DOMAIN DECOMPOSITION TECHNIQUES FOR TREATING MULTISCALE PROPERTIES IN RESERVOIR ENGINEERING APPLICATIONS

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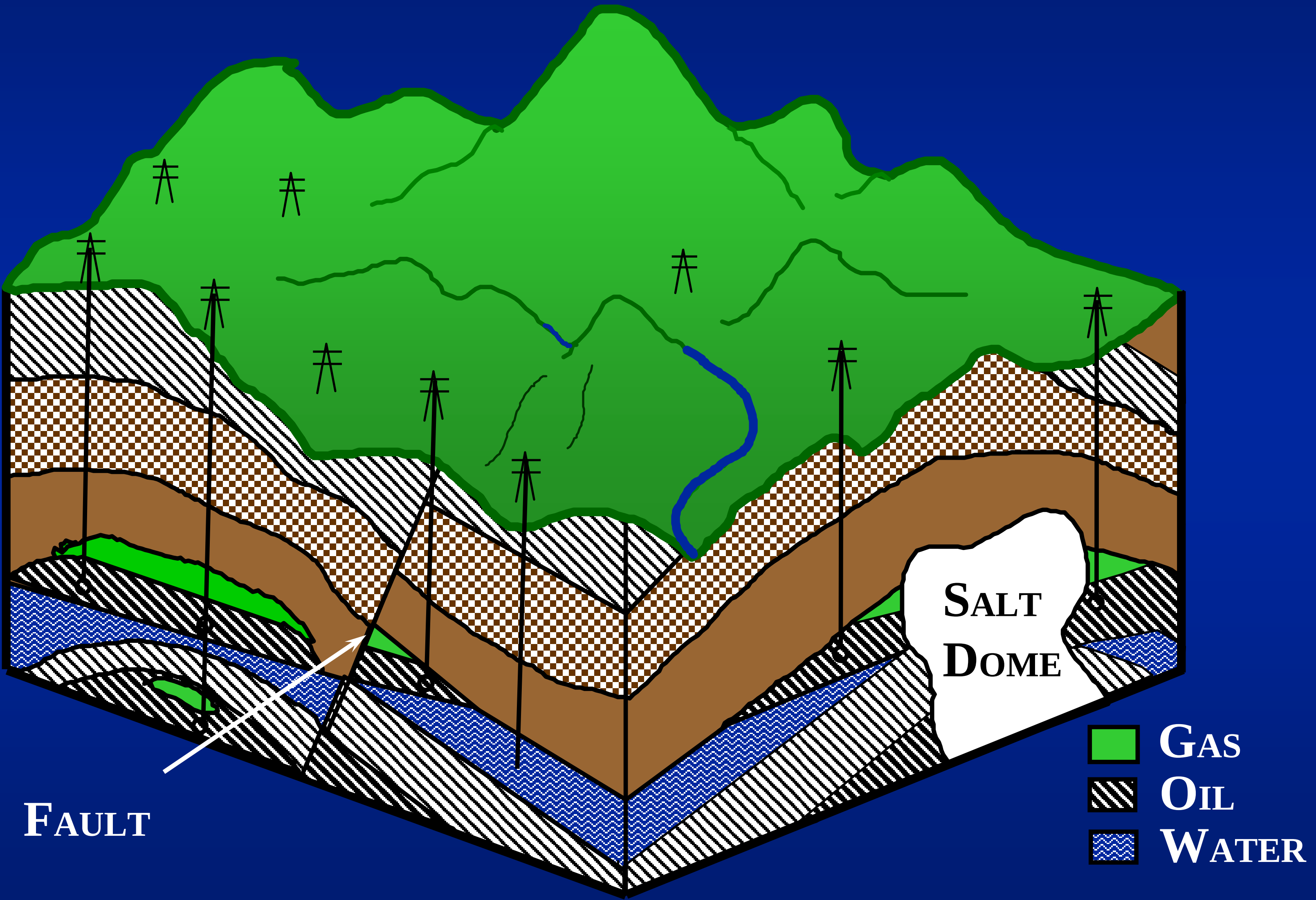
GAS, OIL, WATER



TRANSPORT IN POROUS MEDIA

PHYSICAL RELATIONS:

CONSTITUTIVE LAW (D'ARCY)

$$u = -T\nabla\Phi$$

CONSERVATION OF MASS

$$-\operatorname{div} u = \frac{\partial}{\partial t}\left(\phi S(\Phi)\right) + q$$

COMPONENT BALANCES

$$\frac{\partial}{\partial t}\left(\phi f_1(c)\right) + \operatorname{div}\left(u\, f_2(c)\right) = \nabla \bullet \left(D\nabla c\right) + q(c)$$

EQUATIONS OF STATE

THERMAL BALANCE

UPSCALING

$$\tilde{u} = -\frac{\tilde{K}}{\mu}\nabla \Phi, \qquad \tilde{K} = \text{Effective Permeability}$$

$$\frac{\partial}{\partial t}(\phi c) + \nabla \bullet (\tilde{u} c) - \nabla \bullet \tilde{D}(\tilde{u})\nabla c = \hat{q} c, \qquad \tilde{D} = \text{Macrodispersity}$$

- GELHAR, AXNESS - WATER RES. RES., 1983
- DAGAN - FLOW & TRANS. POROUS MEDIA, 1989
- EWING, RUSSELL, YOUNG - 10 SPE RES. SIM., 1989
- ESPEDAL, LANGLO, SAEVAREID, GISLEFOSS - SPE 21231, 1991
- HANSEN - 11 SPE RES. SIM., 1991
- DURLOFSKY - WATER RES. RES., 1991
- NEUMAN - WATER RESOUR. RES., 1993
- GLIMM, LINDQUIST, PEREIRA, ZHANG - TRANS. POROUS MEDIA, 1993
- AMAZIANE, BOURGEAT, KOEBBE - TRANS. POROUS MEDIA, 1994
- LANGLO, ESPEDAL - ADV. WATER RES., 1994
- BECKIE, ALDAMA, WOOD - WATER RESOUR. RES., 1996
- CHRISTIE - J. PET. TECH., 1996
- WEN, GOMEZ-HERNANDEZ - J. HYDROL., 1996
- CVETKOVIC, DAGAN - PROC. R. SOC. LONDON A, 1996
- RENARD, DE MARSILY - ADV. WATER RES., 1997
- RUBIN, SUN, MAXWELL, BELLIN - J. FLUID MECH., 1999
- EFENDIEV, DURLOFSKY, LEE - WATER RES. RES., 2000
- WU, EFENDIEV, HOU, DCDS-B, 2002.
- GINTING, EWING, EFENDIEV, LAZAROV - JCAM, 2004

SINGLE-PHASE FLOW EQUATIONS

D'ARCY'S LAW:

$$u = -\frac{K}{\mu}(\nabla p - \rho g \nabla z)$$

MASS BALANCE:

$$\frac{\partial}{\partial t}(\phi\rho) + \nabla \bullet (u\rho) = q$$

TRANSPORT:

$$\frac{\partial}{\partial t}(\phi c) + \nabla \bullet uc - \nabla \bullet D(u)\nabla c = \lambda(c) + qc$$

WHERE

$$D_{ij} = \phi(x)\left[d_m I + \frac{d_\ell}{|u|}\begin{pmatrix} u_1^2 & u_1 u_2 \\ u_1 u_2 & u_2^2 \end{pmatrix} + \frac{d_t}{|u|}\begin{pmatrix} u_2^2 & -u_1 u_2 \\ -u_1 u_2 & u_1^2 \end{pmatrix} \right]$$

$$+ \text{ Nonlocal Dispersion } (t)$$

MULTI-COMPONENT MISCIBLE TRANSPORT UPSCALING

We Expand About Course Grid Values Of The C **And** v

$$C = \overline{C} + C', \quad v = \overline{v} + v'$$

And Perform A Perturbation Argument To obtain

$$\frac{\partial \overline{C}}{\partial t} + \overline{v} \cdot \nabla \overline{C} + \overline{v' \cdot \nabla C'} = 0.$$

