### **Preface**

This volume contains a selection of 41 refereed papers presented at the 18<sup>th</sup> International Conference of Domain Decomposition Methods hosted by the School of Computer Science and Engineering (CSE) of the Hebrew University of Jerusalem, Israel, January 12–17, 2008.

## 1 Background of the Conference Series

The International Conference on Domain Decomposition Methods has been held in twelve countries throughout Asia, Europe, the Middle East, and North America, beginning in Paris in 1987. Originally held annually, it is now spaced at roughly 18-month intervals. A complete list of past meetings appears below.

The principal technical content of the conference has always been mathematical, but the principal motivation has been to make efficient use of distributed memory computers for complex applications arising in science and engineering. The leading such computers, at the "petascale" characterized by  $10^{15}$  floating point operations per second of processing power and as many Bytes of application-addressable memory, now marshal more than 200,000 independent processor cores, and systems with many millions of cores are expected soon. There is essentially no alternative to domain decomposition as a stratagem for parallelization at such scales. Contributions from mathematicians, computer scientists, engineers, and scientists are together necessary in addressing the challenge of scale, and all are important to this conference.

Though the conference has grown up in the wake of commercial massively parallel processors, it must be remarked that some important applications of domain decomposition are not massively parallel at all. "Gluing together" just two subproblems to effectively exploit a different solver on each is also part of the technical fabric of the conference. Even as multiprocessing becomes commonplace, multiphysics modeling is in ascendancy, so the International Conference on Domain Decomposition Methods remains as relevant and as fundamentally interdisciplinary as ever.

The conference typically draws between 100 and 200 researchers concerned with the large-scale computational solution of PDEs in areas such as fluid dynamics, structural mechanics, biomechanics, geophysics, plasma physics, radiation transport, electricity and magnetism, flows in porous media, and the like. The conference is led by the International Scientific Committee of DDM.ORG under a set of by-laws that appear at the website www.ddm.org.

While research in domain decomposition methods is presented at numerous venues, the International Conference on Domain Decomposition Methods is the only regularly occurring international forum dedicated to interdisciplinary technical interactions between theoreticians and practitioners working in the creation, analysis, software implementation, and application of domain decomposition methods.

International Conferences on Domain Decomposition Methods:

- Paris, France, 1987
- Los Angeles, USA, 1988
- Houston, USA, 1989
- Moscow, USSR, 1990
- Norfolk, USA, 1991
- Como, Italy, 1992
- University Park (Pennsylvania), USA, 1993
- Beijing, China, 1995
- Ullensvang, Norway, 1996
- Boulder, USA, 1997
- Greenwich, UK, 1998
- Chiba, Japan, 1999
- Lyon, France, 2000
- Cocoyoc, Mexico, 2002
- Berlin, Germany, 2003
- New York City, USA, 2005
- St. Wolfgang/Strobl, Austria, 2006
- Jerusalem, Israel, 2008

International Scientific Committee on Domain Decomposition Methods:

- Petter Bjørstad, Bergen
- Martin Gander, Geneva
- Roland Glowinski, Houston
- Laurence Halpern, Paris
- Ronald Hoppe, Augsburg and Houston
- Hideo Kawarada, Chiba
- David Keyes, New York
- Ralf Kornhuber, Berlin
- Yuri Kuznetsov, Houston and Moscow
- Ulrich Langer, Linz
- Jacques Périaux, Paris
- Alfio Quarteroni, Lausanne

- Zhong-ci Shi, Beijing
- Olof Widlund, New York
- Jinchao Xu, University Park

#### 2 About the Eighteenth Conference

The eighteenth conference was chaired by Michel Bercovier, Bertold Badler Chair of Scientific Computation at the School of Computer Science and Engineering, and held on the Edmond J. Safra Campus of the Hebrew University, at Givat Ram, Jerusalem. 107 scientists from 15 countries attended. The conference included 12 invited plenary lectures, 22 talks given in five Minisymposia, 30 contributed talks, and two special sessions: one dedicated to the memory of Moshe Israeli, who should have been on the organizing committee, and a special collection of ten talks, organized as Minisymposium 5, given as an "Historical Perspective to Milestones in the Development of Domain Decomposition." Conference details remain on the conference web site http://www.cs.huji.ac.il/dd18.

