Convergence theories of the subspace correction methods for singular and nearly singular system of equations

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Many mathematical models lead to singular and/or nearly singular problems. Simple examples include the Laplace equation with the Neumann boundary condition, the nearly incompressible linear elasticity equations, variational problems in electromagnetism at certain parameter ranges (in general variational problems on $\mathbf{H}(\mathrm{div})$ and/or $\mathbf{H}(\mathbf{curl})$). The main goal of this presentation is to report recent research results on the abstract convergence analysis for both singular and nearly singular system of equations. For singular problems, we will present a sharp convergence rate identity for general subspace correction methods. We then apply the abstract theory in the study of the convergence of the multigrid method for certain singular problems.

Our discussion of nearly singular problems will begin with a simple linear system and the difficulties that arise when solving such a system by a classical iterative method. To tackle these difficulties in the simple example, as well as in much more complicated situations, we introduce new abstract assumptions and based upon these assumptions we present a refined convergence analysis of a class of iterative methods via the subspace correction framework. Our new theory clearly shows the crucial role played by the *right* assumptions in obtaining optimal convergence rate estimates. As illustration, we present a convergence analysis of a multilevel method for the linear elasticity system $-\kappa^2 \mathbf{grad}\,\mathrm{div} - \rho^2 \Delta$ discretized by high order conforming finite elements. In particular, our analysis shows that the proposed methods are uniformly convergent regardless of the size of the weights κ^2 and ρ^2.

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