MINISYMPOSIUM 4: Domain Decomposition Methods Motivated by the Physics of the Underlying Problem

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Domain decomposition methods are a powerful tool to handle very large systems of equations. They can however also be used to couple different physical models or approximations, which one might want to do for various reasons: in fluid structure coupling for example, the physical laws in the fluid differ from the physical laws in the structure, and a domain decomposition method could naturally take this into account. Even if the physical model is the same, one might want to use a simplified equation in part of the domain, where certain effects are negligible, like for example in aerodynamics, to save computation time. Or one could simply want to use a much coarser mesh, like in combustion away from the flame front, which again could be taken naturally into account by a domain decomposition method that can handle non-matching grids, possibly in space and time.

In the first paper, Gander, Halpern, Labbé and Santugini present an optimized Schwarz waveform relaxation algorithm for the parallel solution in space-time of the equations of ferro-magnetics in the micro-magnetic model. The algorithm uses Robin transmission conditions, and a numerical study of the dependence of the optimized parameters on the physical properties of the problem is presented.

In the second paper, Halpern and Japhet present a space-time decomposition method for heterogeneous problems, with transmission conditions adapted to the heterogeneity of the physical model. The algorithm is of Schwarz waveform relaxation type with time windows, and discretized using a discontinuous Galerkin method in time, and a classical finite element discretization in space. The performance of this new method is illustrated by numerical experiments.

In the third paper, Halpern and Szeftel present a quasi-optimal Schwarz waveform relaxation algorithm for the one dimensional Schrödinger equation. This algorithm uses non-local interface conditions in time, which are exact for the case of a constant potential. If the potential is not constant, a frozen coefficient approach is used. The authors compare the performance of this new method to the performance of a classical Schwarz waveform relaxation method, and an optimized one with Robin conditions.

In the fourth paper, Haynes, Huang and Russel present an innovative Moving Mesh Schwarz Waveform relaxation method. In this method, evolution problems are decomposed in space, and on each subdomain, a moving mesh method is used to solve the subdomain problem on a given time window, before information is exchanged across the interfaces between subdomains. The method uses classical Dirichlet transmission conditions, and is both adaptive in space, with the moving mesh method, and in time, with a step size control. Its performance is well illustrated by numerical experiments for the viscous Burgers equation.