# Lagrange Sliding-Colliding Interfaces

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Abstract:

A Lagrange Sliding-Colliding Interface enables the free interaction of several meshes during the Lagrange Finite difference time step. It is based on dynamically identifying the boundary edges of the other meshes which may collide with a certain boundary vertex and identifying a "penetration point" for each such edge. From those non-additive possible collisions, the minimal sufficient set of interacting edges are chosen.

The interaction corrects the vertices' velocities to satisfy relative velocity constraints in order to prevent interpenetration while preserving the momentum. Later on, after the coordinates have been advanced to the next time step, the coordinates positions may be corrected (if the need arises).

The interface auto-detects the need for its existence - the user does not have to define it in advance.

Another usage of the interface is correcting small overlaps caused by independent re-mesh operations of several regions.

# Distributed Hypersphere Decomposition in Arbitrary Dimensions

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**David E. Keyes**
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Abstract:

We describe a parallel technique for decomposing the surface of a hypersphere of arbitrary dimension, both exactly and approximately, into a specific number of regions of equal area and small diameter. We provide variations of the algorithm for when the number of regions does not need to be exact, and for when the absolute minimum diameter is not needed. We then analyze the performance of the various algorithms by finding the minimum distance between the centers of neighboring partitions, and use this to estimate the amount of work required to construct a partition with a minimum resolution for each algorithm. An accompanying C++ software package with a MATLAB interface will also be provided.

# Two-Level Schwarz Preconditioners for Super Penalty Discontinuous Galerkin Methods

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Abstract:

In recent years, much attention has been given to domain decomposition methods for linear elliptic problems that are based on a partitioning of the domain of the physical problem. Since the subdomains can be handled independently, such methods are very attractive for coarse-grain parallel computers. In this talk we shall present some non-overlapping additive and multiplica-tive Schwarz domain decomposition methods for the solution of the algebraic linear systems of equations arising from super penalty discontinuos Galerkin approximations of elliptic problems. We provide the convergence estimates, and we show that the proposed Schwarz methods can be successfully accelerated with suitable Krylov iterative solvers. Numerical experiments to validate our theory and to illustrate the performance and robustness of the proposed two-level methods will be presented.

# Linearly implicit domain decomposition methods for time-dependent reaction-diffusion equations

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Abstract:

This work is devoted to the numerical solution of a class of parabolic problems which involve nonlinear reaction and diffusion terms. The spatial discretisation is carried out using the support-operator method (cf. [3]), which constructs a cellcentred finite difference scheme that is naturally adapted to non-Cartesian grids. The resulting nonlinear system of stiff differential equations is then integrated in time by means of a linearly implicit fractional step method which extends the ideas proposed in [2] to the nonlinear case. At each time step, the discrete nonlinear diffusion operator is expressed as the sum of two components: one of them groups the first two terms of its Taylor expansion around the numerical solution at the previous time step, whereas the other involves the remaining higher-order terms of that expansion. The former is a discrete linear operator, which is subsequently split into a number of simpler linear suboperators, whilst the latter remains nonlinear. Such operator splitting is subordinated to a decomposition of the spatial domain into a set of overlapping subdomains in order to obtain an efficient parallel algorithm (cf. [1]). Finally, the linearly implicit method considers implicit time integrations for the linear suboperators while explicitly handling both the remaining discrete nonlinear operator and the discrete nonlinear reaction term. Therefore, the original problem is reduced to the solution of several linear systems per time step which can be trivially decomposed into a set of uncoupled parallelisable linear subsystems. As a difference with classical domain decomposition techniques, our proposal does not require any Schwarz iterative procedure. Numerical results illustrate a second-order convergence in both space and time for the described algorithm.

References:
[1] L. Portero, B. Bujanda, J.C. Jorge. A combined fractional step domain decomposition method for the numerical integration of parabolic problems. *Lecture Notes in Comput. Sci.* 3019 (2004), 1034–1041.
[2] L. Portero, J.C. Jorge. A generalization of Peaceman–Rachford fractional step method. *J. Comput. Appl. Math.* 189 (2006), 676–688.
[3] M. Shashkov. *Conservative finite-difference methods on general grids*. CRC Press, Boca Raton, 1996.