The twelve invited talks were:

- Achi Brandt, Weizmann Institute of Science and University of California at Los Angeles), Principles of Systematic Upscaling
- Michael J. Holst, University of California, San Diego: *Analysis and Convergent Adaptive Solution of the Einstein Constraint Equations*
- Ronald W. Hoppe, University of Houston and University of Augsburg: Adaptive Multilevel Primal-Dual Interior-Point Methods in PDE Constrained Optimization
- Claude Le Bris, Ecole Nationale des Ponts et Chaussées: Domain Decomposition and Electronic Structure Calculations: a New Approach
- Patrick Le Tallec, Ecole Polytechnique: From Domain Decomposition to Homogenization in the Numerical Modelling of Materials
- Jan Martin Nordbotten, University of Bergen and Princeton University: Variational Scale Separation Methods
- Ilaria Perugia, University of Pavia: Plane Wave Discontinuous Galerkin Methods
- Olivier Pironneau, University of Paris-VI: Numerical Zoom for Multi-Scale Problems
- Francois-Xavier Roux, ONERA and University of Paris-VI: Domain Decomposition methods: Industrial Experience at Hutchinson
- Xuemin Tu, University of California at Berkeley: *Balancing Domain Decomposition Methods by Constraints (BDDC)*
- Olof B. Widlund, Courant Institute, New York University: Accommodating Irregular Subdomains in Domain Decomposition Theory
- Jinchao Xu, Pennsylvania State University: Robust Iterative Methods for Singular and Nearly Singular System of Equations

The papers in Part I of these proceedings are ordered alphabetically according to the names of the plenary speakers.

The "Milestone" lectures were:

- Olof Widlund, Courant Institute, New York University: Coarse Space Components of Domain Decomposition Algorithms
- Petter Bjørstad, University of Bergen: To Overlap or not to Overlap
- Roland Glowinski, University of Houston: On Fictitious Domain Methods
- Jinchao Xu, Pennsylvania State University: On the Method of Subspace Corrections
- Alfio Quarteroni, Ecole Polytechnique Fédérale de Lausanne: Heterogeneous Domain Decomposition
- David Keyes, Columbia University: Domain Decomposition and High Performance Computing
- François-Xavier Roux: The FETI Method
- Frédéric Nataf, Ecole Polytechnique: Optimized Schwarz Methods
- Xiao-Chuan Cai, University of Colorado and Boulder: Domain Decomposition Methods for Nonlinear Problems
- Laurence Halpern, University of Paris 13: Space-Time Parallel Methods

These lectures were taped and will remain available at http://www.cs.huji.ac.il/dd18/video.

The papers in Part II of these proceedings are ordered according to the order of the five minisymposia, and inside each such group according to the names of the speakers. Part III is organized similarly.

The session dedicated to the memory of Moshe Israeli (1940-2007) included three lectures:

- Amir Averbuch, Tel Aviv University: Contributions of Prof. Moshe Israeli to Scientific Computing
- Irad Yavneh, Technion: Automated Transformations of PDE Systems
- Roland Glowinski, University of Houston: Clustering Phenomena for Particulate Flow in Spinning Cylinders

The Local Organizing Committee Members were:

- Michel Bercovier (Chairman), Hebrew University of Jerusalem
- Amir Averbuch, Tel Aviv University
- Pinhas Z. Bar-Yoseph (IACMM representative), Technion
- Matania Ben-Artzi, Hebrew University of Jerusalem
- Michael S. Engelman, Corporate VP, ANSYS
- Dan Givoli, Technion
- Raz Kupferman, Hebrew University of Jerusalem
- Zohar Yosibash, Ben Gurion University

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#### 3 About Domain Decomposition Methods