The Term $\overline{v' \cdot \nabla C'}$ **Represents Subgrid Effects And Can Be Computed From The Equation for Fluctuation**

$$\frac{\partial C'}{\partial t} + v \cdot \nabla C' + v' \cdot \nabla \overline{C} - \overline{v' \cdot \nabla C'} = 0.$$

Projecting This Equation Onto $dx(\tau)/d\tau = v$ **, We Have**

$$\frac{dC'}{dt} = -v' \cdot \nabla \overline{C} + \overline{v' \cdot \nabla C'}.$$

Intergrating Over 0 **To** t**, We Obtain The Solution Of This Equation Along The Trajectory**

$$C'(x,t) = \int_0^t \left(-v'(x(\tau),\tau) \cdot \nabla \overline{C}(x(\tau),\tau) + \overline{v'(x(\tau),\tau) \cdot \nabla C'(x(\tau),\tau)} \right) d\tau.$$

MULTI-COMPONENT MISCIBLE TRANSPORT UPSCALING

SUBGRID EFFECTS CAN BE WRITTEN AS

$$\overline{v_i'(x,t)C'(x,t)} = -\overline{v_i'(x,t)\int_0^t v'(x(\tau),\tau)\cdot\nabla\overline{C}(x(\tau),\tau)\,d\tau}.$$

THE EQUATION OBTAINED IS AN INTEGRO-DIFFERENTIAL EQUATION. IF WE ASSUME THAT THE CONCENTRATION VARIES SLOWLY ALONG STREAMLINES, THEN

$$\overline{v_i'(x,t)C'(x,t)} \approx -\sum_j \left(\int_0^t \overline{v_i'(x,t)v_j'(x(\tau),\tau)}\,d\tau\right)\frac{\partial\overline{C}(x,t)}{\partial x_j}.$$

THE COARSE-SCALE EQUATION IS

$$\frac{\partial\overline{C}(x,t)}{\partial t} + \overline{v}\cdot\nabla\overline{C}(x,t) - \nabla\cdot(D(x,t)\nabla\overline{C}(x,t)) = 0,$$

WHERE $D_{ij}(x,t) = \int_0^t \overline{v_i'(x,t)v_j'(x(\tau),\tau)}\,d\tau.$

D **DEPENDS ON THE CONCENTRATION BECAUSE THE VELOCITY DEPENDS ON THE CONCENTRATION VIA FLOW EQUATIONS**

MACRODISPERSION COMPUTATIONS

IF $L_j(x,t) = \int_0^t v_j'(x(\tau),\tau)\,d\tau$ (LENGTH OF TRAJECTORY IN j DIRECTION), THEN

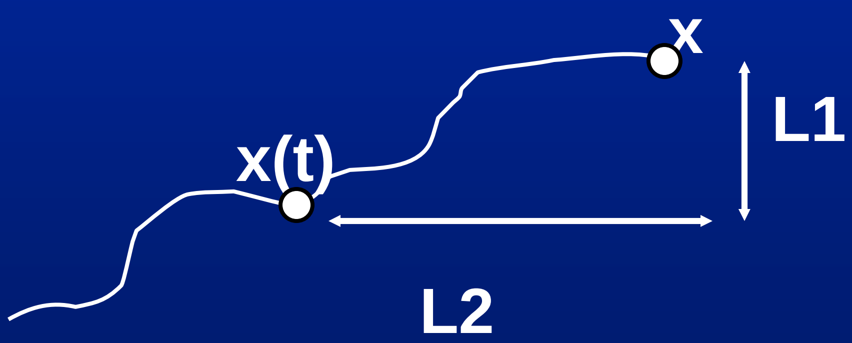
$$D_{ij}(x,t) \approx \overline{v_i'(x,t)\,L_j(x,t)}.$$

WE CAN SHOW THAT

$$L_j(x,t) = L_j(y_p,t_p) + (t - t_p)\,v_j'(x,t),$$

i.e., THE MACODISPERSION CAN BE COMPUTED LOCALLY IN TIME (WE DO NOT NEED THE HISTORY OF THE VELOCITY):

$$D_{ij}(x,t) \approx \overline{v_i'(x,t)\,L_j(y_p,t_p)} + (t - t_p)\,\overline{v_i'(x,t)\,v_j'(x,t)}.$$

