Robust Norm Equivalencies and Preconditioning

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Summary. In this contribution we report on work done in continuation of [1, 2] where *additive multilevel methods* for the construction of preconditioners for the stiffness matrix of the Ritz- Galerkin procedure were considered with emphasis on the model problem $-\nabla\omega\nabla u = f$ with a scalar weight ω.

We present an new approach leading to a preconditioner based on a modification of the construction in [4] using weighted scalar products thereby improving that one in [2]. Further we prove an upper bound in the underlying norm equivalencies which is up to a fixed level completely independent of the weight ω, whereas the lower bound involves an assumption about the local variation the coefficient function which is still weaker than in [1]. More details will be presented in a forthcoming paper.

1 Preliminaries

1.1 Ritz -Galerkin-Method

Let Ω be a bounded domain in $\mathbb{R}^2$ and $H_0^1(\Omega) = Y$ be the Hilbert space defined as the closure of $C_0^\infty(\Omega)$ with respect to the usual Sobolev norm. Further let A be an elliptic operator defined on $H_0^1(\Omega)$ with an associated coercive and symmetric bilinear form $a(u,v)$. The *Lax-Milgram Theorem* guarantees then a unique solution $u \in Y$ of

$$a(u,v) = (f,v) := \int_\Omega f \cdot v \; dx, \qquad \forall v \in Y,$$

for any $f \in L_2(\Omega)$. Define the *Ritz -Galerkin approximation* $u_h \in \mathcal{V}_h \subset Y$ by

$$a(u_h, v) = (f, v), \qquad \forall v \in \mathcal{V}_h.$$

If $\psi_1, \cdots, \psi_N$ is a basis of $\mathcal{V}_h$, u_h is obtained by the equations:

$$\sum_{i=1}^N \alpha_i \; a(\psi_i, \psi_k) = (f, \psi_k), \quad 1 \le k \le N, \qquad u_h := \sum_{i=1}^N \alpha_i \psi_i.$$

These equations are solved *iteratively* in the form

$$u^{(\nu+1)} = u^{(\nu)} - \omega\ \mathcal{C}r^{(\nu)}, \quad \nu = 0, 1, 2, \cdots \tag{1}$$

where $r^{(\nu)} := \mathcal{A}u^{(\nu)} - b$ with *stiffness matrix* $\mathcal{A} \equiv \mathcal{A}_\psi := \Big(a(\psi_i, \psi_k)\Big)_{i,k}$ and $b := \{(f, \psi_k)\}$. Further ω denotes a relaxation factor and $\mathcal{C}$ a *preconditioner matrix.* The goal is to achieve $\kappa(\mathcal{CA}) \ll \kappa(\mathcal{A})$ or at least of order $\mathcal{O}(1)$ independent of N.

A basic fact is: If $\mathcal{C}$ is the matrix associated to operator $C : \mathcal{V} \to \mathcal{V}$ satisfying

$$\gamma\ (u\ ,\ C^{-1}\ u)\ \leq a(u,u)\ \leq\ \Gamma\ (u\ ,\ C^{-1}\ u), \quad u \in \mathcal{V}, \tag{2}$$

then $\kappa(\mathcal{CA}) \leq \Gamma/\gamma$. Thus $\mathcal{C}$ can be taken as a discrete analogue of C or an approximative inverse of $B = C^{-1}$.

In the theory of *Additive Multi-level-Methods* an approach to construct the bilinear form with associated B is to assume a hierarchical sequence of subspaces

$$\mathcal{V}_0 \subset \mathcal{V}_1 \subset \cdots \subset \mathcal{V}_J := \mathcal{V}_h\ \subset Y\ \subset X := L_2(\Omega), \tag{3}$$

and construct *bounded linear projections* $Q_j : \mathcal{V} \longrightarrow \mathcal{V}_j$ with

$$\beta_0\ a(u,u) \leq \sum_{j=0}^{J} d_j^2\ ||Q_j u - Q_{j-1}u||_X^2 \leq \beta_1\ a(u,u), \tag{4}$$

with $Q_0 u := 0$ and suitable coefficients $\{d_j\}$. β_0, β_1 are constants not depending on the d_j, $u \in \mathcal{V}_h$ or J.
Then define the positive definite operator $B = C^{-1}$ by

$$(u\ ,\ B\ u) := \sum_{j=1}^{J} d_j^2\ ||Q_j u - Q_{j-1}u||_X^2, \qquad u \in \mathcal{V}. \tag{5}$$