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# NKS for Fully Coupled Fluid-Structure Interaction Problems

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Abstract:

We study a parallel Newton-Krylov-Schwarz method for solving systems of nonlinear equations arising from the fully coupled, implicit finite element discretization of fluid-structure interaction problems on unstructured dynamic meshes. As expected the coupled system is considerably harder to solve than the individual fluid system or the solid system, but we show that the Schwarz preconditioner is capable of reducing the coupling effect and therefore guarantees the fast and scalable convergence of the Krylov subspace method. We focus on the simulations of blood flows in compliant arteries in 2D modeled by the coupled elastic wave equation and the incompressible Navier-Stokes equations. Numerical results obtained on parallel computers with hundreds of processors will be reported.

# UNIFORM PRECONDITIONING FOR GENERALIZED FINITE ELEMENT METHOD DISCREITATIONS AND ITS APPLICATION

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Abstract:

This poster is on the e_cient solution of linear systems arising in discretizations of second order elliptic PDEs by a generalized _nite element method (GFEM). The multigrid methods with line Gauss-Seidel smoothers apply for GFEM equations on uniform rectangular grids in 2 spatial dimension. We prove that the resulting multigrid methods converge uniformly. Our further results apply for GFEM equations on unstructured simplicial grids in 2 and 3 spatial dimensions. We propose an e_cient preconditioner by using auxiliary space techniques and an additive preconditioner for the auxiliary space problems. We also prove that the condition number of the preconditioned system is uniformly bounded with respect to the mesh parameters. These results have a potential application in the design of a multilevel preconditioner for the pure traction problem of linear elasticity.

# Efficient simulation of multi-body contact problems on complex geometries

**T. Dickopf, R. Krause**
Institute for Numerical Simulation, University of Bonn

Abstract:

In this talk we consider the numerical simulation of non-linear contact problems in elasticity on complex three-dimensional geometries. In the case of curvilinear contact boundaries and non-matching finite element meshes, particular emphasis has to be put on the discretization of the transmission of forces and the non-penetration conditions at the contact interface.

Expressing the discrete contact constraints in a weak sense by means of a non-conforming domain decomposition method allows for a proof of the optimality of the discretization error. However, the computation of the discrete transfer operator requires great effort in implementation and additional analysis.

We develop an efficient method to assemble the discrete coupling operator, which can be regarded as a global approximation of a contact mapping by a composition of local projections and inverse projections. The emerging non-linear system can be solved efficiently by a monotone multigrid method.

We illustrate the effectiveness of our approach by several numerical examples in 3D and consider a biomechanical application. More precisely we present a possible strategy for the examination of the biomechanics of the spine, which has been altered due to the implantation of an artificial disc.

# TaGas-Granular Flow Simulation by Parallel Computation

**Joseph Falcovitz***
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The Hebrew University of Jerusalem, Israel

**Eran Kot**
Department of Physics, Tel Aviv University, Israel

**David Sidilkover**
Propulsion Physics Division, Soreq NRC, Israel

Abstract:

We present a recent stage in an ongoing research effort aimed at achieve a realistic simulation of gas--granular {\em heterogeneous} flow, such as the process that takes place upon igniting a granular charge in a gun chamber. This methodology, named Lagrange-Euler-Gas-Granular-Simulation (LEGGS), is based on explicit gas-to-grain coupling (momentum and heat exchange), as well as grain-to-grain coupling (collisions). A physically sound and accurate description of gas-grain flows with combustion (or even particles-gas 2-phase combustion) can be obtained by LEGGS methodology in complex geometries. The subject of the current presentation is a parallel 3D gas dynamics code based on the GRP scheme.

At the beginning of this work we were faced with the usual dilemma -- how to combine an accurate representation of moving waves (including shocks) with the capability of treating complex geometries--. The configuration we adopted relies on a Cartesian grid, using cut-cells to accommodate contoured boundary surfaces. Test cases involving a shock diffraction past a sphere and a Laval nozzle flow are presented. The computational results are validated by comparison to experimental data or to analytical solutions. Plans for future development will be discussed.