Domain decomposition, a form of divide-and-conquer for mathematical problems posed over a physical domain, as in partial differential equations, is the most common paradigm for large-scale simulation on massively parallel distributed, hierarchical memory computers. In domain decomposition, a large problem is reduced to a collection of smaller problems, each of which is easier to solve computationally than the undecomposed problem, and most or all of which can be solved independently and concurrently. Typically, it is necessary to iterate over the collection of smaller problems, and much of the theoretical interest in domain decomposition algorithms lies in ensuring that the number of iterations required is very small. Indeed, the best domain decomposition methods share with their cousins, multigrid methods, the property that the total computational work is linearly proportional to the size of the input data, or that the number of iterations required is at most logarithmic in the number of degrees of freedom of individual subdomains.

Algorithms whose work requirements are linear or log-linear in the size of the input data in this context are said to be "optimal." Near optimal domain decomposition algorithms are now known for many, but certainly not all, important classes of problems that arise science and engineering. Much of the contemporary interest in domain decomposition algorithms lies in extending the classes of problems for which optimal algorithms are known.

Domain decomposition algorithms can be tailored to the properties of the physical system as reflected in the mathematical operators, to the number of processors available, and even to specific architectural parameters, such as cache size and the ratio of memory bandwidth to floating point processing rate.

Domain decomposition has proved to be an ideal paradigm not only for execution on advanced architecture computers, but also for the development of reusable, portable software. The most complex operation in a typical domain decomposition method — the application of the preconditioner — carries out in each subdomain steps nearly identical to those required to apply a conventional preconditioner to the global domain. Hence software developed for the global problem can readily be adapted to the local problem, instantly presenting lots of "legacy" scientific code for

to be harvested for parallel implementations. Furthermore, since the majority of data sharing between subdomains in domain decomposition codes occurs in two archetypal communication operations — ghost point updates in overlapping zones between neighboring subdomains, and global reduction operations, as in forming an inner product — domain decomposition methods map readily onto optimized, standardized message-passing environments, such as MPI.

Finally, it should be noted that domain decomposition is often a natural paradigm for the modeling community. Physical systems are often decomposed into two or more contiguous subdomains based on phenomenological considerations, such as the importance or negligibility of viscosity or reactivity, or any other feature, and the subdomains are discretized accordingly, as independent tasks. This physically-based domain decomposition may be mirrored in the software engineering of the corresponding code, and leads to threads of execution that operate on contiguous subdomain blocks. These can be either further subdivided or aggregated to fit the granularity of an available parallel computer.

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# **Contents**

Part I Plenary Presentations	
A Domain Decomposition Approach for Calculating the Graph Corresponding to a Fibrous Geometry Achi Brandt, Oleg Iliev, Joerg Willems	3
Adaptive Multilevel Interior-Point Methods in PDE Constrained Optimization	
Harbir Antil, Ronald H.W. Hoppe, Christopher Linsenmann	15
Numerical Homogeneisation Technique with Domain Decomposition Based a-posteriori Error Estimates	27
Patrick Le Tallec	21
Multiscale Methods for Multiphase Flow in Porous Media  Jan M. Nordbotten	39
Mixed Plane Wave Discontinuous Galerkin Methods Ralf Hiptmair, Ilaria Perugia	51
Numerical Zoom and the Schwarz Algorithm Frédéric Hecht, Alexei Lozinski, Olivier Pironneau	63
BDDC for Nonsymmetric Positive Definite and Symmetric Indefinite Problems	
Xuemin Tu, Jing Li	75
Accomodating Irregular Subdomains in Domain Decomposition Theory  Olof B. Widlund	87
Auxiliary Space Preconditioners for Mixed Finite Element Methods	
Ray S. Tuminaro, Jinchao Xu, Yunrong Zhu	99

