An adapted coarse space for Balancing domain decomposition method to solve nonlinear elastodynamic problems

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This work is devoted to present a scalable domain decomposition method to solve nonlinear elastodynamic problems. Large non linear elastodynamic problems represent an appropriate application field for substructuring methods which are efficient on parallel computer with the proviso of using specific preconditioner techniques well adapted to the mechanical modeling. According to this reason, we develop an adapted Balancing domain decomposition method [Man93, LeT94] appropriated to solve this kind of systems. By using the theoretical framework of Schwarz additive decomposition method [LeT94, LMV98] and by using arguments developed in [ABLV00], we propose a two level Neumann-Neumann preconditioner based on the construction of a coarse space of "lower energy" modes adapted to finite deformations problems with dynamic process.

In section 1, nonlinear elastodynamic problems and domain decomposition frameworks are recalled. The section 2 is devoted to present the definition of an adapted coarse space by using Schwarz additive formulation. The construction of the two level Neumann-Neumann preconditioner is detailled in section 3. In last section 4, we test the efficiency of this updated Balancing domain decomposition method on numerical solutions of an academic non linear dynamic problem.

1 Nonlinear elastodynamic problems and domain decomposition frameworks

Dynamic deformable body systems in large deformations are governed by non linear time dependent equations. A typical non linear elastodynamic problem defined in a reference configuration can take the following variational form,

$$\begin{cases} \text{Find } \mathbf{u} \in L^2(]0;T[;U_0) \quad \text{such that for each } t \in]0;T[, \\ \int_\Omega \rho \ddot{\mathbf{u}}(t).\mathbf{v} + \int_\Omega \mathbf{\Pi}(t) : \nabla \mathbf{v} - \int_\Omega \mathbf{f}(t).\mathbf{v} - \int_{\partial_g \Omega} \mathbf{g}(t).\mathbf{v} = 0, \quad \forall \mathbf{v} \in U_0 \end{cases} \tag{1}$$

where ρ denotes the mass density; $\mathbf{\Pi}$ is the first Piola-Kirchoff tensor; $\mathbf{f}$ and $\mathbf{g}$ are the external force densities. A dot superscript indicates the time derivative. The set $U_0 = \{\mathbf{v} \in H^1(\Omega)^{dim}; \mathbf{v} = \mathbf{0} \text{ on } \partial_0\Omega\}$ represents the space of kinematically admissible displacement fields.

The problem (1) can be solved by an energy conservative time integration scheme [Gon00] which is appropriate due to his long term time integration accuracy and stability. In the following, we consider a collection of discrete times $(t_p)_{p=1..P}$ which define a partition of the time interval $[0;T] = \bigcup_{p=1}^{P}[t_p; t_{p+1}]$ with $t_{p+1} = t_p + \Delta t$ and $\Delta t = \frac{T}{P}$. By using a second order time integration scheme (adapted midpoint scheme) [Gon00], the weak form (1) integrated between the times t_p and t_{p+1} gives the following system,

$$\begin{cases} \text{Find } \mathbf{u}_{p+1} \in U_0 \quad \text{such that} \\ \frac{1}{\Delta t}\int_\Omega \rho(\dot{\mathbf{u}}_{p+1} - \dot{\mathbf{u}}_p).\mathbf{v} + \int_\Omega \mathbf{\Pi}_{algo} : \nabla\mathbf{v} - \int_\Omega \mathbf{f}_{p+\frac{1}{2}}.\mathbf{v} - \int_{\partial_g\Omega} \mathbf{g}_{p+\frac{1}{2}}.\mathbf{v} = 0, \end{cases} \tag{2}$$

where $\square_{p+\frac{1}{2}} = \frac{1}{2}(\square_p + \square_{p+1})$ and $\square_p$ denotes the approximation of $\square(t_p)$. The energy conservative scheme (2) used in this work, is characterized by the tensor $\mathbf{\Pi}_{algo}$ proposed by Gonzalez [Gon00]. After a fully discretization step (time and space), we deduce the non linear systems defined by