# Computational Tool for a Mini-Windmill study with SOFT

**M. Garbey*, M. Smaoui, N. De Brye, and C. Picard**

Abstract:

In this paper we present a new concept leading to the completely automated conception of a Vertical Axis Wind Turbine (VAWT). Our tool, SOFT, goes through four steps: **S**imulation, **O**ptimum design, **F**abrication and **T**esting. It works as follows:

1. We do a computational fluid dynamic simulation of the fluid structure interaction flow problem. We derive from that computation the torque and average rotation speed for a given friction coefficient on the rotor shaft and average flow speed. Our objective function is to get the most power out of the windmill.
2. We optimize the shape of the blade wingspan based on a surface response or eventually a genetic algorithm. The objective function corresponds to a direct simulation of the Navier Stokes flow interacting with the rotating turbine, until a steady regime is reached. This simulation takes time and we distribute the computation of the evaluation function for each gene on a network of computers.

So far this procedure is fairly standard in optimum design of turbine or wing shape. The next two steps are more innovative:

3. From the result of the optimization procedure we get a supposedly optimum shape. This shape is sent to a three D printer that fabricates the turbine. This turbine is mounted on a standardize base that has an electric generator.
4. The windmill is tested in a mini wind tunnel. We measure the rotation speed and power output with an electronic tester. This information is analyzed by the computer system and compared to the simulation.
5. The system can then decide to refine the simulation or to restart the SOFT loop for a different class of design, depending on the number of blades, the number of stages in the turbine, the use of a stator to channel the flow etc…

This project has some obvious pedagogic components that can motivate undergraduate students to do science!

However, in reality, a critical step in the process is obviously the CFD method to test the VAWT performance: the numerical simulator should be robust, extremely fast but accurate enough to discriminate bad design from good design.

We will discuss an immersed boundary method and domain decomposition solver that we have tentatively developed to satisfy this ambitious program.

# Component-Averaged Domain Decomposition Techniques

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Abstract:

Component-Averaged Domain Decomposition (CADD) was introduced by Gordon and Gordon through the CARP algorithm and its CG acceleration CARP-CG. In CADD, external grid points of a subdomain are "cloned" (copied) into the subdomain, and the clones are updated by the subdomain solver together with the subdomain's internal points. The final values of all boundary points are taken as the average of their updated values and their clones in neighboring subdomains; this differs from standard DD methods. In CARP and CARP-CG, Kaczmarz row projections are performed in each subdomain, and then the results from the different subdomains are merged by the CADD method. CARP-CG is extremely robust and efficient on stiff elliptic PDE problems. It also produced excellent results in electron tomography, and it has shown a good potential for CFD problems with unstructured grids. The robustness of CARP-CG on grids of varying sizes indicates its potential usefulness for multilevel applications.

# Numerical study on a BDDC algorithm for mortar discretizations of elasticity problems

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Abstract:

A BDDC algorithm with the Neumann-Dirichlet preconditioner has been developed for elliptic problems and shown to be the most efficient for the problems with high contrast ratio of coefficients across sub interfaces.
The algorithm was extended to three dimensional elasticity problems with heterogeneous material parameters on geometrically non-conforming subdomain partitions.
Numerical experiments on the BDDC algorithm for the elasticity problems will be presented. The performance of the method regarding to discontinuous material parameters, to bad aspect ratio of interfaces, and to the selection of primal and non-primal interfaces will be investigated.

# A preconditioner for generalized saddle point problems with an indefinite block.

**Piotr Krzyzanowski**
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Abstract:

Recently there have been developed several preconditioning methods for systems of block structure:
A & B^T
B & 0
where A is symmetric and positive definite. Some of these methods used block preconditioners in order to apply the existing domain decomposition preconditioners in this context. We discuss the construction and properties of preconditioners for symmetric systems as above, but in the case when the A block is indefinite and possibly singular. An example of such a problem is the discrete time-harmonic Maxwell's equation with large wave number. We will show the conditions under which the block diagonal and block triangular preconditioners bulit up from any good preconditioner for the corresponding s.p.d. subproblems are optimal with respect to the mesh size. We shall also discuss similarities of this approach to the augmented Lagrangian technique.

# Lower bounds for eigenvalues of elliptic operators by overlapping domain decomposition

**Yuri Kuznetsov**
Professor, Department of Mathematics, University of Houston, USA

Abstract:

In this presentation, we consider a new approach to the estimation from below the lowest eigenvalues of symmetric positive definite elliptic operators. The approach is based on the overlapping domain decomposition procedure and on the replacement of subdomain operators by the special low rank perturbed scalar operators. The algorithm is illustrated by applications to the elliptic problems with mixed boundary conditions and strongly discontinuous coefficients.

