Numerical Method for Antenna Radiation Problem by FDTD Method with PML

Takashi Kako[1] and Yoshiharu Ohi[2]

[1] The University of Electro-Communications, Department of Computer Science, Chofu, Tokyo 182-8585, Japan, kako@im.uec.ac.jp
[2] The University of Electro-Communications, Department of Computer Science, Chofu, Tokyo 182-8585, Japan, ohi@sazae.im.uec.ac.jp

Summary. In the numerical simulation of electromagnetic wave radiation from an antenna, the antenna is assumed to be a perfectly conducting obstacle. It was shown numerically that the antenna can be effectively modeled by a highly conducting region occupied by it. The Finite Difference Time Domain method combined with Perfectly Matched Layer gives a flexible numerical methodology for this problem. We apply the method to analyze several radiation problems with different types of antennas such as a birdcage and the Yagi types where the delta gap type power supply model is adopted. For treating an unbounded outer region numerically, we apply a newly developed technique to discretize the PML region with little artificial reflection. Theoretical justification of this procedure for a 1D case was presented in DD17, and effectiveness of this technique was also demonstrated numerically for 2D and 3D cases. We observe a good 3D numerical performance of the method and confirm its usefulness though theoretical justification remains as a future problem.

1 FDTD Method and PML

In this paper, we consider a numerical method for electromagnetic wave propagation in an unbounded domain. The standard numerical method for computing an electromagnetic wave is the FDTD (Finite Difference Time Domain) method introduced by [6]. To solve the problem in the unbounded domain, one must truncate the outer unbounded domain appropriately. For this purpose the PML (Perfectly Matched Layer) which was firstly introduced by [1] is popularly used, where one introduces an artificial magnetic conductivity σ^* in this region. When we discretize the equations in the PML, some artificial reflections are observed in a solution by the original scheme of Berenger. We firstly review a new discretization scheme with fewer reflection introduced by the present authors, [3] for a 1D problem and also applied for 2D and 3D problems. This scheme applied to the 1D problem does not cause any artificial reflection at least in the constant σ^* region. Although we have not proved mathematically the non-existence of the artificial reflection for 2D and 3D cases, we have succeeded in validating the method for these cases numerically.

Secondly, we develop a 3D numerical method to simulate propagation of an RF (Radio-Frequency) wave emitted by various antennas such as the Yagi antennas and birdcage coil antennas used for an MRI (Magnetic Resonance Imaging) device.

In the FDTD method, a finite difference method with a space-time staggered mesh is used for discretization of the Maxwell equation. To solve the problem in an unbounded region, we employ the PML and introduce an artificial absorption term σ^* in the equation to attenuate the wave there. In order to make the computational domain finite we impose a perfectly reflecting boundary condition on the outermost boundary of the PML. The additional boundary condition may introduce a further extra artificial reflection, but the reflection is supposed to be controllable within a negligible level in most applications. The Maxwell equation in non-PML region is written as

$$\frac{\partial}{\partial t}E(t,x) = -\frac{\sigma(x)}{\epsilon}E(t,x) + \frac{1}{\epsilon}\nabla \times H(t,x), \tag{1}$$

$$\frac{\partial}{\partial t}H(t,x) = -\frac{1}{\mu}\nabla \times E(t,x), \tag{2}$$

and in the PML region as

$$\frac{\partial}{\partial t}E(t,x) = -\frac{\sigma(x)}{\epsilon}E(t,x) + \frac{1}{\epsilon}\nabla \times H(t,x), \tag{3}$$

$$\frac{\partial}{\partial t}H(t,x) = -\frac{\sigma^*(x)}{\mu}H(t,x) - \frac{1}{\mu}\nabla \times E(t,x), \tag{4}$$

with $E = (E_x, E_y, E_z)$ the electric field, $H = (H_x, H_y, H_z)$ the magnetic field, ϵ the electric permittivity, μ the magnetic permeability, σ the electric conductivity and σ^* the artificial magnetic conductivity. In the followings, we assume without loss of generality, that $\epsilon = \mu = 1$, and also impose an impedance matching condition, $\sigma^* = \sigma$.

By introducing σ^*, the solution in the non-PML region does not change numerically in 1D, 2D and 3D cases, and we can prove it theoretically in 1D case (see [3]).