Part II Minisymposia					
A Multilevel Domain Decomposition Solver Suited to Nonsmooth Mechanical Problems					
Damien Iceta, Pierre Alart, David Dureisseix					
A FETI-2LM Method for Non-Matching Grids François-Xavier Roux					
Truncated Nonsmooth Newton Multigrid Methods for Convex Minimization Problems					
Carsten Gräser, Uli Sack, Oliver Sander					
A Recursive Trust-Region Method for Non-Convex Constrained Minimization					
Christian Groß, Rolf Krause					
A Robin Domain Decomposition Algorithm for Contact Problems: Convergence Results					
Mohamed Ipopa, Taoufik Sassi					
Patch Smoothers for Saddle Point Problems with Applications to PDE-Constrained Optimization Problems					
René Simon, Walter Zulehner					
A Domain Decomposition Preconditioner of Neumann-Neumann Type for the Stokes Equations					
Vitorita Dolean, Frédéric Nataf, Gerd Rapin					
Non-overlapping Domain Decomposition for the Richards Equation via Superposition Operators					
Heiko Berninger					
Convergence Behavior of a Two-Level Optimized Schwarz Preconditioner Olivier Dubois, Martin J. Gander					
An Algorithm for Non-Matching Grid Projections with Linear Complexity					
Martin J. Gander, Caroline Japhet					
A Maximum Principle for $L^2$ -Trace Norms with an Application to Optimized Schwarz Methods					
Sébastien Loisel, Daniel B. Szyld					
An Extended Mathematical Framework for Barrier Methods in Function Space					
Anton Schiela					

Optimized Schwarz Preconditioning for SEM Based Magnetohydrodynamics Amik St-Cyr, Duane Rosenberg, Sang Dong Kim
Nonlinear Overlapping Domain Decomposition Methods  Xiao-Chuan Cai
Optimized Schwarz Waveform Relaxation: Roots, Blossoms and Fruits  Laurence Halpern
Optimized Schwarz Methods F. Nataf
The Development of Coarse Spaces for Domain Decomposition Algorithms
<i>Olof B. Widlund</i>
Part III Contributed Presentations
<b>Distributed Decomposition Over Hyperspherical Domains</b> <i>Aron Ahmadia, David Keyes, David Melville, Alan Rosenbluth, Kehan Tian</i> 251
Domain Decomposition Preconditioning for Discontinuous Galerkin Approximations of Convection-Diffusion Problems Paola F. Antonietti, Endre Süli
Linearly Implicit Domain Decomposition Methods for Nonlinear Time-Dependent Reaction-Diffusion Problems  A. Arrarás, L. Portero, J.C. Jorge
NKS for Fully Coupled Fluid-Structure Interaction with Application  Andrew T. Barker, Xiao-Chuan Cai
Weak Information Transfer between Non-Matching Warped Interfaces  Thomas Dickopf, Rolf Krause
Computational Tool for a Mini-Windmill Study with SOFT  M. Garbey, M. Smaoui, N. De Brye,, C. Picard
On Preconditioners for Generalized Saddle Point Problems with an Indefinite Block
Piotr Krzyżanowski
Lower Bounds for Eigenvalues of Elliptic Operators by Overlapping  Domain Decomposition  Yuri A. Kuznetsov

# XVI Contents

From the Boundary Element Domain Decomposition Methods to Local Trefftz Finite Element Methods on Polyhedral Meshes  Dylan Copeland, Ulrich Langer, David Pusch
An Additive Neumann-Neumann Method for Mortar Finite Element for 4th Order Problems  Leszek Marcinkowski
A Numerically Efficient Scheme for Elastic Immersed Boundaries F. Pacull, M. Garbey
A Domain Decomposition Method Based on Augmented Lagrangian with a Penalty Term  Chang-Ock Lee, Eun-Hee Park
Parallelization of a Constrained Three-Dimensional Maxwell Solver F. Assous, J. Segré, E. Sonnendrücker
A Discovery Algorithm for the Algebraic Construction of Optimized Schwarz Preconditioners  Amik St-Cyr, Martin J. Gander
On the Convergence of Optimized Schwarz Methods by way of Matrix Analysis  Sébastien Loisel, Daniel B. Szyld

**Plenary Presentations**