# Parallel Interface Concentrated Finite Element Tearing and Interconnecting Methods

**Ulrich Langer[*1,2,3], Sven Beuchler[1,2], Clemens Pechstein[2]**
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Abstract:

This talk is devoted to the fast solution of Interface-Concentrated (IC) finite element equations. The IC finite element schemes are constructed on the basis of a non-overlapping domain decomposition where a conforming boundary concentrated finite element approximation is used in every subdomain. Similar to boundary element domain decomposition methods the total number of unknowns per subdomain behaves like $O((H/h)d-1)$, where $H$, $h$, and $d$ denote the usual scaling parameter of the subdomains, the average discretization parameter of the subdomain boundaries, and the spatial dimension, respectively. We propose and analyze tearing and interconnecting methods which asymptotically exhibit the same complexity as the number of unknowns up to a logarithmic factor. In particular, our IC Finite Element Tearing and Interconnecting (FETI) solver is highly parallel and robust with respect to large coefficient jumps. We present numerical results confirming the parallel efficiency and the robustness of our IC-FETI solver, and we compare the IC-FETI with the standard one-level FETI solvers.

The authors gratefully acknowledge the financial support of the FWF (Austrian Science Funds) Special Research Program SFB F013 on “Numerical and Sympolic Scientific Computing”.

# A Neumann-Neumann algorithm for mortar finite element discretization of fourth order problems

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Abstract:

The purpose of the talk is to present a Neumann-Neumann type of method for solving a system of linear equations arising from the discretization of a model fourth order boundary value problem with discontinuous coefficients on locally nonconforming meshes.

The coefficients can be discontinuous with possibly very large jumps across the boundaries of the subregions of the original domain.

The finite element discretization is done by triangulating independently each subdomain and then the final discretization is built by a mortar technique, i.e. local conforming finite element space are introduced in subdomains, and $L_2$ integral conditions on the boundaries of subregions are imposed.

Then we propose a Neumann-Neumann type method for solving the resulting system of linear equations. The coarse space problem is solved independently of the local problems and the convergence rate of the method is quasi-optimal i.e. the number of CG iterations grows polylogarithmically as the sizes of the meshes decrease.

# High Order One-Way Nesting in One and Two Dimensions.

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Abstract:

The problem of one-way nesting, in which global coarse-scale information is incorporated into a regional fine-scale solution, is considered. Carpenter's lateral boundary scheme (Quart. J. R. Met. Soc., vol. 108, 717--719, 1982) and its relation to Sommerfeld's absorbing boundary conditions is presented and analyzed in the context of the scalar wave equation. Carpenter's boundary scheme is then compared in one dimension with other possible boundary conditions and is shown to yield better results. Next, the shortcomings of Carpenter's lateral boundary scheme are demonstrated in the case of a two dimensional model problem and a new lateral boundary scheme, which is based on the Hagstrom-Warburton family of high-order absorbing boundary conditions (Wave Motion, vol. 39, 327--338, 2004), is presented. This lateral boundary scheme is then extended further and investigated using a two-dimensional model problem with wave-guide geometry.

# A Numerically Efficient Scheme for Elastic Immersed Boundaries

**F.Pacull [*](1) and M.Garbey (2)**
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Abstract:

This paper focuses on an efficient numerical scheme for immersed elastic boundary. The immersed bodies considered here have a fixed topology but can encompass large deformation. Our technique takes advantage of a ourier representation of the interface in the Immersed Boundary Method. We will present first an implicit temporal scheme that conserves mass by constrained optimization of the Fourier representation. We will discuss second a fast parallel implementation with domain decomposition in Matlab-MPI. Finally we will give some examples with the bubble test case as well as an innovative artificial motion process of cells in fluids.

# A dual iterative substructuring method with a penalty term

**Chang-Ock Lee and Eun-Hee Park***
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Abstract:

An iterative substructuring method with Lagrange multipliers is considered for the second order elliptic problem, which is a variant of the FETI-DP method. We propose an augmented Lagrangian functional by a penalty term with a positive parameter ή, which measures the jump across the interface. Then, a dual iterative substructuring method is induced from the saddlepoint problem associated with the augmented Lagrangian. Without any preconditioners, it is shown that the proposed method is numerically scalable in the sense that for a large !, the condition number of the resultant dual problem is bounded by a constant independent of the subdomain size and the mesh size. According to the numerical results, the presented method is superior to both of the FETI-DP method and the preconditioned FETI-DP by the Dirichlet preconditioner from many perspectives such as the conditioning of the dual system, the CG iteration number, and the virtual wall clock time.