2 A Diffusion Problem as a Model Problem

2.1 Spectral Equivalencies

Let $\mathcal{T}_0$ be an initial coarse triangulation of a region $\Omega \subset \mathbb{R}^2$. Regular refinement of triangles leads to triangulations $\mathcal{T}_0 \subset \mathcal{T}_1 \subset \cdots \subset \mathcal{T}_J = \mathcal{T}$.

Each triangle in $\mathcal{T}_k$ is geometrically similar to a triangle of $\mathcal{T}_0$. We define then the $\{\mathcal{V}_j\}_{j=1}^{J}$ in (3) as spaces of piecewise linear functions with respect to these triangulations. Also its elements have to satisfy Dirichlet boundary conditions. In particular there exists a nodal basis $\psi_k^{(j)}$ for $\mathcal{V}_j = \text{span}\{\psi_k\}$.

In the following we consider the model problem

$$a(u,v) := \int_\Omega \omega\ (\nabla u, \nabla v) \tag{6}$$

which correspond s to the differential operator $A = \nabla \cdot \omega \nabla$.
Observe that in case $u, v \in \mathcal{V}_j,\ j = 0, 1, \cdots, J$ the bilinear form $a(u,v)$ reduces in view of $\nabla v = \text{const.}$ on $T \in \mathcal{T}_j$ to

$$a(u,v) = a_j(u,v) := \sum_{T\in\mathcal{T}_j} \omega_T \int_T (\nabla u, \nabla v) \tag{7}$$

with average weights

$$\omega_T := \frac{1}{\mu(T)} \int_T \omega dx, \qquad \mu(T) \ = \ \text{area of } T.$$

This leads to *weighted norms*

$$\|v\|_{j,\omega}^2 := \sum_{T\in\mathcal{T}_j} \omega_T \int_T |v|^2, \qquad \|v\|_\omega := \|v\|_{J,\omega} \, . \tag{8}$$

Instead of the usual orthogonal projections $Q_j : \mathcal{V} \to \mathcal{V}_j$ we define now in contrast to [1] operators $Q_j^\omega : \mathcal{V} \to \mathcal{V}_j$ with *level-depending weights* by

$$(Q_j^\omega u, v)_{j,\omega} = (u,v), \qquad \forall v \in \mathcal{V}_j \tag{9}$$

and $A_j^\omega : \mathcal{V}_j \to \mathcal{V}_j$ for $u \in \mathcal{V}_j$ by

$$(A_j^\omega u, v)_{j,\omega} = a(u,v), \qquad \forall v \in \mathcal{V}_j. \tag{10}$$

Then the following modification of a well-known result in the theory of multilevel methods (cf. surveys [3, 5] of J. Xu and H. Yserentant) can be proved.

Theorem 1. *Suppose that there exists a decomposition* $u = \sum_{k=0}^J u_k$ *for* $u \in \mathcal{V}$ *with* $u_k \in \mathcal{V}_k$ *and positive definite operators* $B_k^\omega : \mathcal{V}_k \to \mathcal{V}_k$ *satisfying*

$$\sum_{k=0}^J (B_k^\omega u_k, u_k)_{k,\omega} \le K_1 \ a(u,u), \tag{11}$$

then $C^\omega := \sum_{k=0}^J (B_k^\omega)^{-1} Q_k^\omega$ *satisfies* $\lambda_{\min}\ (C^\omega A) \ge 1/K_1$.
If the operators B_k^ω *further satisfy*

$$a(\sum_{k=0}^J w_k, \sum_{l=0}^J w_l) \ \le \ K_2 \ \sum_{k=0}^J (B_k^\omega w_k, w_k)_{k,\omega}, \qquad w_k := (B_k^\omega)^{-1} Q_k^\omega A u \tag{12}$$

then $\lambda_{\max}\ (C^\omega A) \le K_2$*, i.e. the operator* C^ω *is spectrally equivalent to* A*.*

The proof will be given in a forthcoming paper by M. Griebel and M.A. Schweitzer.