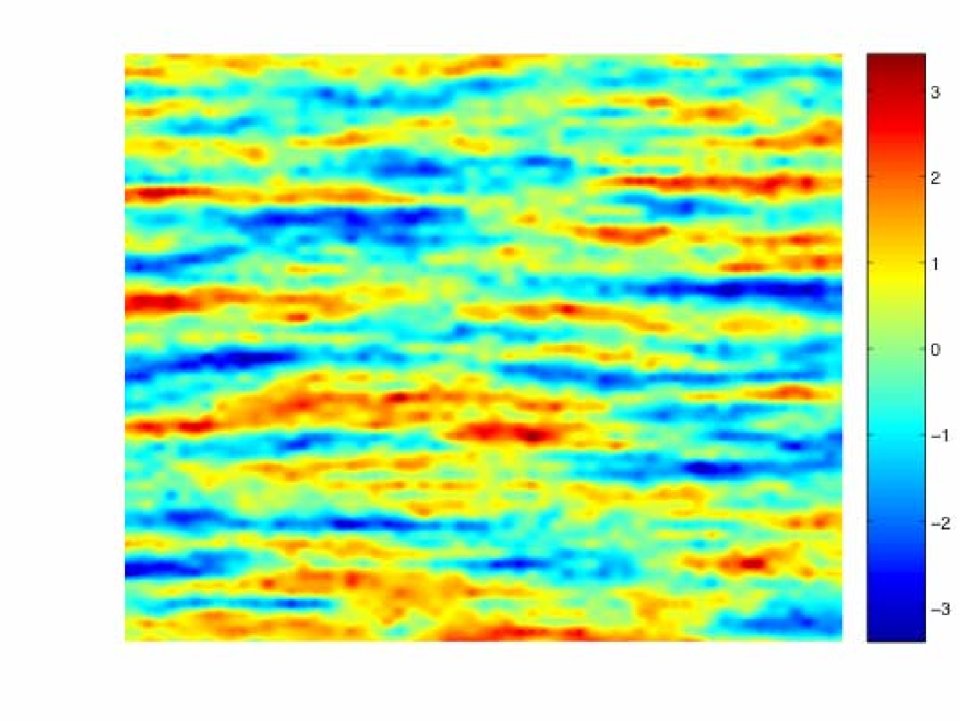


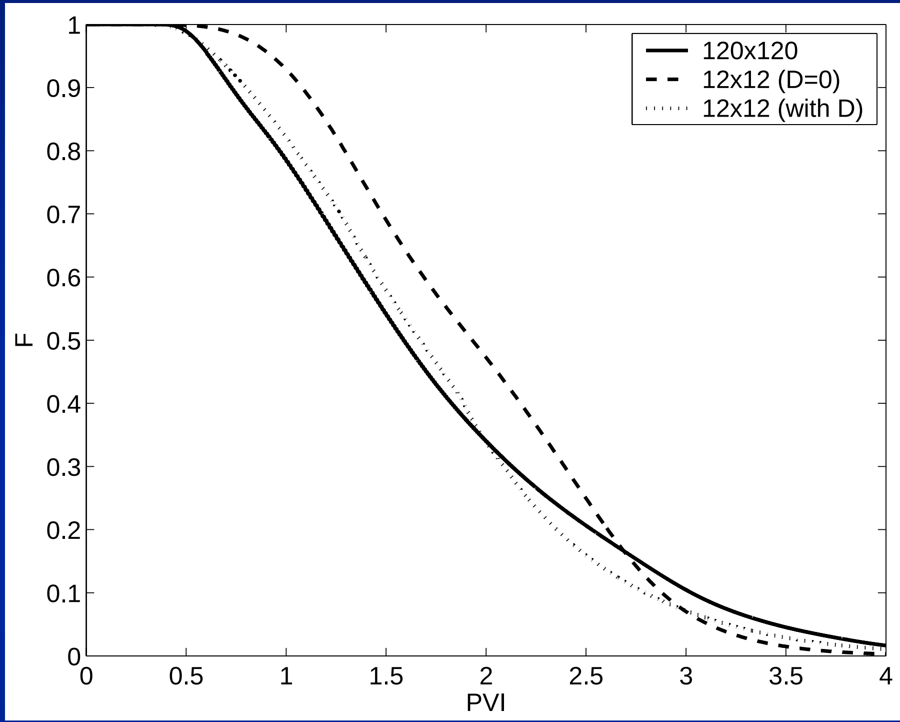
UPSCALING OF FLOW EQUATIONS

- For Upscaling Of The Pressure Equation, Multiscale Finite Volume Element Method Is Used. The Idea Of This Method Is To Capture The Subgrid Effects Using Multiscale Basis Functions. This Allows Us To Recover Subgrid Effects Of The Velocity Field Locally And Compute The Macrodispersion.

- Adaptive Local Grid Re Finement Can Be Used Near Sharp Fluid Interfaces To Resolve The Finer Scale And More Complex Physics.

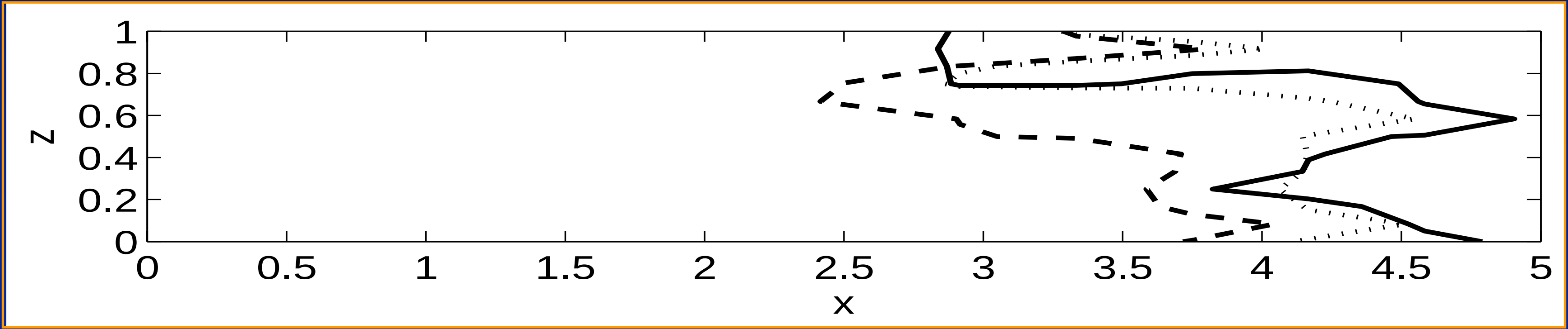
NUMERICL RESULTS

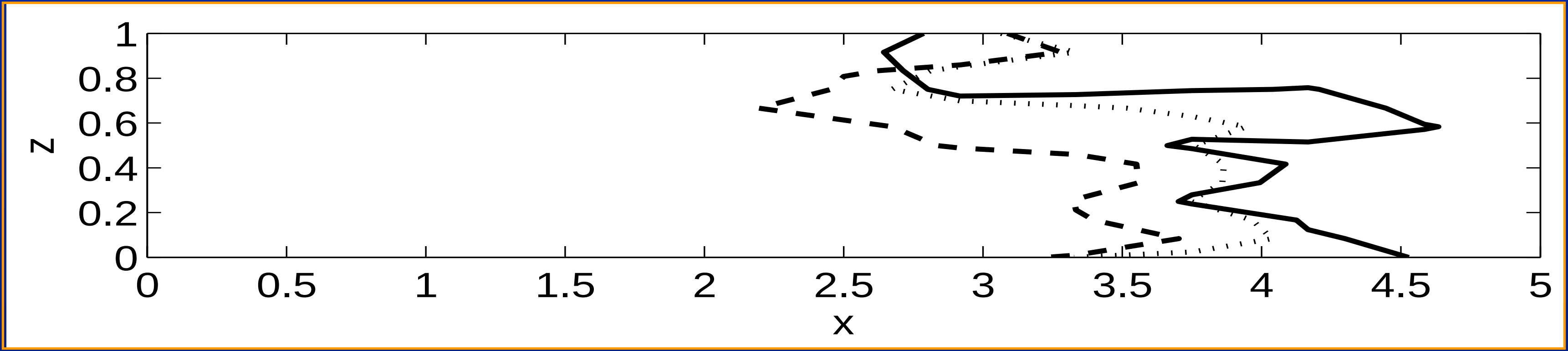




$$l_x = 0.20,\ l_z = 0.02,\ \sigma = 1.5,\ \mathrm{M{=}2}$$

NUMERICAL RESULTS





TWO-PHASE FLOW

D'ARCY'S LAW:

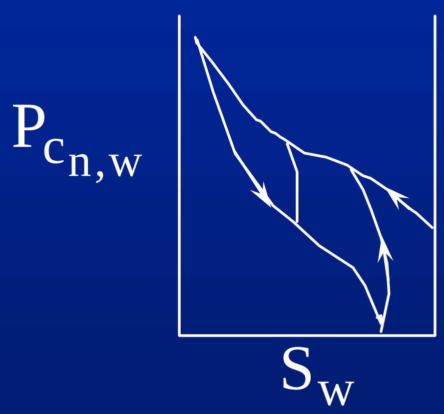
$$u_i = -\frac{K k_{ri}}{\mu_i}\left(\nabla p_i - \rho_i g \nabla z\right) \;,\; i = w,n$$

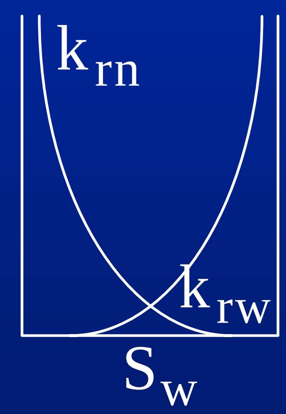
MASS BALANCE:

$$\frac{\partial}{\partial t}\left(\phi \rho_i S_i\right) + \nabla \bullet \left(\rho_i u_i\right) = q_i \rho_i \;,\; i = w,n$$

$$p_{c_{n,w}} = p_n - p_w \;,\; S_n + S_w = 1$$

w = Wetting Phase
n = Nonwetting Phase





MULTIPHASE FLOW EQUATIONS

$$S_a c_a \frac{dp}{dt} + \nabla \bullet u + S_a c_a f_w \frac{dp_c}{dt} = -\frac{\partial \phi}{\partial t} + q_t$$

$$\phi \frac{\partial S_w}{\partial t} + \nabla \bullet f_w u + \nabla \bullet K f_a f_w \lambda (\nabla p_c - G_l) = -S_w \frac{\partial \phi}{\partial t} + q_w$$

WHERE

$$u = -K\lambda(\nabla p + G), u = u_a + u_w$$

$$p = S_a p_a + S_w p_w + \frac{1}{2}\int_0^{p_c} (f_a - f_w) d\eta$$

$$\lambda_i = k_{ri} / \mu_i, \ f_i = \lambda_i / \lambda, i = a, w$$

$$\frac{\partial}{\partial t} \equiv \phi \frac{\partial}{\partial t} + u_a / S_a \bullet \nabla, \lambda = \lambda_a + \lambda_w$$