# The Algebraic Optimized RAS Method

**Amik St-Cyr [1,*], Martin J. Gander[2]**
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[2] Universite de Geneve, Section de Mathématiques 2-4 rue du Lièvre, CP 64CH-1211 Genève

Abstract:

In this work it is shown that no a priori knowledge of the underlying PDE, represented by a matrix at the discrete level, is necessary in order to transform a restricted additive Schwarz (RAS) preconditioner to an optimized RAS (ORAS). A discovery algorithm automatically detecting the elliptic nature of the problem is proposed therein. It is also shown how to extract from the subdomain matrices the various metrics required to apply the optimized transmission conditions (originally discovered at the continuous level for various PDEs). A formula representing a first order approximation to the normal derivative is then applied to the subdomain matrices where connectivity was changed with respect to the global problem. Numerical experiments using elliptic problems discretized with Q1-FEM, P1-FEM, FDM and FVM demonstrate the effectiveness of the method and its algebraic nature.

# Dune: The Distributed and Unified Numerics Environment

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Abstract:

Dune is a set of libraries for grid-based numerical computations. Its main feature is the introduction of an abstract interface which separates grid implementations from the algorithms that use them. Applications are written for the interface instead of for a specific grid implementation. Hence grid implementations can be changed at any moment in algorithm development with minimal effort. Several such implementations are available. Some of them are grids specifically written for Dune, others encapsulate existing well-known finite element codes such as Alberta and UG. Due to the use of modern programming techniques the extra abstraction layer comes at very little additional cost.

# Convergence and Implementation of Robin Domain Decomposition Algorithm for contact Problems

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Abstract:

Contact problems take an important place in computational structural mechanics. Many numerical procedures have been proposed in the literature. They are based on standard numerical solvers for the solution of global problem in combination with a special implementation of the nonlinear contact conditions. The numerical treatment of such non classical contact problems leads to very large and ill-conditioned systems.
Domain decomposition methods are a good alternative to overcome this difficulties (see [2] for example).We propose a and study Robin domain decomposition algorithm for solving unilateral problems arising in contact mechanics [1]. We prove the convergence of this algorithm independently of the discretization step h. Numerical realizations which illustrate the performance of the proposed method will be discussed and results of some numerical examples will be shown.

[1] Ipopa, M., Sassi, T., Generalization of Lions' nonoverlapping domain decomposition method for contact problems, Lect. Notes comput. Sci. Eng., Srobl, 2008.
[2]* Krause, R. H., Wohlmuth, B., *A Dirichlet-Neumann type algorithm for contact problems with friction, Comput. Vis. Sci. 5 (2002), no. 3, 139--148.

# Parallelization of a constrained three-dimensional Maxwell solver

**Franck ASSOUS**
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**Jacques SEGRE***
CEA-Saclay, DEN/DM2S/SFME, France

**Eric SONNENDRUCKER**
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Abstract:

The numerical solution of very large three-dimensional electromagnetic field problems are challenging for various applications in the industry. In this presentation, we describe a nonoverlapping domain decomposition approach for solving the three-dimensional Maxwell equations on MIMD computers, based on a mixed variational formulation. It is especially well adapted for the resolution of the Vlasov-Maxwell equations, widely used to simulate complex devices like particle injectors or accelerators. This approach has the important property, that it leads to reuse without modification most of an existing sequential code. The continuity at the interfaces are imposed by duality using Lagrange multipliers. Hence, the resulting parallel algorithm requires only to add an external preconditioned Uzawa solver. We present the results of some numerical experiments on a parallel distributed memory machine that show the efficiency of the method in particular for very large (real-life) problems.

# Algebraic multigrid applied to the transonic small disturbances equation.

**Shlomy Shitrit*** **and David Sidilkover**
Propulsion Physics Division, Soreq NRC, Israel

Abstract:

Capability of computing efficiently compressible potential flow can still be of a significant practical value, for instance, in aerodynamic design problem. Moreover, such a capability becomes crucial in the context of the factorizable methods, which were introduced a few years ago.

