MS12 Trefftz-Methods

Organized by: Ismael Herrera, Robert Yates

Trefftz originally proposed his method in 1926 [1], but it has been in the last decades, especially since Jirousek and his collaborators originated hybrid-Trefftz (HT) finite element (FE) model [2-4] that it has become a highly efficient computational tool for the solution of difficult boundary value problems. More recently, Herrera interpreted Trefftz method as a domain decomposition method (DDM) and proposed a unified theory of DDMs based on it [5-7]. Trefftz methods, when seen from this perspective, are classified into two broad categories: direct (or Trefftz-Jirousek) and indirect methods. This latter approach is frequently called Trefftz-Herrera method because was originated by Herrera and his collaborators [8,9]. The unified theory expands considerably Trefftz method scope and yields new avenues of thought for numerical methods of partial differential equations. In particular, it has produced an elegant and very general formulation of DDMs [10],together with broad families of numerical approximations such as Localized Adjoint Methods (LAM) [11,12].

Trefftz methods have received important contributions for their development from many different fields of application [13]; among them: potential problems, plane elasticity, plate bending (thin, thick, post-buckling), heat conduction, advective diffusive transport. In spite of their obvious connections, the community that studies DDMs and that which studies Trefftz method have remained apart and, to a large extent, unaware of each other. Clearly, putting them in contact is a very worthwhile endeavor since an exchange of experiences would be very profitable for both of them. The purpose of this minisymposium is to present and discuss recent progress of Trefftz method, and its implications in different fields of application,when it is interpreted as a domain decomposition method. Key representatives of the communities mentioned above will make such presentations and discussions.

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Ismael Herrera, Robert Yates

A New and More General Version of the Hybrid-Trefftz Finite Element Model, Derived by Application of th-Domain Decomposition

Location: Room 049, **Time:** Tuesday, 22 July, 11:00

In recent years the hybrid-Trefftz finite element (hT-FE) model, which originated in the work by Jirousek and his collaborators [1,2], and makes use of an independently defined auxiliary inter-element frame, has been considerably improved. It has indeed become a highly efficient computational tool for the solution of difficult boundary value problems [3-5]. In parallel and to a large extent independently, a general and elegant theory of Domain Decomposition Methods (DDM) has been developed by Herrera and his coworkers [6-8], which has already produced very significant numerical results [8-10]. Theirs is a general formulation of DDM, which subsumes and generalizes the standard approaches. In particular, it is the natural theoretical framework for Trefftz methods, as has been recognized by some of the most conspicuous researchers of this area [11-13]. To clarify further this point, it is important to spell out in greater detail than has been done so far, the relation between Herrera's theory and the procedures studied by researchers working in standard approaches to Trefftz method (Trefftz-Jirousek approach). As a contribution to this end, in this paper the hybrid-Trefftz finite element model is derived in considerable detail, from Herrera's theory of DDM. In addition, by so doing, almost automatically, the hT-FE model is generalized to non-symmetric systems (actually, to any linear differential equation, or system of such equations, independently of its type) and for boundary value problems with prescribed jumps.

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Robert Yates, Ismael Herrera

Trefftz-Herrera Method: Highly Accurate Numerical Algorithms for Parabolic Equations

Location: Room 049, **Time:** Tuesday, 22 July, 11:25

Trefftz-Herrera method, or indirect method, has attracted much interest recently due to its generality and flexibility in deriving solutions on the internal boundary of the domain decomposition. For the case of elliptic equations, of both second and higher order, the method has been shown to give excellent results. In this paper, for the first time, the method has been successfully applied to parabolic equations and a family of numerical procedures which yield solutions to any desired accuracy is presented. A particular member of this family is the well-known Crank-Nicolson second order procedure -thus, the family of numerical algorithms here presented may be thought as a generalization of the Crank-Nicolson procedure-.

Ernesto Rubio-Acosta, Ismael Herrera, Robert Yates

Parallel Implementation of Indirect Collocation Methods

Location: Room 049, **Time:** Tuesday, 22 July, 11:50

Implementing collocation methods by parallel processing, presents complications that must be overcome in order to profit from the advantages of these resources [1,2]. However, when a TH-Discretization approach is used, a very novel and effective manner of incorporating parallel processors in the computational models has recently being introduced. These new procedures are also based on TH-Domain Decomposition. In particular, a novel, simple and systematic manner of producing the Schur complement and the local matrices, which completely reduces the global equation, is here reported. A discussion of the implications of these new procedures, as well as comparisons with other approaches, will also be presented.

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[2] Guarnaccia, J., Herrera, I. y Pinder, G. *Solution of Flow and Transport Problems by a Combination of Collocation and Domain Decomposition Procedures*, Computational Methods in Water Resources X, Vol. 1, Eds. A. Peters, et.al., Kluwer Academic Publishers, International Conference on Computational Methods in Water Resources, Heidelberg, pp. 265-272, Julio, 1994.

Martin Diaz-Viera, Ismael Herrera

Trefftz-Herrera Domain Decomposition Method for Biharmonic Equation
Location: Room 049, **Time:** Tuesday, 22 July, 12:15

Trefftz-Herrera Domain Decomposition Method, that were introduced in previous papers [1-3], are formulated and applied to the biharmonic equation in two dimensions. The basic unifying concept of that theory consists on interpreting domain decomposition methods as procedures for obtaining information about the sought solution at the 'internal boundary' (Σ, which separates the subdomains from each other), sufficient for defining well-posed problems in each one of the subdomains (to be referred as 'local problems'). The distinguishing feature of the Trefftz-Herrera Method is the use of specialized test functions which have the property of yielding any desired information on Σ. The guidelines for the construction of such weighting functions are supplied by a special kind of Green's formulas (Green-Herrera formulas), formulated in Sobolev spaces of discontinuous functions, which permit analyzing the information on Σ, contained in approximate solutions. Some preliminary numerical experiments about efficiency and convergence of this procedure are presented.

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