A Newton Based Fluid–Structure Interaction Solver with Algebraic Multigrid Methods on Hybrid Meshes

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Summary. Fluid–structure interaction problems arise in many application fields such as flows around elastic structures or blood flow problems in arteries. One method for solving such a problem is based on a reduction to an equation on the interface, involving the so-called Steklov–Poincaré operators. This interface equation is solved by a Newton iteration for which directional derivatives with respect to the interface perturbation have to be evaluated appropriately. One step of the Newton iteration requires the solution of several decoupled linear sub-problems in the structure and the fluid domains. These sub-problems are spatially discretized by a finite element method on hybrid meshes containing different types of elements. For the time discretization implicit first-order methods are used for both sub-problems. The discretized equations are solved by algebraic multigrid methods.

1 Problem Setting of the Fluid–Structure Interaction

1.1 Geometrical Description

Let Ω_0 denote the initial domain at time $t = 0$ consisting of the structure and the fluid domains Ω_0^{s} and Ω_0^{f}, respectively. The domain $\Omega(t)$ at time t is composed of the deformable structure domain $\Omega^{\mathrm{s}}(t)$ and the fluid domain $\Omega^{\mathrm{f}}(t)$. The corresponding interface $\Gamma(t)$ is evolving from the initial interface Γ_0.

The evolution of $\Omega(t)$ is obtained by two families of mappings:

$$\mathcal{L}_t : \Omega_0^{\mathrm{s}} \times R^+ \to \Omega^{\mathrm{s}}(t) \quad \text{and} \quad \mathcal{A}_t : \Omega_0^{\mathrm{f}} \times R^+ \to \Omega^{\mathrm{f}}(t).$$

The maps $\mathcal{L}_t = \mathcal{L}(\cdot, t)$ and $\mathcal{A}_t = \mathcal{A}(\cdot, t)$ track the structure and the fluid domains in time (see Fig. 1 for an illustration). They satisfy the continuity condition of the velocity on the interface $\Gamma(t)$, i.e.

$$\mathcal{L}_t = \mathcal{A}_t \quad \text{on} \quad \Gamma(t). \tag{1}$$

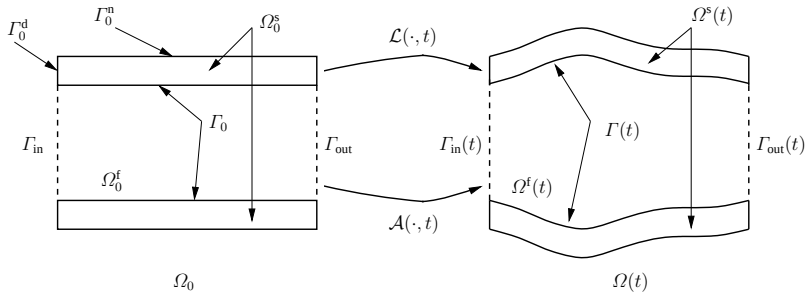


Fig. 1. Two families of mappings.

The structure problem is described in a Lagrangian framework. Therefore, the position of a point $x_0 \in \Omega_0^{\mathrm{s}}$ at time t is given by

$$x(x_0,t) \equiv \mathcal{L}(x_0,t) = x_0 + d^{\mathrm{s}}(x_0,t),$$

where $d^{\mathrm{s}}(x_0,t)$ denotes the displacement $d^{\mathrm{s}}(x_0,t)$ of the structure domain.

Correspondingly, the position of any point $x_0 \in \Omega_0^{\mathrm{f}}$ at time t is given by

$$x(x_0,t) \equiv \mathcal{A}(x_0,t) = x_0 + d^{\mathrm{f}}(x_0,t),$$

where $d^{\mathrm{f}}(x_0,t)$ denotes the displacement of the fluid domain. The fluid problem is stated in an Arbitrary-Lagrangian-Eulerian (ALE) framework. Using the continuity condition (1), $d^{\mathrm{f}}(x_0,t)$ is determined by an arbitrary extension of its value on the interface $d^{\mathrm{f}} = \mathrm{Ext}(d^{\mathrm{s}}|_{\Gamma_0})$, e.g. the harmonic extension:

$$-\Delta d^{\mathrm{f}} = 0 \text{ in } \Omega_0^{\mathrm{f}},\ d^{\mathrm{f}} = d^{\mathrm{s}} \text{ on } \Gamma_0, \text{ and } d^{\mathrm{f}} = 0 \text{ on } \Gamma_{\mathrm{in}}(t) \cup \Gamma_{\mathrm{out}}(t). \tag{2}$$

