has been extended to the one and two-level methods in [4]. Also, its convergence for the constraint minimization of the non-quadratic convex functionals is proved in [2]. The goal of this paper is the extension of the results in [2] to the study of the multilevel methods. The error estimates we give show the same dependence of the error on the mesh and overlapping parameters as in the linear cases. Numerical examples are given to illustrate the convergence of the methods.

[1] L. BADEA, *On the Schwarz alternating method with more than two subdomains for nonlinear monotone problems*, SIAM J. Numer. Anal., 28 (1991), pp. 179-204.

[2] L. BADEA, Convergence rate of a multiplicative Schwarz method for strongly nonlinear inequalities, in *Analysis and optimization of differential systems*, V. Barbu, I. Lasiecka, D. Tiba and C. Varsan Eds., Kluwer Academic Publishers, Boston/Dordrecht/London, ISBN 1-4020-7439-5, 2003.

[3] L. BADEA, *Domain decomposition method for strongly nonlinear inequalities*, Preprint series of the Instituite of Mathematics of the Romanian Academy, nr. 6, 2002.

[4] L. BADEA, X.-C. TAI AND J. WANG, *Convergence rate analysis of a multiplicative Schwarz method for variational inequalities*, SIAM J. Numer. Anal., accepted for publication, 2002.

[5] R. GLOWINSKI, G.H. GOLUB, G.A. MEURANT, Eds. 1998, *First Int. Symp. on Domain Decomposition Methods*, SIAM, Philadelphia.

[6] K.H. HOFFMANN AND J. ZOU, *Parallel solution of variational inequality problems with nonlinear source terms*, IMA J. Numer. Anal. 16, 1996, pp. 31-45.

[7] R. KORNHUBER, *Monotone multigrid methods for elliptic variational inequalities I*, Numer. Math. 69 (1994), pp. 167-184.

[8] S.-H. Lui, *On monotone Schwarz alternating methods for nonlinear elliptic Pdes*, Modél. Math. Anal. Num., ESIAM:M2AN, vol. 35, no. 1, 2001, pp. 1-15.

[9] J. Mandel, *A multilevel iterative method for symmetric, positive definite linear complementary problems*, Appl. Math. Optimization, 11 (1984), pp. 77-95.

[10] J. Mandel, Hybrid domain decomposition with unstructured subdomains, *Proceedings of the 6th International Symposium on Domain Decomposition Methods*, Como, Italy, 1992, Contemporary Mathematics, 157, 103-112.

[11] F.S. Barry, P.E. Bjørstad and W. Gropp, *Domain decomposition: Parallel Multilevel Methods for Elliptic Differential Equations*, Cambridge University Press, 1996.

[12] X.-C. Tai and M. Espedal, *Rate of convergence of some space decomposition methods for linear and nonlinear problems* SIAM J. Numer. Anal., vol. 35, no. 4 (1998), pp. 1558-1570.

[13] X.-C. Tai and J. Xu, *Global and uniform convergence of subspace correction methods for some convex optimization problems*, Math. of Comp., electronically published on May 11, 2001.

[14] J. Zeng and S. Zhou, *Schwarz algorithm for the solution of variational inequalities with nonlinear source terms*, Appl. Math. Comput., 97, 1998, pp.23-35.

Ralf Kornhuber, Susanna Gebauer, Harry Yserentant

Hierarchical Decomposition of Domains with Fractures
Location: Room 049, **Time:** Thursday, 24 July, 16:25

We consider a diffusion problem with strongly varying diffusion coefficients. More precisely, the diffusion k_0 in a subdomain consisting of a network of fractures is much larger than it is in the remaining porous matrix. The network of fractures consists of long thin rectangles with width $\varepsilon > 0$. Such kind of problems occur not only in hydrology (where the above notions come from) but also has other applications in engineering sciences or medicine. The main difficulty in constructing fast solvers is that the permeability k_0 may become arbitrary large while the width ε may become arbitrary small.

In order to reduce the degrees of freedom, we use an anisotropic quadrilateral partition of the network of fractures while the porous matrix is partitioned in the usual way by isotropic triangles. For the resulting discrete problem we present an hierarchical domain decomposition algorithm with appropriate multigrid solvers for the subproblems. It turns out that under reasonable assumptions on the mesh parameters our algorithm converges robustly for arbitrary large permeability k_0 and vanishing width ε. These theoretical findings are illustrated by numerical computations.

Xue-Cheng Tai

Nonlinear Positive Intepolation Operators for Analysis with Multilevel Grids
Location: Room 049, **Time:** Thursday, 24 July, 16:50

In order to analyse the convergence of multigrid and domain decomposition methods for obstacle type of problems, we need to use some some special interpolation operators. The following properties are typically needed:

1. It should have the standard approximation errors.
2. It should preserve positive functions.
3. It should be monotone, i.e. the interpolation for a given functions on a coarser mesh should not less than the interpolation on a finer mesh.
4. The interpolation operator should be "stable" in some suitable norm.

We shall construct a nonlinear interpolation operator which satisfies all the above properties and show how can we use it for the analysis for multigrid methods for obstacle type of problems.

[1] Nochetto, R.H. and Wahlbin, Lars B.: *Positivity preserving finite element approximation.* Math. Comp. 71 (2002), no. 240, 1405–1419.

[2] Lori Badea, Xue-Cheng Tai and Junping Wang; *Convergence rate analysis of a multiplicative Schwarz method for variational inequalities.* To appear in: SIAM J. Numer. Anal..

[3] Xue-Cheng Tai: *Rate of convergence for some constraint decomposition methods for nonlinear variational inequalities*, Numer. Math., 2003, 93:755-786.

[4] X.-C. Tai, B. Heimsund and J. Xu: *Rate of Convergence for Parallel Subspace* Correction Methods for nonlinear variational inequalities. "Thirteenth international domain decomposition conference", 2001, pp. 127–138, CIMNE.

Jinchao Xu, Long Chen, Pengtao Sun

Anisotropic grid adaptation and multigrid methods
Location: Room 049, **Time:** Thursday, 24 July, 17:15

Some recent studies are reported in this talk on using multilevel subspace correction techniques in grid adaptation. Results presented include gradient and Hessian recovery schemes by using averaging and smoothing (as in multigrid), interpolation error estimates for both isotropic and anisotropic grids and multilevel techniques for global grid moving and local grid refining/moving. Some applications will also be given.

Izaskun Garrido, Magne S. Espedal, G. E. Fladmark

A Convergent Algorithm for Time Parallelization Applied to Reservoir Simulation
Location: Room 049, **Time:** Thursday, 24 July, 17:40

Parallel methods are usually applied to the space domain because the sequential nature of time is considered to be a handicap for the development of competitive algorithms. However, this sequential nature can also play to our advantage by ensuring convergence within a given number of iterations. This new algorithm for time parallelization is derived from the classical Alternating Schwarz method and acts as a predictor corrector improving both speed and accuracy with respect to the sequential solvers. It reduces significantly the numerical effort for the computation of the molar masses in our complex reservoir simulator `Athene`, which will be illustrated with several numerical examples.

[1] G. Bal, Y. Maday. *A parareal time discretization for non-linear PDE's with application to the pricing of an American put*

[2] Jacques-Louis Lions, Y. Maday, Gabriel Turinici. *Rèsolution d'EDP par un schéma en temps pararéel.* In C. R. Acad. Sci. Paris , **332**, No. 1, pages 1-6. 2001