COMPOSITIONAL MODEL

$$\frac{\partial}{\partial t}\left(n_w \phi\right) + \nabla \bullet \left(\frac{n_w}{V_w} u_w \right) = q_w$$

$$\frac{\partial}{\partial t}\left(n^i \phi\right) + \nabla \bullet \left(\frac{n_o^i}{V_o} u_o + \frac{n_g^i}{V_g} u_g \right) - \nabla J^i = q^i \qquad i = 1, \ldots, N_C$$

$$u_\alpha = \frac{K k_{r\alpha}}{\mu_\alpha} \left(\nabla p_\alpha - \rho_\alpha g \nabla z \right) \qquad \alpha = o, w, g$$

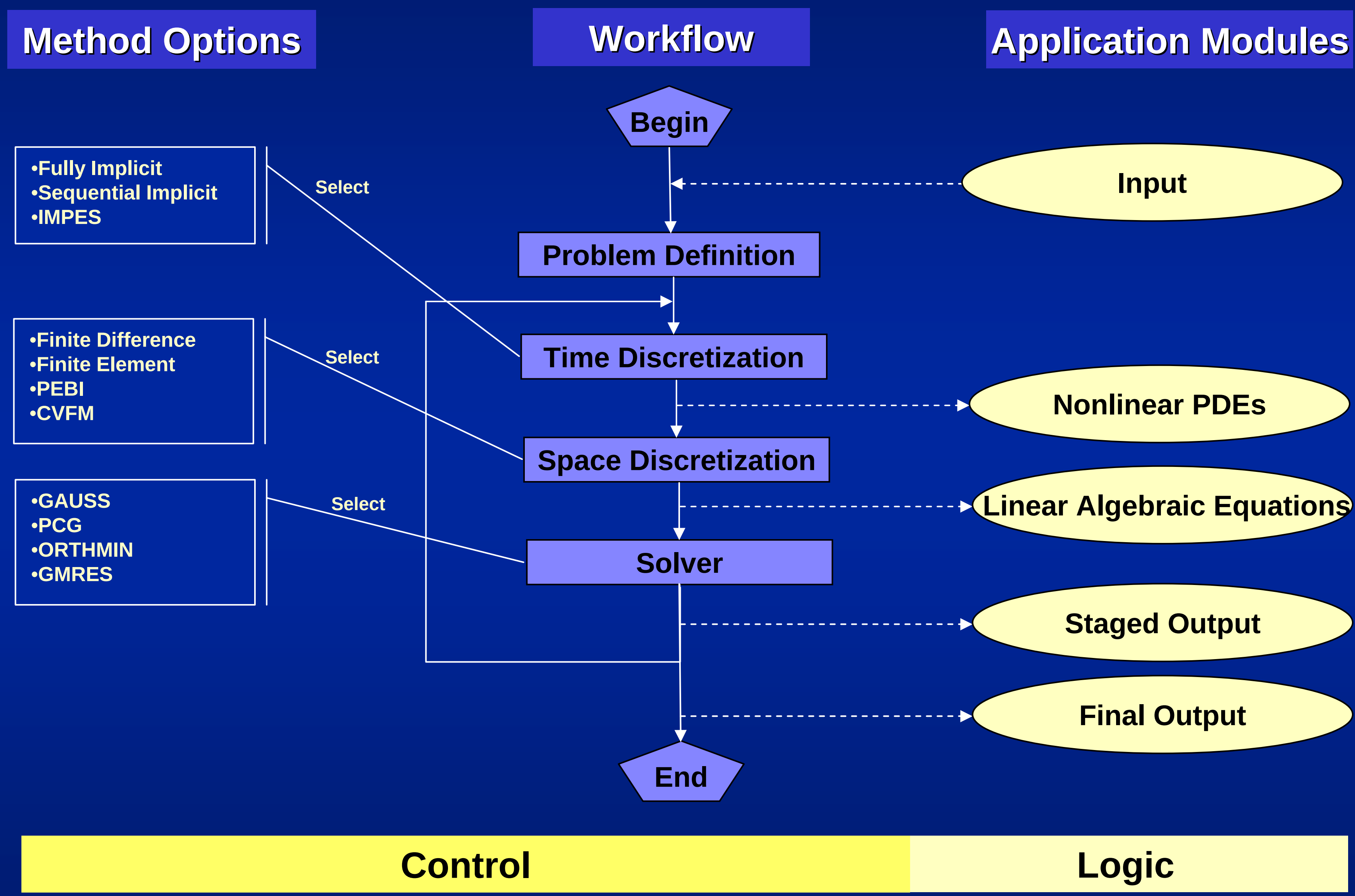
PHASE PACKAGE:

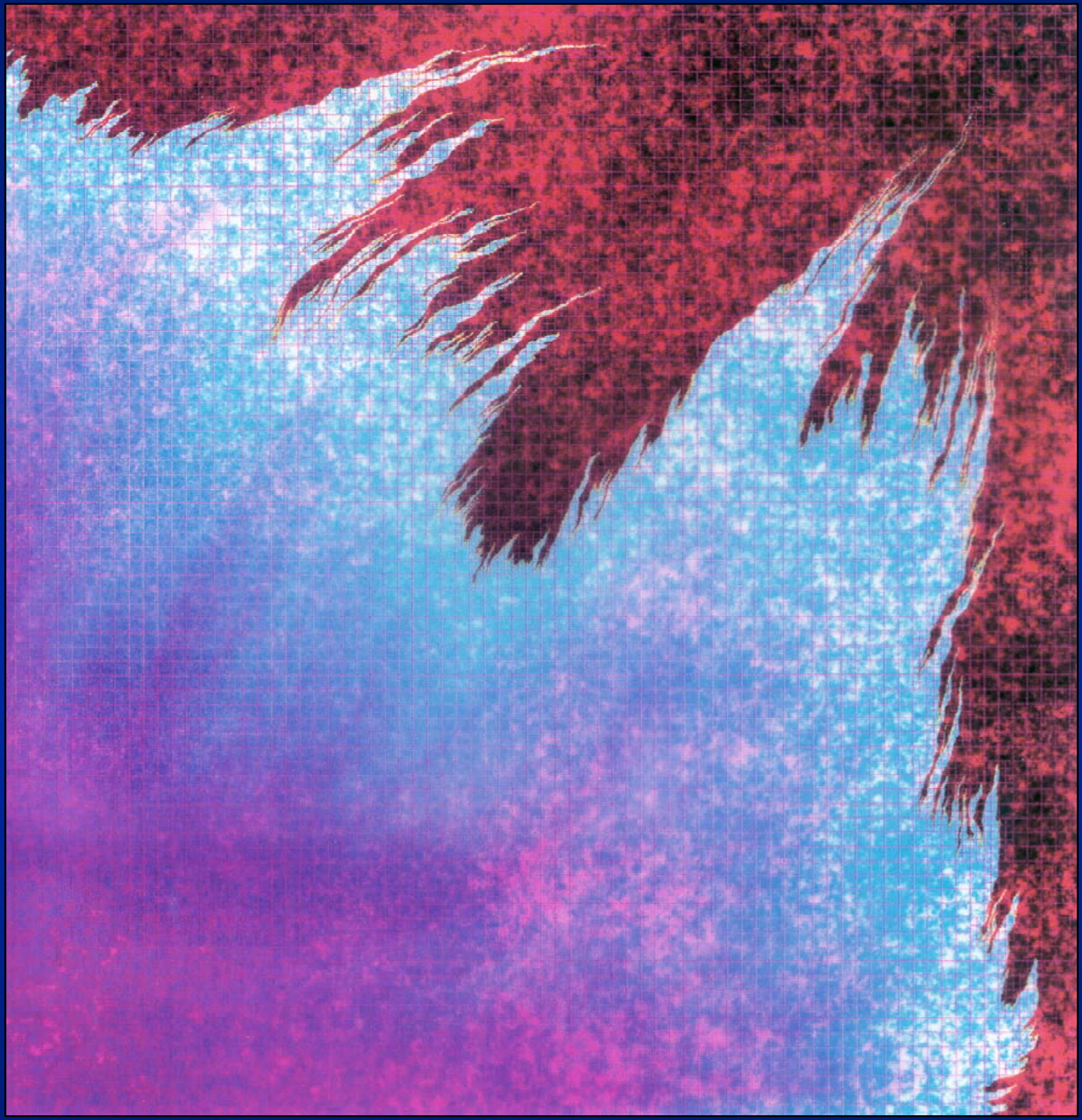
- **PARTITION COEFFICIENTS**
- **CUBIC EQUATION OF STATE**