The differential operator under consideration (the so-called full-potential operators) is of elliptic type in subsonic flow and its anisotropy becomes more significant as the flow velocity increases and approaches the speed of sound. In the case the flow turns supersonic, the type of the operator changes to hyperbolic.

The algebraic multigrid approach was shown to be very efficient for elliptic problems, both isotropic and anisotropic. Therefore, they would be the natural choice for the purpose of treating the subsonic/sonic flows. However, their extension to the supersonic flow regime appeared to be a difficult problem. First, we had to design a stable relaxation procedure that is efficient and fits the framework of the algebraic multigrid. We required such a relaxation to be
--pointwise (not a block relaxation of any kind);
-- direction free (not depending on any particular ordering of the points);
-- easily parallelizable.

The basic discretization we adopted is based on the original Murman & Cole idea. We shall describe the whole relaxation scheme together with our extension of the algebraic multigrid method to the supersonic flow regime.

We shall demonstrate the performance of the algebraic multigrid method based on the developed relaxation on a variety of test-cases concerning the variety of flow regimes starting from the low speed and up to supersonic.

# Numerical Solutions of an Interface Problem in a Heat Conduction Process Using Some Nonlinear Solvers

**Antony Siahaan[*1], Choi-Hong Lai[2], Koulis Pericleoous[3]**
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Abstract:

A Nonoverlapping Domain Decomposition based on the defect method is developed here. We test these methods on some nonlinear heat conduction processes taking place in a multi-chip module which has several geometrically structured subdomains. The arising interface problem is taken care of by imposing a defect equation along subdomain interfaces where the choice of defect equation will be significant in the solution accuracy. The enforcement of the defect equation leads the problem into a system of nonlinear equations which are then solved iteratively by means of some methods in the class of Quasi-Newton and Nonlinear Conjugate Gradient. These nonlinear solvers are developed such that the direct computation of Jacobian matrix can be avoided, thus giving a lighter computation. The performance of these nonlinear solvers will be compared and some issues concerning the development will be discussed.

# Compressible flow equations: decompositions, auxiliary variables and fast solvers

**David Sidilkover**
Propulsion Physics Division, Soreq NRC, Israel

Abstract:

Factorizable methods reflect the mixed character of the fluid flow equations and, therefore, allow to distinguish between the differential operators of different types present in the original system at the discrete level in the same way as at the PDE level. This property makes it possible to address each of the operators can be utilized to design highly efficient solvers by treating each of the operators (co-factors) in the most efficient way, thus leading to a highly efficient solver for the entire system.

The realization of this possibility relies on a certain judiciously chosen ``decomposition'' of the (locally linearized) originalsystem - a variable substitution, that ``separates'' the operators. This decomposition is used then to guide the design of a distributive relaxation . This variable substitution should be as fundamental and general as possible.

Our ``auxiliary variables of choice'' are the classical vorticity—stream function and potential. However, a simple analysis shows that this set of variables is still insufficient for our purpose: it provides no mechanism to drive to zero a certain part of momentum equations residual vector field. This difficulty can be removed by augmenting the classical set of the auxiliary variables by yet another vector, which was given a provisional name ``dual velocity''. This newly ``augmented'' set of variables leads also to some interesting formalism, which will be presented

# The solving of boundary value problems by domain decomposition method without intersection on rectangular quasistructured grids

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Abstract:

The variant of domain decomposition method for the solving of boundary value problems on rectangular quasistructured grids, which combine simplicity of rectangular grids with adaptive properties of quasistructured grids is offered. The boundary value problem is formulated as a problem of the solving of the operational equations containing a difference of normal derivatives on interface boundary of subdomains. The numerical algorithm contains three stages. On first of them on a grid, which determined on interface boundary, the system of the algebraic equations concerning values of required function in the grid nodes is constructed. For this purpose a series of auxiliary problems in subdomains is solved. The second stage of algorithm is the solving of the constructed system. At the third stage a final solution of the boundary value problem in subdomains is found. The suggested algorithm is a direct method of decomposition in the sense of it has no iterations on subdomains. Besides it is parallel and consequently is applicable for the solving of boundary value problems on multiprocessing supercomputers. Estimations of the algorithm efficient and the results of numerical experiments are given.

