MINISYMPOSIUM 5: FETI, Balancing, and Related Hybrid Domain Decomposition Methods

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The FETI and Balancing domain decomposition algorithms form two important families of iterative methods. They have been implemented and tested for very large applications and have been used extensively in academia as well as in national laboratories in Europe and the United States. Over the years, these methods have been improved and in the FETI family, FETI–DP (dual-primal finite element tearing and interconnection) has proven to be a very robust algorithm which also, when carefully implemented, respects the memory hierarchy of modern parallel and distributed computing systems. This is essential for approaching peak floating point performance.

While the coarse component of these preconditioners typically only has a dimension which is a small multiple of the number of subdomains of the decomposition of the domain, it has become increasingly clear that it can become a bottleneck when the number of subdomains is very large. Solutions of this problem are quite nontrivial. Inexact, rather than exact, solvers of the coarse problem have been developed successfully; see the contribution by Klawonn et al. in which FETI-DP is applied to spectral elements. The problem can also be approached by introducing a third or even more levels. This is demonstrated for BDDC in the paper by Mandel et al..

The FETI–DP methods are very closely related to the BDDC (balancing domain decomposition by constraints) algorithms and, in fact, it has been established that the eigenvalues of the relevant operators essentially are the same, given a pair of methods defined by the same set of constraints; see the paper by Brenner. As demonstrated in the paper by Dostál, et al., the FETI–DP algorithms have also proven quite successful for difficult mechanical contact problems. Another extension from the original studies of linear elliptic problems and lower order finite elements is exemplified by the work by Klawonn et al., which demonstrates that these algorithms perform very well also for spectral element approximations. Still another extension of the BDDC methods is the study of Dryja et al., which develops and analyzes Discontinuous Galerkin Methods.

The basic ideas of the FETI algorithms have also inspired work on new iterative methods for boundary integral methods. The contribution by Of provides a sample of this important development.

Finally, there are two papers by Dohrmann et al., which represent a different development. They concern two issues. The first is the development of a new family of two-level overlapping Schwarz methods with traditional local solvers on overlapping subdomains but where the coarse level solver is inspired by those of iterative substructuring methods, i.e., methods which are based on a partition into non-overlapping subdomains. While the development of theory is just beginning, these methods have proven successful in a number of different applications. The second issue and paper concerns the extension of the theory for one of these methods to the case when the subdomains have quite irregular boundaries; so far these results are for two dimensions only.

We also note that the invited plenary talk by Hyea Hyun Kim concerned BDDC and FETI–DP algorithms for mortar finite elements.