APPLICATION/MODULES IN RESERVOIR SIMULATION

- **Models**
  - Black Oil Model
  - Compositional Model
  - Dual Porosity Model
  - Thermal Model
  - Chemical Model
- **Numerical Methods**
  - Space Discretization Methods
    - Finite Difference (FD)
    - Finite Element (FM)
    - PEBI
    - CVFM
  - Well Coupling Methods
    - Implicit BHP
    - Explicit BHP
    - Recycling
  - Grids
    - Rectangle
    - Unstructured
    - 2.5 D Unstructured
- **Solution Component**
  - Time Discretization
    - Fully Implicit
    - Sequential Implicit
    - IMPES
  - Space Discretization
    - Finite Difference
    - Finite Element
    - PEBI
    - CVFM
  - Solver
    - GAUSS
    - PCG
    - ORTHMIN
    - GMRES
- **Advanced Components**
  - Local Grid Refinement
    - Rectangular
    - Triangular
    - Polygon
  - Near Well Bore Flow
    - Hybrid Grid
    - Turbulence
    - Horizontal Well
  - Fault
    - Finite Difference
    - VORONOI Grids
    - 3D Cases
  - Corner Point Correction
    - Well
    - Fault
    - Boundary

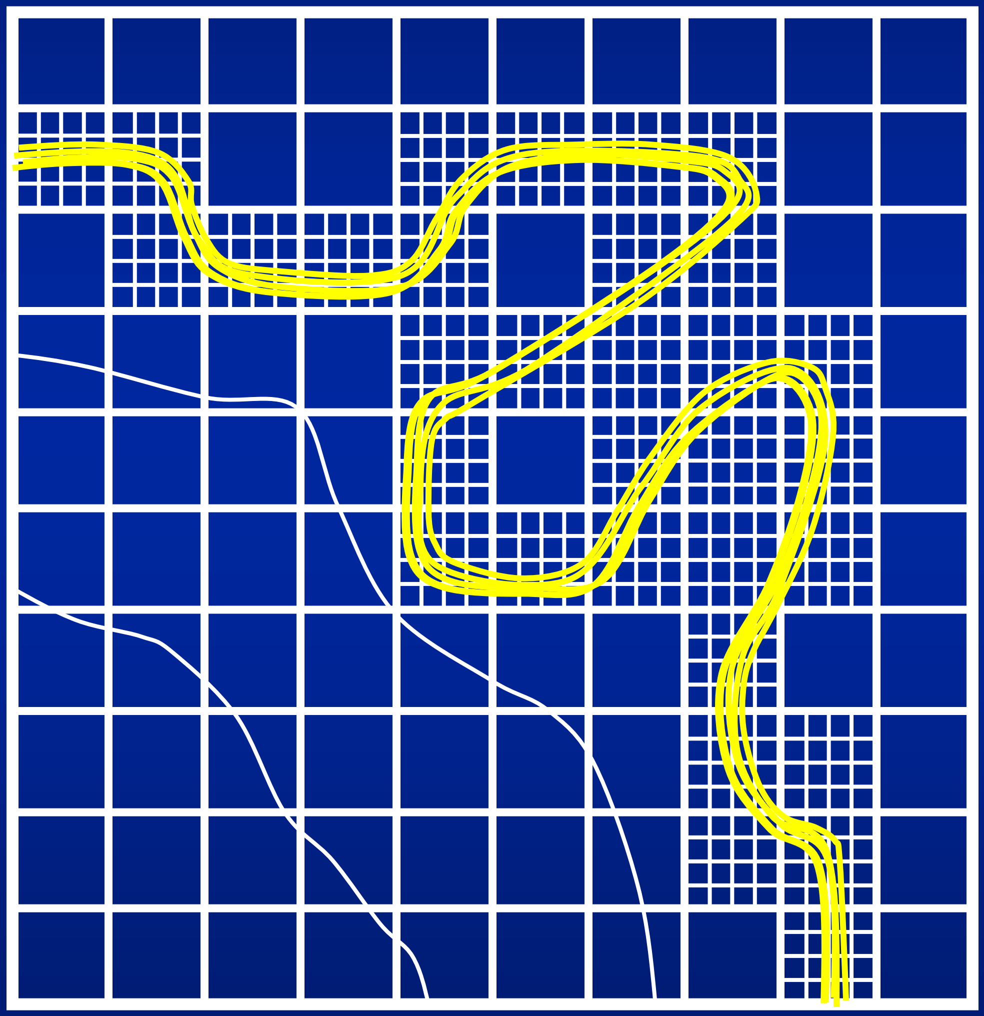
EXAMPLE SCENARIO IN RESERVOIR SIMULATION



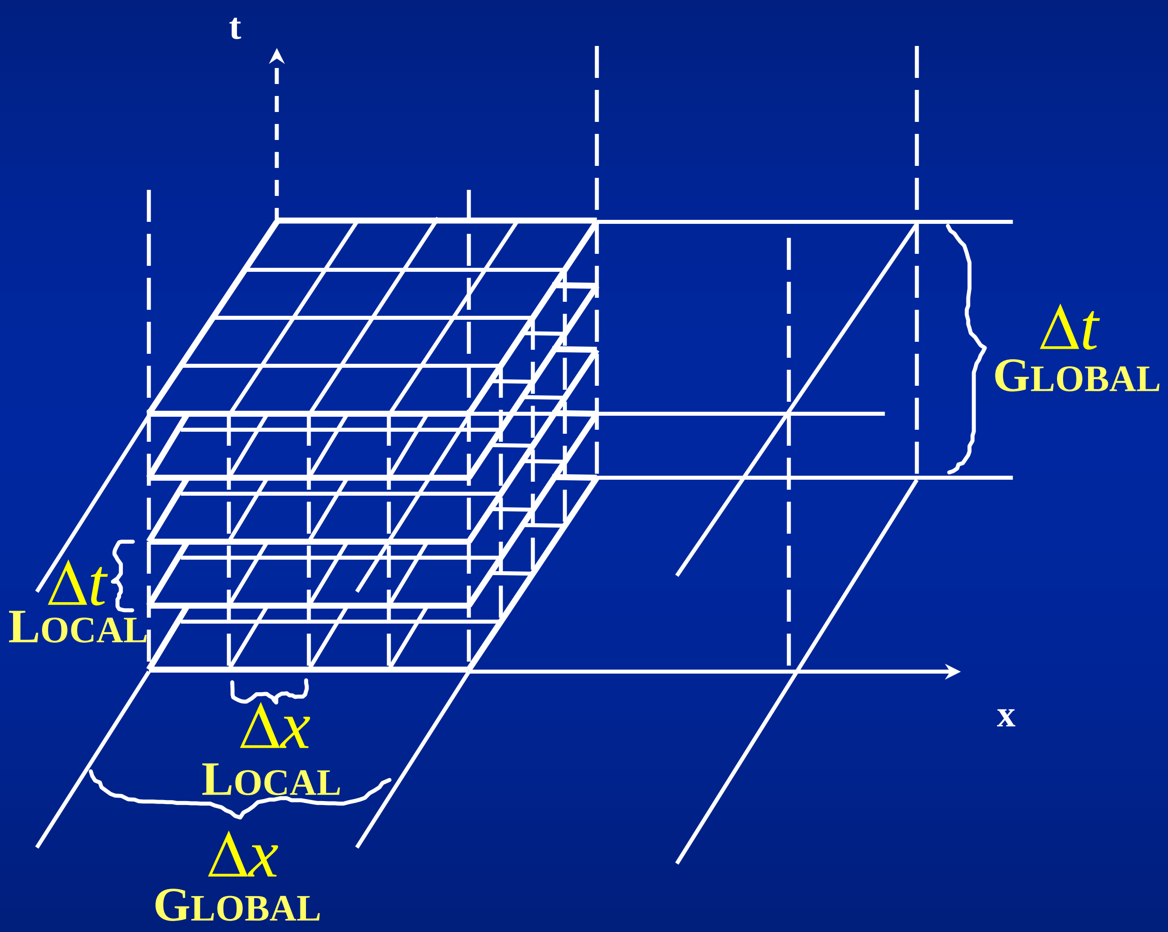


Saevareid, Espedal, *et al.* 1992.