Furthermore, we introduce the domain velocities by

$$w^{\mathrm{s}}(x_0,t) := \frac{\partial d^{\mathrm{s}}}{\partial t}(x_0,t) \quad \text{and} \quad \hat{w}^{\mathrm{f}}(x_0,t) := \frac{\partial d^{\mathrm{f}}}{\partial t}(x_0,t)$$

for the structure and the fluid domains, respectively.

1.2 The Physical Model

The Lagrange formulation of the pure displacement model of linearized elasticity is defined in the reference material configuration Ω_0^{s}. The state variable d^{s} satisfies the momentum balance law

$$\rho_{\mathrm{s}} \frac{\partial^2 d^{\mathrm{s}}}{\partial t^2} - \mathrm{div}(\sigma_{\mathrm{s}}(d^{\mathrm{s}})) = f_{\mathrm{s}} \qquad \text{in } \Omega_0^{\mathrm{s}}, \tag{3}$$

and the boundary conditions

$$\sigma_{\mathrm{s}}(d^{\mathrm{s}})n_{\mathrm{s}} = 0 \text{ on } \Gamma_0^{\mathrm{n}} \quad \text{and} \quad d^{\mathrm{s}} = 0 \text{ on } \Gamma_0^{\mathrm{d}}, \tag{4}$$

where ρ_s is the density, σ_s the first Piola-Kirchoff stress tensor, f_s is the external force density, and n_s is the outward normal of Ω_0^s. We use the linear Saint-Venant Kirchoff elastic model: $\sigma_s(d^s) = 2\mu^l \varepsilon(d^s) + \lambda^l \mathrm{div}(d^s) I$ with $\varepsilon(d^s) = (\nabla d^s + (\nabla d^s)^T)/2$, and the Lamé constants λ^l and μ^l.

The system of equations for the incompressible fluid problem in the ALE framework is obtained from the balance law of momentum

$$\rho_f \frac{\partial u}{\partial t}\bigg|_{x_0} + \rho_f \left((u - w^f) \cdot \nabla\right) u - 2\mu \mathrm{div}\varepsilon(u) + \nabla p = 0 \quad \text{in } \Omega^f(t), \tag{5}$$

mass conservation

$$\mathrm{div} u = 0 \quad \text{in } \Omega^f(t), \tag{6}$$

and properly chosen boundary conditions

$$\sigma_f(u,p) n_f = g_{\text{in}} \text{ on } \Gamma_{\text{in}}(t) \quad \text{and} \quad \sigma_f(u,p) n_f = 0 \text{ on } \Gamma_{\text{out}}(t), \tag{7}$$

where ρ_f is the fluid density, μ is the dynamic viscosity, $\sigma_f(u,p) = -pI + 2\mu\varepsilon(u)$ and $\varepsilon(u) = (\nabla u + (\nabla u)^T)/2$ are the Cauchy stress tensor σ_f and the strain rate tensor ε, respectively. Here the ALE time derivative of $u(x,t)$ is introduced:

$$\frac{\partial u}{\partial t}\bigg|_{x_0} := \frac{\partial}{\partial t}(u \circ \mathcal{A}_t) \circ (\mathcal{A}_t)^{-1} = \frac{\partial u}{\partial t} + (w^f \cdot \nabla) u$$

for $x \in \Omega^f(t)$, where $w^f(x,t) = \hat{w}^f \circ (\mathcal{A}_t)^{-1}(x)$.

When coupling the two sub-problems together, interface conditions are needed. In particular, no-slip conditions on the interface Γ_0 are explicitly imposed at time t on Γ_0 between the structure and the fluid domains:

$$u \circ \mathcal{A}_t|_{\Gamma_0} = \frac{\partial d^s}{\partial t}\bigg|_{\Gamma_0}. \tag{8}$$

The second interface condition is the equilibrium of normal stresses:

$$(\sigma_f(u,p) n_f) \circ \mathcal{A}_t + \sigma_s(d^s) n_s = 0. \tag{9}$$

To summarize, the complete model consists of problem (2), Eqs. (3), (5), (6), boundary conditions (4), (7), and interface conditions (8), (9) for the state variables d^s, u, p, d^f.