For the diffusion problem (6) we can choose the operator B_k^ω now simply as $B_k u_k := 4^k u_k$ for $u_k \in \mathcal{V}_k$, hence

$$C\ u := \sum_{k=0}^J 4^{-k} Q_k^\omega u. \tag{13}$$

This has several advantages over the approach in [1] which uses direct norm equivalencies like in (4). For spectral equivalence of C with A one needs to prove the upper inequality (11) in the form

$$\sum_{k=0}^{J} 4^k (u_k, u_k)_{k,\omega} \leq K_1 \; a(u,u), \tag{14}$$

only for *some* decomposition $u = \sum_{k=0}^{J} u_k$. However (12) has to be verified in the form

$$a(\sum_{k=0}^{J} w_k, \sum_{l=0}^{J} w_l) \;\leq\; K_2 \; \sum_{k=0}^{J} 4^k (w_k, w_k)_{k,\omega}, \tag{15}$$

for *any* decomposition $v = \sum_{k=0}^{J} w_k, w_k \in \mathcal{V}_k$.
These *weighted Jackson- and Bernstein inequalities* will be verified in the next sections in a *robust form*, i.e. the constants depend only weakly from the diffusion coefficient ω.

Another advantage of the above theorem is that (13) leads to a practical form for the preconditioning matrix $\mathcal{C}$ in (1), namely one shows that the operator C above is spectrally equivalent to the operator

$$\tilde{C} := \sum_{k=0}^{J} 4^{-k} M_k^{\omega}, \qquad M_k^{\omega} v := \sum_{i \in \mathcal{N}_k} \frac{(v, \psi_i^{(k)})}{(1, \psi_i^{(k)})_{k,\omega}} \psi_i^{(k)}$$

where $\{\psi_i^{(k)}\}_{i=1}^{N_k}$ denotes the nodal basis of $\mathcal{V}_k$ for $k \geq 1$. Thus the operators $M_k^{\omega} u$ replace the operators Q_k^{ω} defined as in (9). The reason for this is that one can show (up to an absolute constant)

$$(Q_k^{\omega} u, u) \approx \sum_{i \in \mathcal{N}_k} \frac{(u, \psi_i^{(k)})^2}{(1, \psi_i^{(k)})_{k,\omega}} = \left(u, \sum_{i \in \mathcal{N}_k} \frac{(u, \psi_i^{(k)}) \psi_i^{(k)}}{(1, \psi_i^{(k)})_{k,\omega}}\right) = (M_k^{\omega} u, u).$$

Details as well as the realization of this conditioner in optimal complexity will be presented in the forthcoming paper by M. Griebel and M.A. Schweitzer.

We remark that it can be modified still further to obtain a preconditioner

$$\hat{C} u := \sum_{k=0}^{J} \sum_{i \in \mathcal{N}_k} \frac{(u, \psi_i^{(k)})}{a(\psi_i^{(k)}, \psi_i^{(k)})} \psi_i^{(k)}.$$

2.2 A Weighted Bernstein-Type Inequality

According to (15) we consider here arbitrary decompositions

$$u = \sum_{k=0}^{J} w_k, \qquad w_k \in \mathcal{V}_k \tag{16}$$

of an element $u \in \mathcal{V}_J$. In the following we employ the $a-$orthogonal operators $Q_j^a : \mathcal{V}_J \to \mathcal{V}_j$ defined by

$$a(Q_j^a u, v) = a(u, v), \qquad u \in \mathcal{V}_J, v \in \mathcal{V}_j,$$

so that the elements $v_j := Q_j^a u - Q_{j-1}^a u, v_0 := Q_0^a u$ satisfy

$$u = \sum_{=0}^{J} v_j, \qquad a(v_k, v_j) = \delta_{j,k}, \qquad a(u,u) = \sum_{=0}^{J} a(v_j, v_j). \tag{17}$$