$$\frac{1}{\Delta t}\mathcal{M}(\dot{\mathbf{u}}_{p+1} - \dot{\mathbf{u}}_p) + \mathcal{G}_{algo}(\mathbf{u}_{p+1}, \mathbf{u}_p) - \mathbf{q}_{p+\frac{1}{2}} = \mathbf{0} \tag{3}$$

where $\mathcal{M}$ comes from the discretization of the inertia term $\frac{1}{\Delta t}\int_\Omega \rho(\dot{\mathbf{u}}_{p+1} - \dot{\mathbf{u}}_p).\mathbf{v}$ and $\mathcal{G}_{algo}$ is due to the discretization of the hyperelastic part $\int_\Omega \mathbf{\Pi}_{algo} : \nabla\mathbf{v}$ and $\mathbf{q}_{p+\frac{1}{2}}$ comes from the discretization of the external forces $\int_\Omega \mathbf{f}.\mathbf{v} + \int_{\partial_g\Omega} \mathbf{g}.\mathbf{v}$. The non linear system (3) can be solved by a iterative linearization scheme indexed by i which leads to the solution of linear systems:

$$\mathbf{Ka}_{i,p+1}\Delta\mathbf{u}_{i,p+1} = -\frac{1}{\Delta t}\mathcal{M}(\dot{\mathbf{u}}_{i,p+1} - \dot{\mathbf{u}}_p) - \mathcal{G}_{algo}(\mathbf{u}_{i,p+1}, \mathbf{u}_p) + \mathbf{q}_{p+\frac{1}{2}} \tag{4}$$

$$\text{with} \quad \mathbf{Ka}_{i,p+1} = \frac{2}{\Delta t^2}\mathbf{Ma} + \mathbf{K}_{i,p+1} \quad \text{and} \quad \Delta\mathbf{u}_{i,p+1} = \mathbf{u}_{i+1,p+1} - \mathbf{u}_{i,p+1}$$

where $\mathbf{Ma} = \partial_{\dot{\mathbf{u}}_{p+1}}\mathcal{M}$ represents the mass matrix and $\mathbf{Ka}_{i,p+1} = \partial_{\mathbf{u}_{p+1}}\mathcal{G}_{algo}$ the hyperelastic tangent matrix. We can insist on the fact that the matrix $\mathbf{Ka}_{i,p+1}$ of system (4) is **non symmetric**; the non symmetry comes from the form of tensor $\mathbf{\Pi}_{algo}$ (see [Gon00]).

The linear systems (4) can be solved by a domain decomposition method [LeT94] which has to be adapted to the non symmetry but also to the presence of inertia terms. Before giving the adaptations to non linear dynamic problems (sections 2 and 3), we present briefly now the principal features of the Balancing domain decomposition method [Man93, LeT94]. We choose to adopt a primal Schur complement method written with displacement variables. The basic idea in nonoverlapping domain decomposition methods is to split the domain Ω of study into N small nonoverlapping subdomains Ω^n

and interfaces Γ^n $(n = 1, N)$. The Schur complement method consist then in reducing the global system to an interface problem by a block Gaussian elimination of internal degrees of freedom. The interface problem can take the following variational form:

$$\exists \bar{\mathbf{u}} \in \bar{V} \ / \ < \mathbf{S}_{i,p+1}\bar{\mathbf{u}}, \bar{\mathbf{v}} >=< \bar{\mathbf{f}}_{i,p+1}, \bar{\mathbf{v}} > \ \forall \bar{\mathbf{v}} \in \bar{V} = tr(V)|_{\Gamma}, \tag{5}$$

where V is the discrete set defined from the space U_0 and $\Gamma = \bigcup_{n=1}^{N} \Gamma^n$. The matrices $\mathbf{S}_{i,p+1} = \sum_{n=1}^{N} \mathbf{R}^n \mathbf{S}^n_{i,p+1} (\mathbf{R}^n)^t$ denote the global Schur complement matrices defined on Γ; $(\mathbf{R}^n)^t$ is the restriction operator which goes from Γ to Γ^n. The local Schur complement matrices $\mathbf{S}^n_{i,p+1}$ are defined on Γ^n by

$$\mathbf{S}^n_{i,p+1} = \bar{\mathbf{Ka}}^n_{i,p+1} - (\mathbf{B}^n_{i,p+1})^t (\mathring{\mathbf{Ka}}^n_{i,p+1})^{-1} \mathbf{B}^n_{i,p+1}. \tag{6}$$