MULTIPHASE FLOW SATURATION CONTOURS



LOCAL TIME – STEPPING



LOCAL TIME – STEPPING METHODS

EWING, LAZAROV, VASSILEVSKI – I – COMPUTING, 1990

EWING, LAZAROV, VASSILEVSKI – II – LNFS, 1990

DAWSON, DU, DUPONT – UC – 1989

DAWSON, DU, DUPONT – UC – 1989

HEROX, THOMAS – PROC. CMMG – 1989

ERIKSON, JOHNSON – UG – 1989

EWING, LAZAROV, PASCIAK, VASSILEVSKI – SINUM – 1993

EWING, JACOBS, PARASHKEVOV, SHEN – SIAM 1991

BOYETT, EL-MANDOUH, EWING – SIAM – 1991

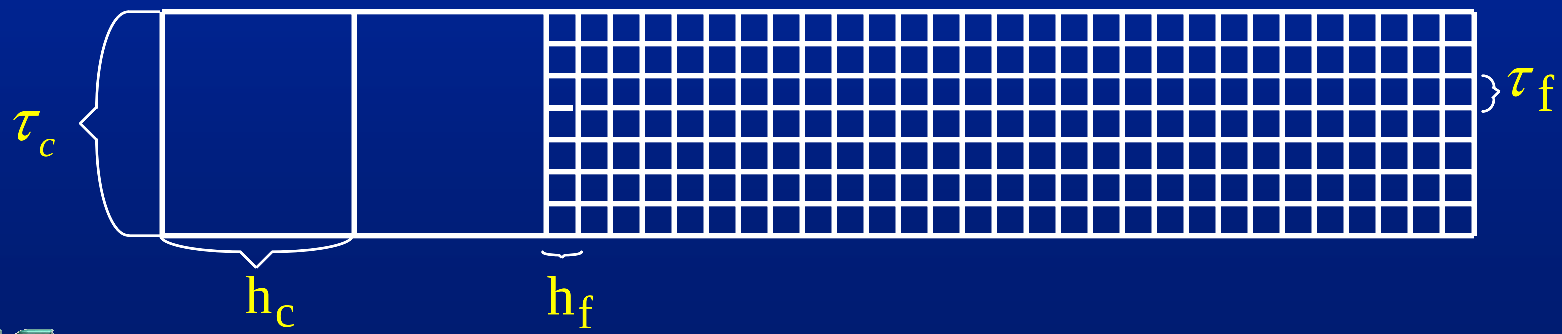
EWING, LAZAROV, VASSILEV – CMAME – 1992

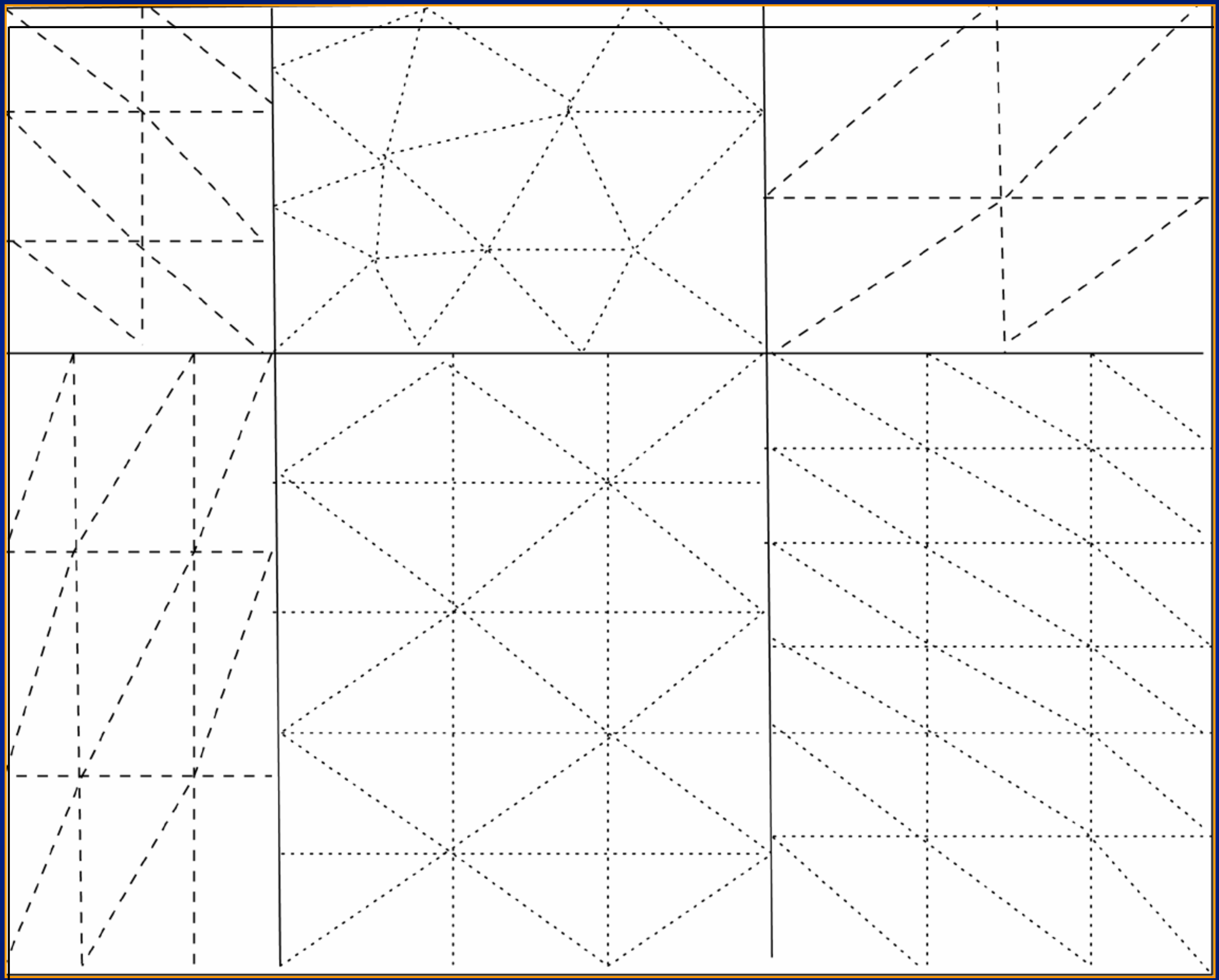
JACOBS – PH.D. THESIS – 1995

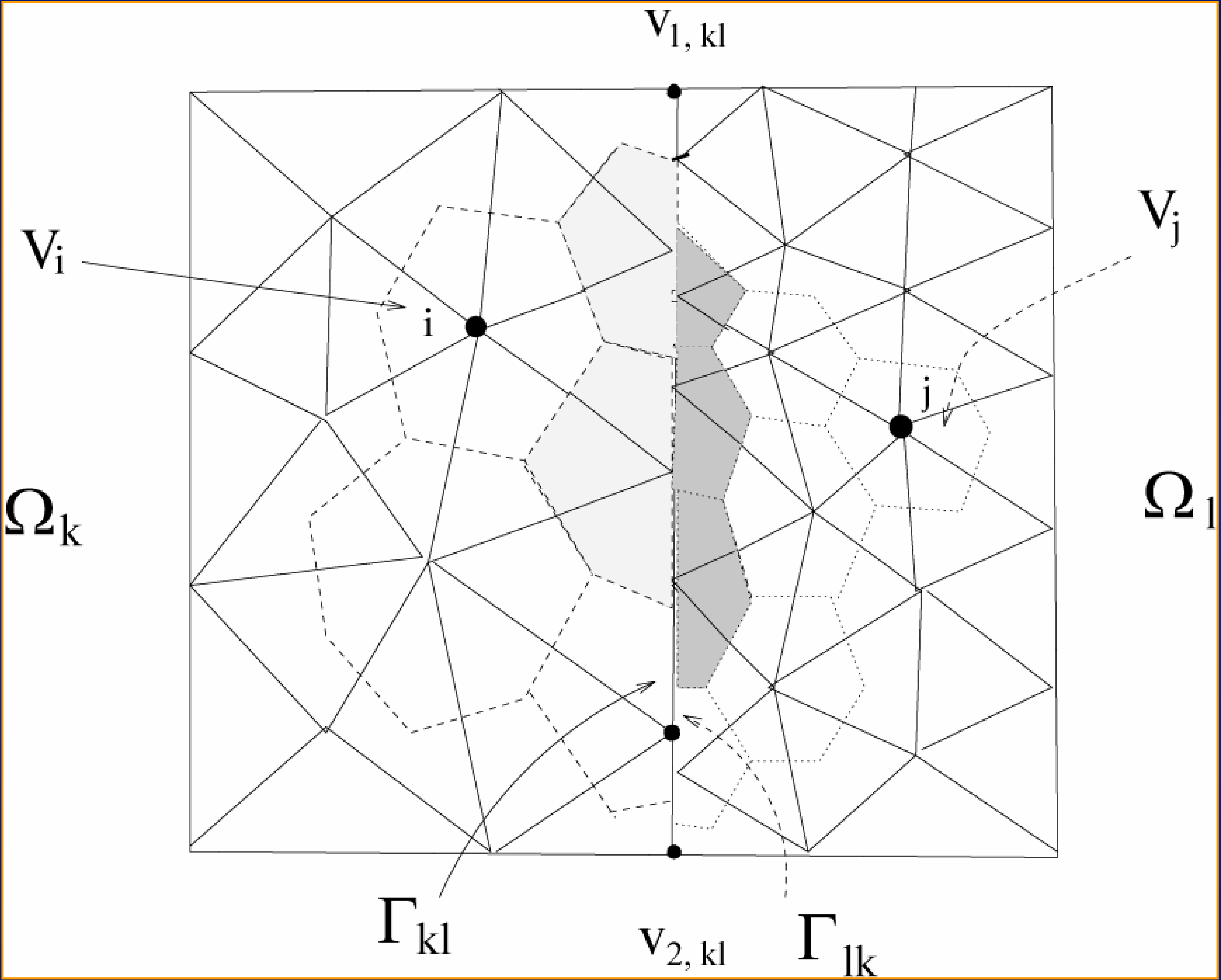
EWING, LAZAROV, VASSILEV – SINUM – 1994

THEOREM: (E,L,V)

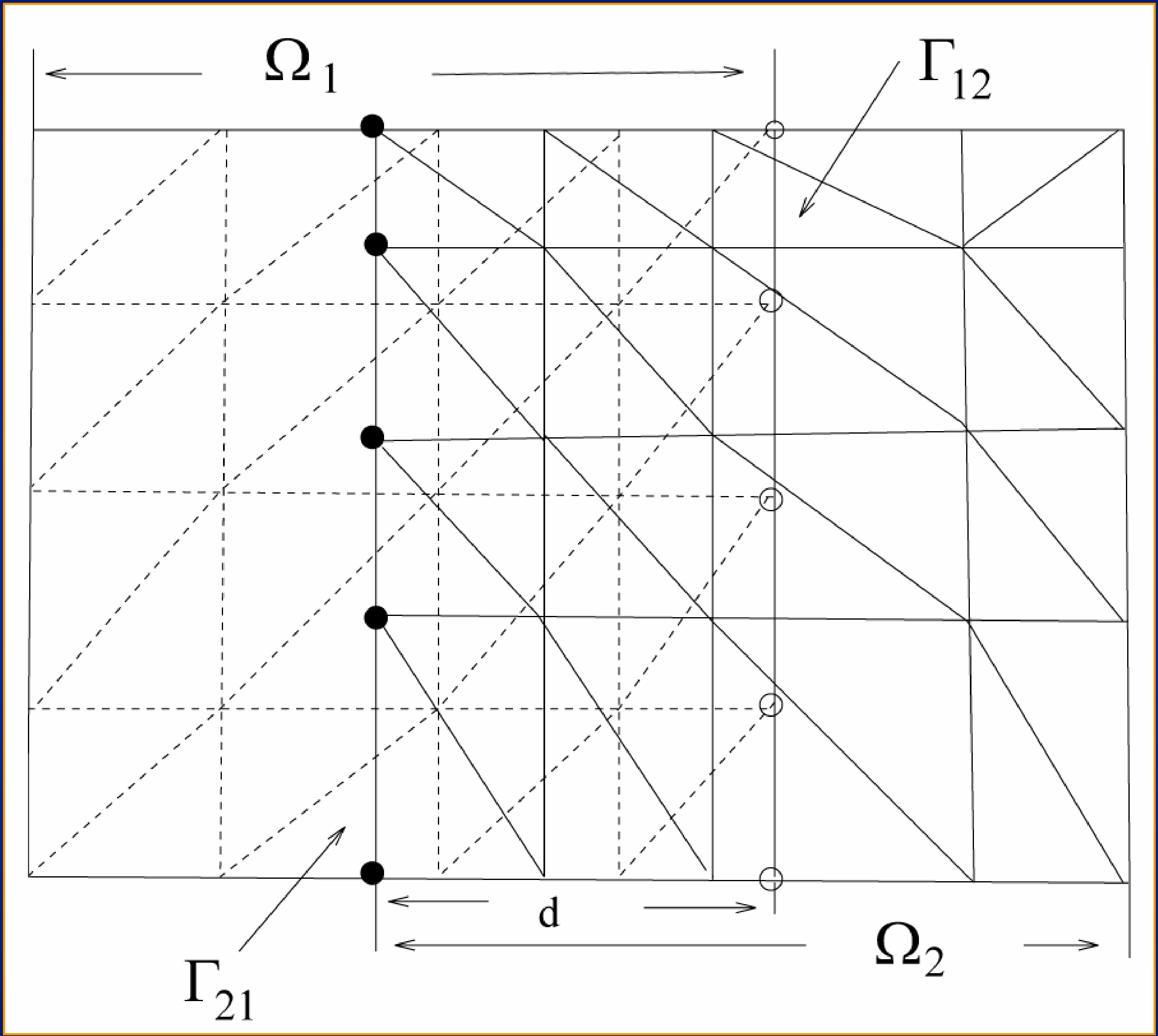
$$\max_{(x,t)\in J} \left| e(x,t) \right| \leq C_c\left(\tau_c + h_c^2\right) + C_f\left(\tau_f + h_f^2\right) + C_I\left(\tau_c + h_c^2 + \frac{\tau_c^2}{h_c}\right)$$