In both PML and non-PML regions, in accordance with the idea of [1], we split the variables E and H into two components as $E_x = E_{xy} + E_{xz}$, $H_x = H_{xy} + H_{xz}$ and so on. By using these variables the Maxwell equation is expressed as

$$\frac{\partial}{\partial t}E_{xy}(t,x) = -\sigma_y(x)E_{xy}(t,x) + \frac{\partial(H_{zx}(t,x) + H_{zy}(t,x))}{\partial y}, \tag{5}$$

$$\frac{\partial}{\partial t}E_{xz}(t,x) = -\sigma_z(x)E_{xz}(t,x) - \frac{\partial(H_{yz}(t,x) + H_{yx}(t,x))}{\partial z}, \tag{6}$$

and similar equations derived by permutating x, y, z cyclically and changing the roles of E and H. The formulation for one dimensional case is seen in [3].

New FDTD discretization scheme for the 3D equations is

$$E_{xy}^{n+1}(i+\frac{1}{2},j,k) = A_{xy}E_{xy}^{n}(i+\frac{1}{2},j,k) + B_{xy}\{H_{zx}^{n+\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k) + H_{zy}^{n+\frac{1}{2}}(i+\frac{1}{2},j+\frac{1}{2},k) - H_{zx}^{n+\frac{1}{2}}(i+\frac{1}{2},j-\frac{1}{2},k) - H_{zy}^{n+\frac{1}{2}}(i+\frac{1}{2},j-\frac{1}{2},k)\}, \quad (7)$$

with the coefficients,

$$A_{xy} = e^{-\sigma_y(j)\Delta t} \quad \text{and} \quad B_{xy} = \frac{\Delta t}{\Delta y}e^{-\sigma_y(j)\Delta t/2}, \quad (8)$$

and so on. In the previous standard scheme by Berenger, the corresponding coefficients are

$$A_{xy} = e^{-\sigma_y(j)\Delta t}, \quad \text{and} \quad B_{xy} = \frac{1-e^{\sigma_y(j)\Delta t}}{\sigma_y(j)\Delta y}. \quad (9)$$

There is another simplified scheme where the coefficients are given as

$$A_{xy} = \frac{1-\frac{\sigma_y(j)\Delta t}{2}}{1+\frac{\sigma_y(j)\Delta t}{2}} \quad \text{and} \quad B_{xy} = \frac{\Delta t}{\Delta y(1+\frac{\sigma_y(j)\Delta t}{2})}. \quad (10)$$

We show comparison of performance among these schemes by numerical examples in Fig. 1. It can be concluded that our new scheme is superior to others.

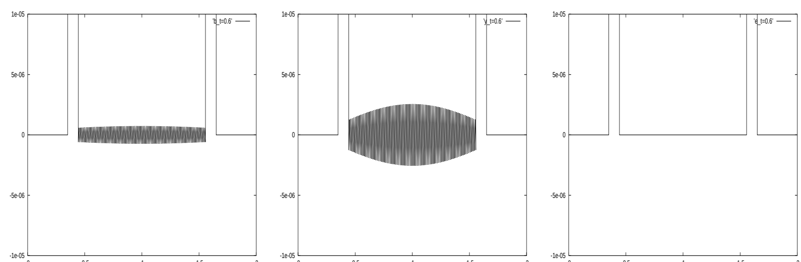


Fig. 1. Comparison of reflection waves at $t = 0.6$ by Berenger's scheme (*left*), simplified scheme (*middle*) and new scheme (*right*).

To check the validity of our method in 3D case, we show a time evolution of the absolute value of the Poynting's vector at an observation point for an initial value problem with a delta function like initial profile. Figure 2 shows that the artificial reflection from the PML region is negligible, although we observe some small reflection wave from the PML region as well as from the outermost boundary.

2 Basic Formulation of Antenna Problem

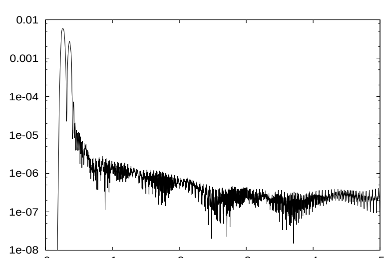
Among variety of electromagnetic radiation and scattering problems the antenna problem is a special case where a scatterer or an obstacle is a low dimensional singular object.