To do that, we have considered the subdomain stiffness matrix formulated by $\mathbf{Ka}^n_{i,p+1} = \begin{pmatrix} \mathring{\mathbf{Ka}}^n_{i,p+1} & \mathbf{B}^n_{i,p+1} \\ (\mathbf{B}^n_{i,p+1})^t & \bar{\mathbf{Ka}}^n_{i,p+1} \end{pmatrix}$. The blocks $\mathring{\mathbf{Ka}}^n_{i,p+1}$ and $\bar{\mathbf{Ka}}^n_{i,p+1}$ correspond respectively to the internal and interface degrees of freedom. The matrix $\mathbf{B}^n_{i,p+1}$ represents the contribution connecting Γ^n to Ω^n. The interface problem (5) can be solved by a GMRES method (non symmetric cases) with the multilevel Neumann-Neumann preconditioner [LeT94, ABLV00]. This iterative technique needs to form the matrix vector products $\mathbf{S}\bar{\mathbf{p}}$ and $\mathbf{M}^{-1}\bar{\mathbf{r}}$ by solving independent auxiliary Dirichlet and Neumann problems on the local subdomains and a global coarse problem defined on a space of singular (rigid body) motions. The adaptation of the Balancing Method to solve linear systems issued from non linear elastodynamic problems [Bar05] can be realized by using the theoritical frameworks of Schwarz additive decomposition method with introduction of an adapted coarse space.

2 Schwarz additive formulation: towards a definition of an adapted coarse space

The two-level Neumann-Neumann preconditioner may be interpreted as a standard additive Schwarz algorithm [LeT94]. This method consists in decomposing the interface space $\bar{V}$ into a coarse and a fine component: $\bar{V}_G$ and $\bar{V}_f$. The coarse space $\bar{V}_G = \sum_{n=1}^{N} D^n Z^n$ (D^n is a given partition of unity defined on the interface, $\sum_{n=1}^{N} D^n R^n = Id|_{\bar{V}}$) can be defined by adding local lower energy components and the fine space $\bar{V}_f$ is defined by duality as follows:

$$\bar{V}_f = \sum_{n=1}^{N} D^n \bar{V}^n_f \ \text{ where } \bar{V}^n_f = \{\bar{\mathbf{v}}^n_f \in \bar{V}^n, < \mathbf{SR}^n \bar{\mathbf{v}}^n, \mathbf{R}^n \mathbf{z}^n >= 0, \ \forall \mathbf{z}^n \in Z^n\}. \tag{7}$$

The key point is the construction of the local spaces Z^n of rigid motions. This construction must, if that is necessary, regularize local Neumann problems

but more especially set the constants or the lower energy modes (like rigid body motions) to zero in the solution of local Neumann problems. For more details on the presentation of the Schwarz additive method, we can refer to [LeT94, LMV98] for symmetric cases and [ABLV00] for non symmetric cases.

With finite deformations and dynamic problems, some lower energy modes cannot be detected in the factorization step of the local tangent matrices of Neumann problems [Bar05]. These drawbacks come from the finite deformations modeling but also from the regularizing contribution of the mass matrix. Moreover, we also need to improve the continuity between subdomains by taking into account specific modes (like corners modes ...). So we have to introduce a specific construction of these lower energy modes.