We introduce then the following assumption on the weight ω : there exists a constant C_ω independent of j and a number $\rho < 2$ such that for all $T \in \mathcal{T}_j$

$$\sup\{\omega_U/\omega_T : U \in \mathcal{T}_k, U \subset T\} \ \leq \ C_\omega \ \rho^{k-j}, \qquad j \leq k. \tag{18}$$

Lemma 1. *Under the above assumption on the weight ω there holds the "hybrid" Bernstein type inequality*

$$\|v_j\|_a \leq \ 6\sqrt{6}\ C_1 \sqrt{C_2 C_\omega}\ (2/\rho)^{j/2} \sum_{k=j}^{J} (2\rho)^{k/2}\ \|w_k\|_{k,\omega} \tag{19}$$

where $C_1 := \max_{T\in\mathcal{T}_0} \operatorname{diam}(T) \geq \max_{T\in\mathcal{T}_0} \sqrt{\mu(T)}$, and
$C_2 := \max_{T\in\mathcal{T}_0} \operatorname{diam}(T)/\sqrt{\mu(T)}$ are constants which depend on the initial triangulation $\mathcal{T}_0$ only.

Proof. In view of the representation $u = \sum_{k=0}^{j-1} w_k$ we have by (17)

$$a(v_j, v_j) = a(v_j, u) = \sum_{k=j}^{J} a(v_j, w_k)\,. \tag{20}$$

By integration by parts we obtain, keeping in mind that $\nabla w_k, \nabla v_j$ are constant on $U \in \mathcal{T}_k$ and $T \in \mathcal{T}_j$, respectively,

$$\begin{aligned} a(v_j, w_k) &= \sum_{U\in\mathcal{T}_k} \omega_U \int_U (\nabla v_j, \nabla w_k) = \sum_{U\in\mathcal{T}_k} \omega_U \int_{\partial U} w_k(\nabla v_j, n_{\partial U}) \\ &= \sum_{T\in\mathcal{T}_j} \sum_{U\subset T} \omega_U \int_{\partial U} w_k(\nabla v_j, n_{\partial U}) = \sum_{T\in\mathcal{T}_j} \sum_{U\in S_k(T)} \omega_U \int_{\partial U} w_k(\nabla v_j, n_{\partial U}), \end{aligned}$$

where $S_k(T)$ denotes the boundary strip along ∂T consisting of triangles $U \in \mathcal{T}_k, U \subset T$. Applying the Cauchy-Schwarz inequality gives

$$a(v_j, w_k) \leq \Big(\sum_{T\in\mathcal{T}_j} \sum_{U\in S_k(T)} \omega_U \int_{\partial U} |w_k|^2 \Big)^{1/2} \Big(\sum_{T\in\mathcal{T}_j} \sum_{U\in S_k(T)} \omega_U \int_{\partial U} \|\nabla v_j\|^2 \Big)^{1/2}. \tag{21}$$

Concerning the first double sum we note that

$$\int_{\partial U} |w_k|^2 \ \leq \ \operatorname{diam}(U)[b_1^2 + b_2^2 + b_3^2] \leq 12\ C_2 C_1\ 2^k \ \int_U |w_k|^2,$$

where we have used $\operatorname{diam}(U) \leq C_2 C_1\ 2^k\ \mu(U)$ and the formula

$$\int_U |w_k|^2 = \frac{\mu(U)}{12} [b_1^2 + b_2^2 + b_3^2 + (b_1 + b_2 + b_3)^2]$$

for linear functions v on U with vertices b_1, b_2, and b_3. It follows that

$$\sum_{T\in\mathcal{T}_j}\omega_T\int_{\partial T}|w_k|^2 \le \sum_{T\in\mathcal{T}_j}\omega_T\sum_{U\in S_k(T)}\int_{\partial U}|w_k|^2 \le 12C_2C_1\;2^k\|w_k\|^2_{k,\omega}. \qquad (22)$$