The antenna is usually modeled as a perfectly conducting obstacle, which constitutes a lower dimensional region in the computational domain such as a flat plate or a parabola panel as 2D region and a line or an array of lines as 1D region.

To make the numerical simulation, we need to calculate the electric current density profile on the line antennas. For this purpose, Pocklington's integral equation for electric current is already known in the case of a straight line antenna, for which the standard numerical methodology is the moment method. There are, however, several demerits of the method, i.e., it is effective only for the time harmonic problem and is not so easy to extend it to more general antenna configurations.

On the other hand, the new method developed by us is free from these demerits as our methodology is based on the FDTD method combined with the PML for solving time dependent problems, and treats the antenna as an electrically highly conductive region, and the current density on the antenna can be calculated afterwards if necessary.

A typical example of an array of line antennas is the Yagi antenna consisting of a power supplier, reflectors and guiders (see [2]). In Fig. 3, the energy density profiles of the wave on x–y plane and x–z plane are shown. The computational region is approximately $2.2 \times 2.2 \times 2.2$ with the PML having the thickness of $6h = 6 \times 2^{-6} \approx 0.1$ with the mesh size $h = 2^{-6}$. The antenna lengths of the supplier, the reflector and two guiders are $29h$, $31h$, $29h$ and $27h$, respectively. At the midpoint of the supplier, we assume a delta gap power supply, i.e., an external source which supplies a time harmonic electric field $\sin(2\pi f t)/h$ with frequency $f = 1.0$ on one mesh point. The spatial mesh size h is $h = 2^{-6} = 1/64$ as stated before and the temporal mesh size τ is $\tau = h/2 = 2^{-7} = 1/128$. Figure 3 shows four spatial profiles of electromagnetic energy density of the radiating wave from a Yagi antenna at time $t = t_0$, $t_0 + (1/4)f$, $t_0 + (1/2)f$ and $t_0 + (3/4)f$ with sufficiently large $t_0 = 5.0 \gg f = 1.0$. One of the most interesting and important problems is to arrange the components of the line antennas so that the best performance of electromagnetic wave radiation to the desirable direction is attained. We leave this problem to be solve in our future study.



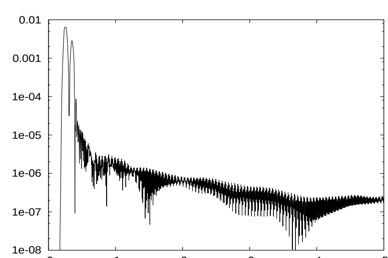


Fig. 2. Time evolution of the absolute value of Poynting's vector for an initial value problem.

3 Application to MRI Problem

As an application of our method, we show an example of the electromagnetic radiation and scattering problems appearing in MRI (Magnetic Resonance Imaging) which is an imaging technology based on NMR (Nuclear Magnetic Resonance). There are many researches on human susceptibility related to a mobile phone (see for example, [4, 5]) because use of electromagnetic wave of radio frequency range is considered to have some unfavorable heating effect on the human body. Though the calculation of SAR (Specific Absorption Rate) is important for this purpose, only a few studies have been carried out on this problem up to now concerning MRI. Hence it is challenging to develop a methodology for the estimation of SAR concerning MRI.

For this purpose we first simulate numerically the propagation of the electromagnetic wave excited by a birdcage coil in MRI device by the FDTD method with the PML. Then by putting a phantom of a human body inside the birdcage coil we estimate SAR in the phantom, by which the possible change of SAR under different coil configuration can be studied.

In Fig. 4 we show examples of numerical simulation on heating of a phantom in MRI with birdcage antennas. The size of computational domain is $2.2 \times 2.2 \times 2.2\,\mathrm{m}^3$ and the thickness of PML is 0.1 m, and the frequency of power supply is 64 MHz. In this example, some specific parts of the phantom body are heated more in comparison with other parts. For example, we observed several typical phenomena, i.e., SAR becomes higher at the positions nearest to the coil as a head and a waist, and also at edges of the body, especially at the edges of convex shape as a head, a shoulder and