In the following, we present in detail the construction of the coarse space $\bar{V}_G$ for non linear dynamic problems. The orthogonality relation used in (7) characterizing the space $\bar{V}_f^n$ permits to obtain informations in order to define the local coarse space Z^n. Indeed the expansion of this orthogonality relation by using $\mathbf{S}_{i,p+1} = \sum_{n=1}^{N} \mathbf{R}^n \mathbf{S}_{i,p+1}^n (\mathbf{R}^n)^t$ involves only terms issued from the subdomains neighbouring to the n^{th} and one term from the n^{th} itself,

$$\underbrace{\sum_{l=1}^{neigh(n)} < \bar{\mathbf{v}}^n \ , \ (\mathbf{R}^n)^t \mathbf{R}^l (\mathbf{S}_{i,p+1}^l)^t (\mathbf{R}^l)^t \mathbf{R}^n \mathbf{z}^n >}_{(8i)} + \underbrace{< \bar{\mathbf{v}}^n \ , \ (\mathbf{S}_{i,p+1}^n)^t \mathbf{z}^n >}_{(8ii)} = 0 \tag{8}$$

The relation (8) can be verified by imposing the terms $(8i)$ and $(8ii)$ to zero. Let us see now what the use of these relations imply:

- use of the term $(8ii)$: this term may be eliminated by imposing that the kernel of $(\mathbf{S}_{i,p+1}^n)^t$ is included in the local coarse space Z_n, $(Ker(\mathbf{S}_{i,p+1}^n)^t \subset Z^n)$. Such a choice leads to the same simplification than those obtained with the kernel of $\mathbf{S}^n$ for the more common symmetric case [LeT94]. For non symmetric cases, this choice leads to the introduction of Dual Rigid Modes (DRM) defined through the kernel of $(\mathbf{S}_{i,p+1}^n)^t$ (see [ABLV00] for more precisions). In a pratical point of view and according to the form $\mathbf{S}_{i,p+1}^n$ given in (6), the dual rigid modes (noted by $\mathbf{v}_{G\alpha}^n$) defined on Ω^n can be calculated through the local matrices $(\mathbf{Ka}_{i,p+1}^n)^t$ by the solution of this following Neumann systems:

$$\mathbf{v}_{G\alpha}^n \in V^n \ \ / \ \ (\mathbf{Ka}_{i,p+1}^n)^t \mathbf{v}_{G\alpha}^n = \mathbf{0}, \quad \alpha = 1, nbDRM^n. \tag{9}$$

where $nbDRM^n$ represents the total number of dual rigid modes of subdomains Ω^n. One can easily prove that the modes $\mathbf{v}_{G\alpha}^n \in Ker(\mathbf{Ka}_{i,p+1}^n)^t$ are connecting to the elements $\mathbf{z}^n \in Ker(\mathbf{S}_{i,p+1}^n)^t$ by the relation $\mathbf{z}^n = \bar{\mathbf{v}}_{G\alpha}^n$ (where $\bar{\mathbf{v}}_{G\alpha}^n$ represents the contribution of $\mathbf{v}_{G\alpha}^n$ on Γ^n).

- contribution of the term $(8i)$: a simple manner to cancel the term $(8i)$ is to fix all the terms of the sum to zero; the elements $\mathbf{z}^n$ of Z^n could then be caracterized by the solution of $(\mathbf{R}^n)^t \mathbf{R}^l (\mathbf{S}_{i,p+1}^l)^t (\mathbf{R}^l)^t \mathbf{R}^n \mathbf{z}^n = 0$. That makes it possible to ensure the continuity of the coarse space elements through the interface Γ^n of Ω^n by considering the contributions relating to corners, edges

and faces of the neighbouring subdomains Ω^l. Indeed, the elements $\mathbf{z}^n$ can be found respectively by these following relations :

$$\mathbf{z}^l \in Z^l \quad / \quad (\mathbf{S}^l_{i,p+1})^t \mathbf{z}^l = \mathbf{0} \quad \forall\, l = 1, neigh(n) \tag{10}$$

$$\mathbf{z}^n \in Z^n \quad / \quad (\mathbf{R}^l)^t \mathbf{R}^n \mathbf{z}^n = \mathbf{z}^l \quad \forall\, l = 1, neigh(n) \tag{11}$$