# New conditions for non-stagnation of GMRES, and corresponding convergence bounds

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Abstract:

A well-established condition guaranteeing that GMRES makes some progress, i.e., that it does not stagnate, is that the symmetric part of the coefficient matrix, $(A + A^T)/2$, be positive definite [Elman, 1982]. This condition results in a bound of the convergence rate for the iterative method which depends on the minimum eigenvalue of $(A + A^T)/2$ and of the norm of A. This bound is usually very pessimistic. Nevertheless, it has been extensively used by the DD community to show that preconditioned problems have a convergence bound for GMRES which is independent of the underlying mesh size of the discretized partial differential equation. In this talk we discuss new and more general conditions on the coefficient matrix so that one can guarantee that there is no stagnation of GMRES. These conditions do not require the symmetric part of the coefficient matrix to be positive definite. Thus, we enlarge the class of matrices for which a bound of the convergence rate for GMRES is available. We present several examples for which the new conditions are satisfied, while the Elman bound is not. It is hoped that the new bounds can be used to show mesh independence of preconditioners for which the Elman bound is not applicable, and similarly to encourage development of new optimal or nearly optimal preconditioners. (joint work with Valeria Simonicini, Universitá di Bologna)

# Risolv - Robust Iterative Solver

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Abstract:

Domain decomposition is a popular approach for designing preconditioning algorithm. In order to use it for solving large linear system Ax = b one need also an adequate accelerator. Gmres, for example, is such an algorithm which is used in cases where A is general, non Hermitian matrix. Unfortunately, it suffers from lack of robustness. The algorithm can exhibit very slow rate of convergence or complete stagnation. In this talk we would like to present a new accelerator, named Risolv, which overcomes this drawback. Whenever a theoretical polynomial (in the preconditioned matrix) algorithm can solve the linear system, so does Risolv. The algorithm is specially efficient in cases where there is a need to solve many systems which share the same matrix and differ by the right hand side vectors. In this case, implementing Risolv for most of the systems is almost free of inner-products. This feature can result in significant saving of cpu, specially in parallel computing

# A domain decomposition method for discontinuous Galerkin discretizations of Helmholtz problems with Lagrange multipliers

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Abstract:

A nonoverlapping domain decomposition method is described for Helmholtz problems discretized by a discontinuous Galerkin finite element method. The discretization uses plane wave basis functions and Lagrange multipliers to enforce a weak continuity of solution over element interfaces. A system of linear equations is formulated for the Lagrange multipliers on the subdomain interfaces. This poorly conditioned system is solved iteratively with a local preconditioner after it has been projected onto the complement of a coarse space in the same way as in the FETI-H method. Numerical experiments study the iterative solution of two-dimensional and three-dimensional model problems and compare the convergence and accuracy to the FETI-DPH method.

# AN EFFICIENT APPROACH FOR UPSCALING PROPERTIES OFCOMPOSITE MATERIALS WITH HIGH CONTRAST OF COEFFICIENTS

**R. EWING[1], O. ILIEV[2,6], R. LAZAROV[3], I. RYBAK[4], AND J. WILLEMS[5*]**

Abstract:

An efficient approach for calculating the effective heat conductivity for a class of industrial composite materials, such as metal foams, fibrous glass materials, and the like, is discussed. These materials, used in insulation or in advanced heat exchangers, are characterized by a low volume fraction of the highly conductive material (glass or metal) having a complex, network-like structure and by a large volume fraction of the insulator (air). We assume that the composite materials have constant macroscopic thermal conductivity tensors, which in principle can be obtained by standard up-scaling techniques, that use the concept of representative elementary volumes (REV), i.e. the effective heat conductivities of composite media can be computed by post-processing the solutions of some special cell problems for REVs. We propose, theoretically justify, and numerically study an efficient approach for calculating the effective conductivity for media for which the ratio δ of low and high conductivities satisfies δ << 1. We start from the known Domain Decomposition approaches for such problems, separating the subdomains in the way that each subdomain contain only one material. We show that for our purposes, one essentially only needs to solve the heat equation in the region occupied by the highly conductive media. For the considered class of problems, we show that under certain conditions on the microscale geometry, the proposed approach produces an upscaled conductivity that is O(δ) close to the exact upscaled permeability. A number of numerical experiments are presented in order to illustrate the accuracy and the limitations of the proposed method. Applicability of the presented approach to upscaling other similar problems, e.g. flow in fractured porous media, is also discussed.

Keywords: effective heat conductivity, permeability of fractured porous media, numerical upscaling, fibrous insulation materials, metal foams.

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