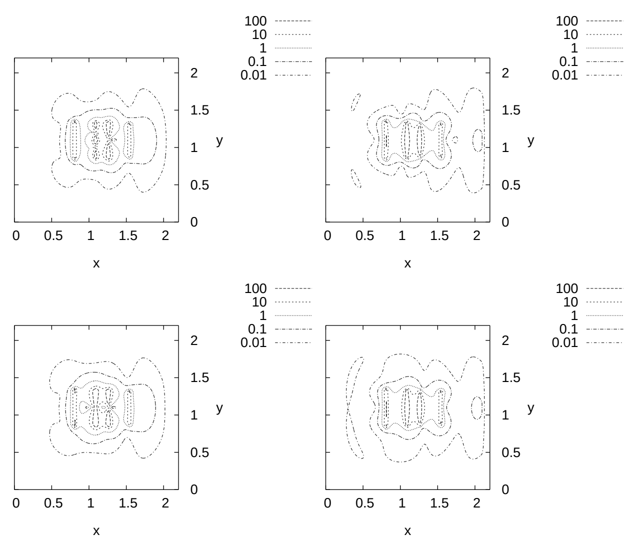


Fig. 3. Examples of numerical simulation on spatial profiles of electromagnetic energy density for a Yagi antennas at different times.

a waist. Naturally, we see that the SAR decreases with increasing the coil dimension (length and radius) and increasing the distance between the body and the coil. The detailed analysis including the optimization of the antenna coil configurations and others based on this methodology is our future problem.

4 Summary and Future Problems

We now summarize our study as follows:

(i) We tested the efficiency of our methodology through a basic antenna configuration such as the Yagi antennas.
(ii) We applied our new scheme to the 3D MRI problem with a source birdcage coil antenna, and computed SAR for a phantom body inside the coil.

Future problems are

(i) the optimal design of various line antennas by using appropriate optimization algorithm (such as gradient method and/or GA);
(ii) the investigation of possible variation of SAR when we increase or decrease the number of leading wires connecting two circular wires of birdcage coil;
(iii) the study of the effect of static background magnetic field configuration as well as the effect of the way of impressing a source voltage through background electric circuit;
(iv) the usage of more realistic CAD models of a human body in its geometric shape and with physical and/or physiological parameters.

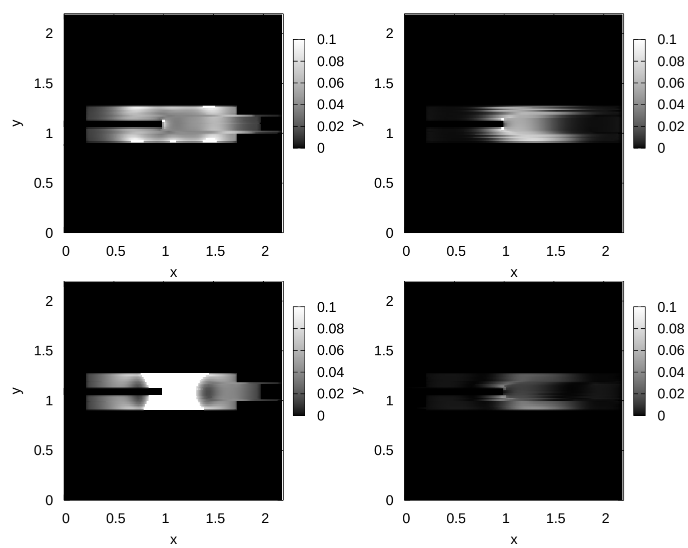


Fig. 4. Examples of numerical simulation on heating of a phantom in MRI with bird cage antennas.

References

1. J.-P. Berenger. A perfectly matched layer for the absorption of electromagnetic waves. *J. Comput. Phys.*, 114(2): 185–200, 1994.
2. E. Jones and W.T. Joines. Design of Yagi-Uda antennas using genetic algorithms. *IEEE Trans. Antennas Propag.*, 45(9): 1386–1392, 1997.
3. T. Kako and Y. Ohi. Numerical method for wave propagation problem by FDTD method with PML. In *Domain Decomposition Methods in Science and Engineering XVII, volume 60 of Lecture Notes in Computational Science and Engineering*, 551–558, 2007.
4. K. Sugimura. *Principle of MRI and Imaging Methodology*. Medical view Co. Ltd, 2000.
5. T. Uno. *Electromagnetic Wave and Antenna Analysis by FDTD Method*. Corona Publishing Co. Ltd, Tokyo, 1998.
6. K.S. Yee. Numerical solution of initial boundary value problems involving Maxwell's equation in isotropic media. *IEEE Trans. Antennas Propag.*, 14(3): 302–307, 1966.