The use of the $(8ii)$ and (10) permits to calculte the dual rigid modes of the subdomains Ω^n and his neighbours Ω^l; furthermore the relation (11) represents the continuity constraint of dual modes through the interface (corners, edges and faces) between Ω^n and Ω^l. This last point makes it possible to connect this approach with the Balancing Domain Decomposition Method with Constraints [MD03]; the enforcement of these kind of constraint leads to expensive computational cost. An inexpensive way, inspired by [LMV98] and [MD03] would be to impose only the continuity on the corners of subdomains Ω^n. That can be done by the computation of the $nbDCM^n$ Dual Corner Modes (DCM) of subdomains Ω^n by enforcing a same arbitrary Dirichlet boundary value on the corner interface degrees of freedom for all concerned subdomains Ω^n and Ω^l (where $nbDCM^n$ is the total number of DCM).

In conclusion, the coarse space Z^n can be generated by considering the $nbDRM^n$ dual rigid modes defined by solutions of the systems (12) and particulary by the $nbDCM^n$ dual corner modes given by the systems (13) (see the next section 3 for more details on the computation of these modes).

3 Adaptation of the 2-level Neumann-Neumann preconditionner

According to the definition of the coarse space introduced in section 2, we propose an adapted construction of the two level Neumann-Neumann preconditioner based on the following steps:

1. Preliminary step : We identify the local internal degrees of freedom $\{Pr^n_\alpha;\ \alpha = 1, nbDRM^n\}$ which cancel all $nbDRM^n$ rigid motions of subdomain Ω^n. This detection can be realized during the factorization of the stiffness matrix $(\mathbf{K}^n_e)$ coming from the linear elastostatic system associated to the non linear elastodynamic problem (3). Notice that we can also make arbitrary this detection.
2. For each Newton iteration i of each time step p
   a) We construct the local regularized matrices $\widetilde{\mathbf{Ka}}^n_{i,p+1}$ by using the degrees of freedom $\{Pr^n_\alpha\}$ detected in step (1). These matrices can be written by using the contributions according to internal and interface degrees of freedom; then only the internal contribution $\mathring{\mathbf{Ka}}^n_{i,p+1}$ of the matrix $\mathbf{Ka}^n_{i,p+1}$ is regularized by using the matrix $\mathring{\mathbf{Q}}^n_\alpha$:

$$\widetilde{\mathring{\mathbf{Ka}}}^n_{i,p+1} = \mathring{\mathbf{Ka}}^n_{i,p+1} + \mathring{\mathbf{Q}}^n_\alpha \quad \text{where} \quad (\mathring{\mathbf{Q}}^n_\alpha)_{jk} = \begin{cases} BV & \text{if } j = k = Pr^n_\alpha \\ 0 & \text{else} \end{cases}$$

where BV is an arbitrary big value (like 10^{30} for example). This regularization is not necessary for dynamic problems due to the contribution of the inertia terms $\frac{2}{\Delta t^2}\mathbf{Ma}$ which ensures the matrices $\mathbf{Ka}_{i,p+1}^{n}$ to be non singular. On the other hand, we need to construct the regularized matrices $\widetilde{\bar{\mathbf{K}}\mathbf{a}}_{i,p+1}^{n}$ in order to impose the boundary conditions on the corners degrees of freedom noted by $\{Pc_{\gamma}^{n};\ \gamma = 1, nbDCM^{n}\}$:

$$\widetilde{\bar{\mathbf{K}}\mathbf{a}}_{i,p+1}^{n} = \bar{\mathbf{K}}\mathbf{a}_{i,p+1}^{n} + \bar{\mathbf{Q}}_{\gamma}^{n} \ \text{ where } \ (\bar{\mathbf{Q}}_{\gamma}^{n})_{jk} = \begin{cases} BV & \text{if } j = k = Pc_{\gamma}^{n} \\ 0 & \text{else} \end{cases}$$