For the second factor in (21) note that by assumption (19) and by the fact that $\mu(S_k(T))/\mu(T) \le 6\cdot 2^{j-k}$ (cf. [5])

$$\begin{aligned}\sum_{T\in\mathcal{T}_j}\sum_{U\in S_k(T)}\omega_U\int_{\partial U}\|\nabla v_j\|^2 &\le C_\omega\;\rho^{k-j}\sum_{T\in\mathcal{T}_j}\sum_{U\in S_k(T)}\omega_T\int_{\partial U}\|\nabla v_j\|^2\\ &\le 3C_1 2^k\;C_\omega\;\rho^{k-j}\sum_{T\in\mathcal{T}_j}\sum_{U\in S_k(T)}\omega_T\int_U\|\nabla v_j\|^2\\ &\le 18C_1 2^k\;C_\omega\;(\rho/2)^{k-j}\sum_{T\in\mathcal{T}_j}\omega_T\int_T\|\nabla v_j\|^2\end{aligned}$$

Inserting this and (22) into (21) inequality (19) follows by (20).

With the help of this lemma the Bernstein-type inequality (15) can be established. It improves the corresponding ones in [1, 2] in that assumption (18) is weaker and at the same time more simple than those made there.

Theorem 2. *Consider a sequence of uniformly refined triangulations $\mathcal{T}_j$ and the respective sequence of nested spaces $\mathcal{V}_j$ of linear finite elements. Then, under assumption (18) on the weight ω with $\rho < 2$ in (6) there holds the upper bound*

$$a(u,u)\;\le\;432C_1^2C_2C_\omega\;\frac{2}{(\sqrt{2}-\sqrt{\rho})^2}\;\sum_{j=0}^{J}\;2^{2j}\;\|w_j\|^2_{j,\omega} \qquad (23)$$

for w_j given in (16).

Proof. By summing the estimate (18) according to (17) we get

$$a(u,u)=\sum_{j=0}^{J}\|v_j\|_a^2\;\le\;216C_1^2\;C_2C_\omega\;(2/\rho)^j\Big(\sum_{k=j}^{J}(2\rho)^{k/2}\;\|w_k\|_{k,\omega}\Big)^2. \qquad (24)$$

From this an upper bound for $a(u,u)$ follows by application of a Hardy inequality to the latter double sum. If quantities s_j, c_j are defined by

$$s_j := \sum_{k=j}^{J}\;a_k,\quad s_{-1}:=0,\qquad c_j:=\sum_{l=0}^{j}b^l,\quad c_{J+1}:=0$$

with $a_k\ge 0$ and $b>1$ such an inequality reads

$$\Big(\sum_{j=0}^{J}\;b^j\;s_j^2\Big)^{1/2}\le\;\frac{\sqrt{b}}{\sqrt{b}-1}\;\Big(\sum_{j=0}^{J}\;b^j\;a_j^2\Big)^{1/2}.$$

Application of this with $a_k := (2\rho)^{k/2}\|w_k\|_{k,\omega}$ and $b = 2/\rho$ to yields

$$\sum_{j=0}^{J}\;(2/\rho)^j\Big(\sum_{k=j}^{J}(2\rho)^{k/2}\;\|w_k\|_{k,\omega}\Big)^2\le\;\frac{2}{(\sqrt{2}-\sqrt{\rho})^2}\;\sum_{j=0}^{J}\;2^{2j}\;\|w_j\|^2_{j,\omega}$$

and after insertion into (24) the bound (23) for $a(u,u)$.

2.3 A Weighted Jackson-Type Inequality

The goal here is to establish inequality (11), i.e. to prove

$$\sum_{k=0}^{J} 4^k \|v_k\|_{k,\omega} \leq K_1 \; a(u,u), \qquad u \in \mathcal{V}_J. \tag{25}$$

By Theorem 2.1 we can employ a particular decomposition of u. We choose

$$u = \sum_{j=0}^{J} v_k, \qquad v_k := Q_j^a u - Q_{j-1}^a u \;\text{ as in }\; (17). \tag{26}$$

The *basic idea* is as in [1] to prove a local estimate for $\|v_j\|_{j,\omega}$ on subdomains $U \subset \Omega$ by modifying the duality technique of Aubin-Nitsche. The following result gives an estimate which improves the corresponding one in [1] in that the constant does not depend on the weight ω.