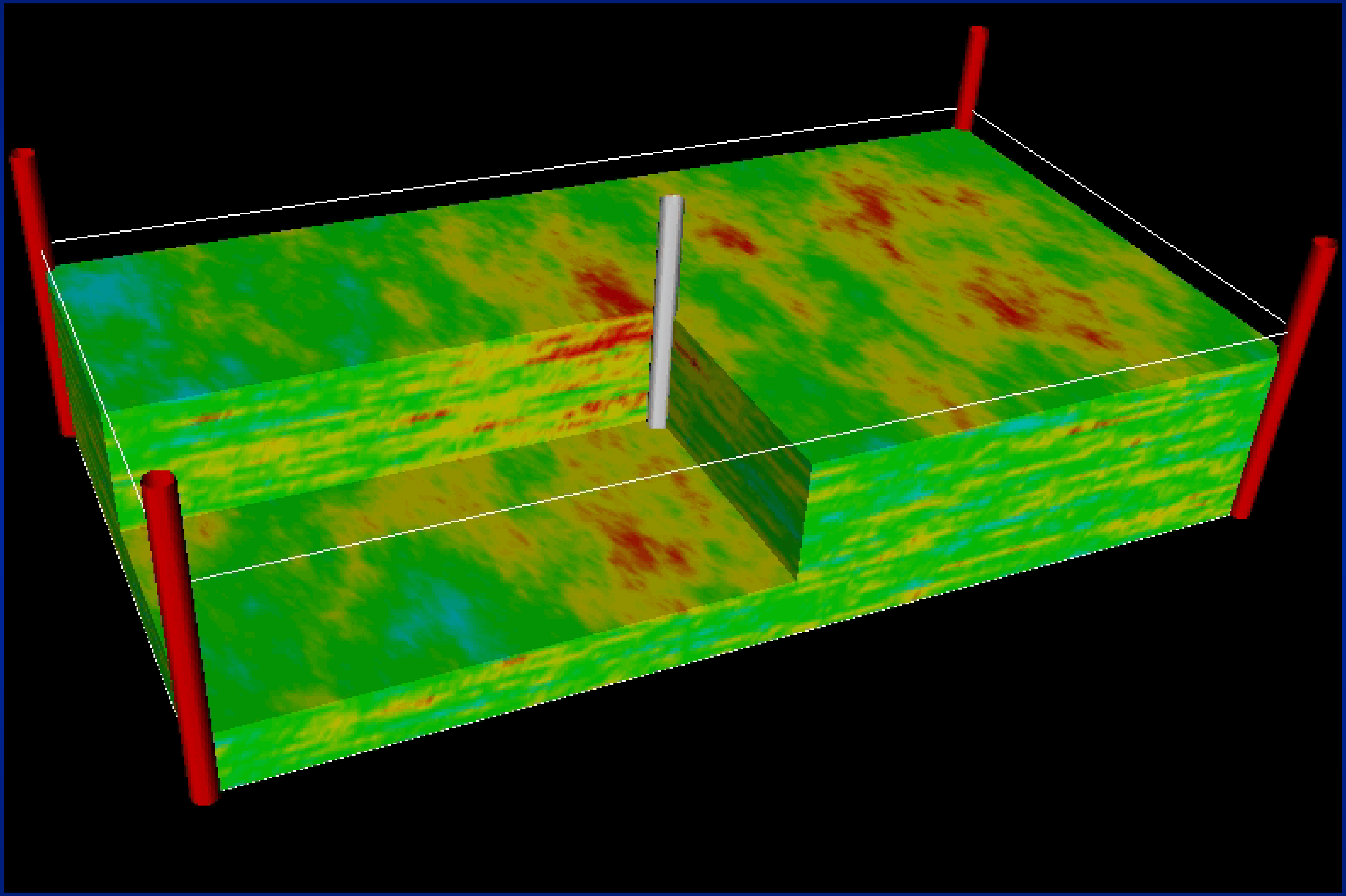


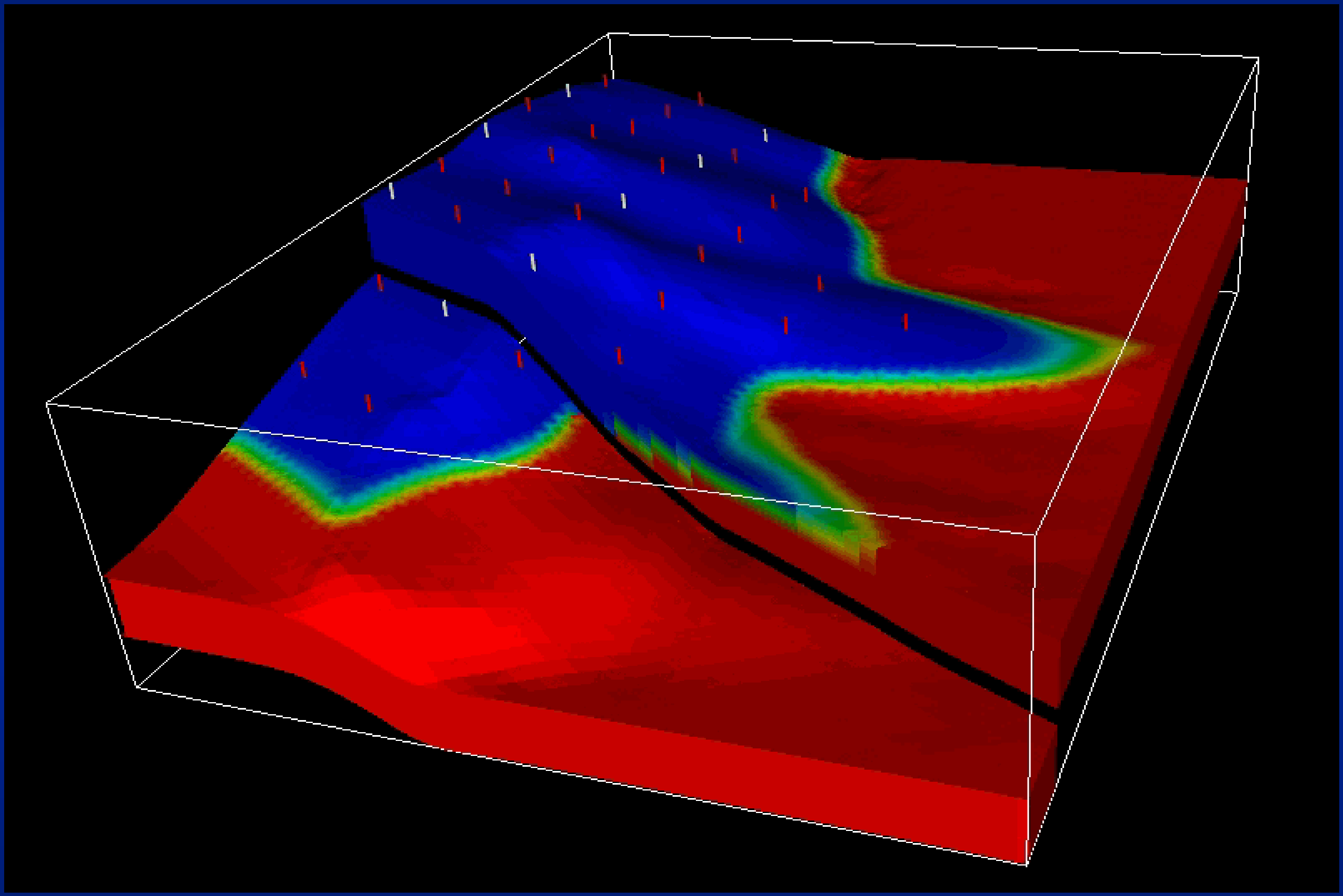


$V_{1,\,kl}$
V_i
i
V_j
j
Ω_k
Ω_l
Γ_{kl}
$V_{2,\,kl}$
Γ_{lk}



Ω_1
Γ_{12}
d
Ω_2
Γ_{21}





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