b) We compute the dual rigid modes $\{\mathbf{v}_{G\alpha}^{n};\ \alpha = 1, nbDRM^{n}\}$ by solving the (regularized) local Neumann problems set on the space V^{n} of subdomain displacements functions,

$$\begin{pmatrix} \widetilde{\mathring{\mathbf{K}}\mathbf{a}}_{i,p+1}^{n} & \mathbf{B}_{i,p+1}^{n} \\ (\mathbf{B}_{i,p+1}^{n})^{t} & \bar{\mathbf{K}}\mathbf{a}_{i,p+1}^{n} \end{pmatrix} \begin{pmatrix} \mathring{\mathbf{v}}_{G\alpha}^{n} \\ \bar{\mathbf{v}}_{G\alpha}^{n} \end{pmatrix} = \begin{pmatrix} \mathring{\mathbf{e}}_{\alpha}^{n} \\ \mathbf{0} \end{pmatrix};\ \alpha = 1, nbDRM^{n} \qquad (12)$$

where the j^{th} component $(\mathring{\mathbf{e}}_{\alpha}^{n})_{j}$ of the vector $\mathring{\mathbf{e}}_{\alpha}^{n}$ is equal to the arbitrary big value BV if $j = Pr_{\alpha}^{n}$ and to the value zero if not.

c) We compute the dual corner modes $\{\mathbf{v}_{G\gamma}^{n};\ \gamma = 1, nbDCM^{n}\}$ by solving local Neumann problems in which the continuity of modes on corners can be realized by enforcing the same arbitrary Dirichlet boundary value (1 for example) on the corners interface degrees of freedom $\{Pc_{\gamma}^{n};\ \gamma = 1, nbDCM^{n}\}$ for all concerned subdomains Ω^{n} :

$$\begin{pmatrix} \mathring{\mathbf{K}}\mathbf{a}_{i,p+1}^{n} & \mathbf{B}_{i,p+1}^{n} \\ (\mathbf{B}_{i,p+1}^{n})^{t} & \widetilde{\bar{\mathbf{K}}\mathbf{a}}_{i,p+1}^{n} \end{pmatrix} \begin{pmatrix} \mathring{\mathbf{v}}_{G\alpha}^{n} \\ \bar{\mathbf{v}}_{G\alpha}^{n} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \bar{\mathbf{e}}_{\gamma}^{n} \end{pmatrix};\ \gamma = 1, nbDCM^{n} \qquad (13)$$

where the j^{th} component $(\bar{\mathbf{e}}_{\gamma}^{n})_{j}$ of the vector $\bar{\mathbf{e}}_{\gamma}^{n}$ is equal to the arbitrary big value BV if $j = Pc_{\gamma}^{n}$ and to the value zero if not.

3. We define the local coarse space by:

$$Z^{n} = \text{vect}\left(\{\bar{\mathbf{v}}_{G\alpha}^{n};\ \alpha = 1, nbDRM^{n}\},\ \{\bar{\mathbf{v}}_{G\gamma}^{n};\ \gamma = 1, nbDCM^{n}\}\right).$$

With this construction of lower energy modes, the two-level Neumann-Neumann preconditioner is classically defined for each time step p and each Newton iteration i by

$$\mathbf{M}_{i,p+1}^{-1} = \mathbf{P}_{G} + \sum_{n=1}^{N} (\mathbf{I} - \mathbf{P}_{G})\mathbf{D}_{i,p+1}^{n}(\widetilde{\mathbf{S}}_{i,p+1}^{n})^{-1}(\mathbf{D}_{i,p+1}^{n})^{t}(\mathbf{I} - \mathbf{P}_{G})^{t}, \qquad (14)$$

where $(\widetilde{\mathbf{S}}_{i,p+1}^{n})^{-1}$ is the regularized Schur inverse matrix and $\mathbf{P}_{G}$ denotes the orthogonal $\mathbf{S}$-projection of $\bar{V}$ on $\bar{V}_{G}$.