Lemma 2. *Let $U = \operatorname{supp} \psi_l^{(j-1)}$ be the support of a nodal function in $\mathcal{V}_{j-1}$. There holds*

$$\|v_j\|_{j,\omega,U} \;\leq\; \operatorname{diam}(U) \left(\|\nabla v_j\|_{j,\omega,U} \;+\; 18 C_R \; \|v_j\|_{j,\omega,U} \right), \tag{27}$$

where C_R is an absolute constant.

Proof: We give only a rough idea of it. For triangles $S \in \mathcal{T}_j$ with $T \subset U$ consider the Dirichlet problems

$$-\Delta \phi_S = v_j \qquad \text{on } S, \qquad \phi_S|_{\partial S} = \psi_l^{(j-1)}|_{\partial S}.$$

Then $|v_j|^2 = -v_j \cdot \Delta \phi_S$ on U. Partial integration on each $S \subset U$ gives

$$||v_j||^2_{j,\omega,U} = \left| \sum_{S \subset U} \omega_S \int_S (\nabla \phi_S, \nabla v_j) - \sum_{S \subset U} \omega_S \int_{\partial S} v_j (\nabla \phi_S, n_{\partial S}) \right|.$$

The rest of the proof consists in a careful estimate of both terms on the right hand side. Concerning details we refer again to the forthcoming paper with by M. Griebel and M.A. Schweitzer. We mention only that the constant C_R above arises from the well-known regularity result

$$\|\phi_S\|_{2,2,S} \;\leq\; C_R \; \|v_j\|^2_{0,S}.$$

□

Now by the assumption made on the triangulations there holds $\operatorname{diam}(U) \leq C_0 2^{-j}$ with a constant C_0 depending only on the initial triangulation. Then choose j_0 as the smallest integer with $2^{j_0} = 27\sqrt{3} C_R C_0$ and the second term on the right hand side in (27) is $\leq (2/3) \|v_j\|_{j,\omega,U}$ for all $j \geq j_0$.
If we insert this, square and multiply the resulting inequality with the factor 4^j, the summation with respect to U and $j \geq j_0$ yields

Theorem 3. *There holds the Jackson-type inequality*

$$\sum_{j=j_0}^{J} 4^j \|v_j\|_{j,\omega}^2 \le 9C_0^2 \quad \sum_{j=j_0}^{J} \sum_U a(v_j, v_j)_U \le \; 9C_0^2 \; \sum_{j=j_0}^{J} a(v_j, v_j) \tag{28}$$

for $j_0 = \log_2 \; (27\sqrt{3}C_R C_0)$.

If one solves at first the Ritz-Galerkin equations up to level $j_0 - 1$ the preconditioning to the levels $j \ge j_0$ would be robust under condition (18) on the weight ω.

Another possibility would be to establish a bound of the remaining sum on the left hand side up to level $j_0 - 1$. Here one has to use a different argument at the expense of a dependence of the corresponding constant on ω. However one can achieve this under a condition which is weaker than (18).

Corollary 1. *Under the condition (18) on the weight* ω *the discretized version of the operator* $\tilde{C}$ *in (13) yields a robust preconditioning in (1) for the diffusion problem.*

3 Concluding Remarks

The results represented here are concerned with the classical additive multi-level method for solving Ritz-Galerkin equations with piecewise linear elements by preconditioning. The proofs given or indicated here for the necessary norm equivalencies simplify and improve those in [1]. They show that for the diffusion problem a simple modification (13) of the classical preconditioner makes it robust for a large class of diffusion coefficients ω. It covers all piecewise constant functions independent of the location of jumps, their number or their frequency. In particular we do not require the jumps to be aligned with the mesh on any level , i.e. no mesh must resolve the jumps.

However the constants in the Jackson- and Bernstein type inequalities involve the height of the maximal jump. If we assume that $m_\omega := \min_{x\in\Omega} \omega(x) = 1, M_\omega := \max_{x\in\Omega} \omega(x) = \epsilon^{-1}$ a bound for the constant C_ω in assumption (13) is given by ϵ^{-1}. For most practical purposes it is therefore necessary to assume that M_ω is not too big. By the form of (13) one sees that even singularities of maximal height ρ^J and exponential growth limited by ρ are admissible.

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