4 A nonlinear dynamic problem: the cantilever beam

In this section, we illustrate numerically the previous adaptations in the case of the solution of a 2-dimensional non linear elastodynamic problem. The application relates to the dynamic evolution of a cantilever beam in plane displacements. To do that, we consider an elastic beam clamped on one of

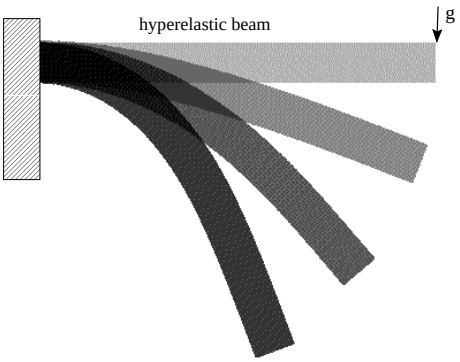


Fig. 1. Deformed sequence of a cantilever beam.

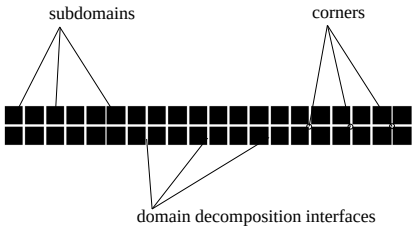
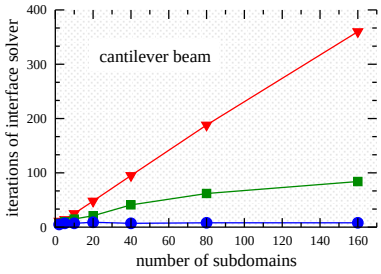


Fig. 2. Substructuration of the beam in 40 subdomains.

its tips and an external time independent loading g on the opposite tip. The compressible material response considered is governed by an Ogden constitutive law. The mesh and its deformed configurations during the time are presented in figure 1. From this numerical experiment, we analyse the scalability of the interface solver (GMRES) with some versions of the two-level Neumann-Neumann preconditioners. The considered preconditioners are :
- the non symmetric Neumann-Neumann preconditioner given in [ABLV00] (curve ▼) without non linear dynamic adaptations,
- the non symmetric Neumann-Neumann preconditioner given in [Bar05] (curve ■) presented in section 3 but without dual corner modes (step (c)),
- the improved non symmetric Neumann-Neumann preconditioner introduced in section 3 (curve ●) with all the features (steps (a), (b) and (c)).

The figure 3 gives the evolution of average number of GMRES iterations (per Newton iterations) for a beam decomposed in 2, 5, 10, 20, 40, 80 and 160 subdomains (see figure 2 for a decomposition in 40 subdomains). We observe that the number of iterations obtained with the 2-level Neumann-Neumann preconditioner without adaptations (curve ▼) grows up with respect to the number of subdomains. So the interface solver with this preconditioner loses its classical scalability. We can remark that the preconditioner (curve ■) given in [Bar05] (without corners modes) permits to strongly decrease the solver iterations but the dependence with respect to the number of subdomains is already present. On the other hand, the improved Neumann-Neumann preconditioner



N.-N. preconditioner without adaptations
N.-N. preconditioner given in [Bar05] (without corner modes)
improved N.-N. preconditioner (presented in section 3)

N.-N. preconditioners \ number of subdomains	2	5	10	20	40	80	160
▼—▼	11	14	25	48	95	188	359
■—■	7	10	15	21	41	62	84
●—●	5	7	7	9	7	8	7

Fig. 3. Numerical scalability with Neumann-Neumann (N.-N.) preconditioners.

(curve ●) leads to recover the numerical scalability properties i.e. the independence of the solver iterations with respect to the number of subdomains. Furthermore, the performances of this preconditioner are pratically the same as the ones obtained to solve linear elastostatic problems. Indeed, the average number of iterations is equal to 7 for a decomposition in 160 subdomains (see table in figure 3) and if we consider the associated linear elastostatic problem the number of iterations is equal to 6. In order to validate these performances, we must test this preconditioner on other less academic simulations.

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