1.3 Reformulation of the Model

As in [1], we express the interface conditions in terms of the so-called Steklov–Poincaré operators for which we introduce the interface variable $\lambda(t)$ by $d^s = d^f = \lambda$ for time t at Γ_0. Then the no-slip interface condition is automatically satisfied.

Let $S_s(\lambda)$ denote the Neumann data $\sigma_s(d^s) n_s$ of the structure problem, where the displacement $d^s := d^s(x_0, t)$ satisfies the Eqs. (3) and (4) with prescribed Dirichlet data $d^s = \lambda$ on the interface Γ_0.

Let $S_{\mathrm{f}}(\lambda)$ denote the Neumann data $\sigma_{\mathrm{f}}(u,p)n_{\mathrm{f}} \circ \mathcal{A}_t$ of the fluid problem, where u and p are determined in the following way: We first compute the harmonic extension $d^{\mathrm{f}} := d^{\mathrm{f}}(x_0,t)$ by solving (2) with Dirichlet condition $d^{\mathrm{f}} = \lambda$ on Γ_0. Then the fluid domain is given by $\Omega^{\mathrm{f}}(t) = d^{\mathrm{f}} + \Omega_0^{\mathrm{f}}$ and we compute u and p by solving (5), (6), (7) with prescribed Dirichlet data $u \circ \mathcal{A}_t = \partial\lambda/\partial t$ on the interface Γ_0.

Then the coupled problem is reduced to the following equation

$$S(\lambda) := S_{\mathrm{s}}(\lambda) + S_{\mathrm{f}}(\lambda) = 0,$$

which is the so-called Steklov–Poincaré equation.

1.4 Time Semi-Discretized Weak Formulations

We need the following function spaces $V^{\mathrm{s}} = [H^1(\Omega_0^{\mathrm{s}})]^3$, $V_0^{\mathrm{s}} = \{v^{\mathrm{s}} \in V^{\mathrm{s}} | v^{\mathrm{s}} = 0 \text{ on } \Gamma_0^{\mathrm{d}} \bigcup \Gamma_0\}$, and $V_g^{\mathrm{s}}(t) = \{v^{\mathrm{s}} \in V^{\mathrm{s}} | v^{\mathrm{s}} = \lambda(t) \text{ on } \Gamma_0\}$ for the structure. For the fluid, we define $D^{\mathrm{f}} = [H^1(\Omega_0^{\mathrm{f}})]^3$, $D_0^{\mathrm{f}} = \{d \in D^{\mathrm{f}} | d = 0 \text{ on } \Gamma_0\}$, $D_g^{\mathrm{f}}(t) = \{d \in D^{\mathrm{f}} | d = \lambda(t) \text{ on } \Gamma_0\}$, $V^{\mathrm{f}}(t) = \{v^{\mathrm{f}} | v^{\mathrm{f}} \circ x_t^{\mathrm{f}} \in [H^1(\Omega_0^{\mathrm{f}})]^3\}$, $V_0^{\mathrm{f}}(t) = \{v^{\mathrm{f}} \in V^{\mathrm{f}}(t) | v^{\mathrm{f}} \circ x_t^{\mathrm{f}} = 0 \text{ on } \Gamma_0\}$, $V_g^{\mathrm{f}}(t) = \{v^{\mathrm{f}} \in V_0^{\mathrm{f}} | v^{\mathrm{f}} \circ x_t^{\mathrm{f}} = w^{\mathrm{f}} \circ x_t^{\mathrm{f}} \text{ on } \Gamma_0\}$, and $Q^{\mathrm{f}}(t) = \{q^{\mathrm{f}} | q^{\mathrm{f}} \circ x_t^{\mathrm{f}} \in L^2(\Omega_0^{\mathrm{f}})\}$, where $H^1(\Omega_0^{\mathrm{s}})$ and $H^1(\Omega_0^{\mathrm{f}})$ denote the standard Sobolev spaces.

Time Semi-discretized Structure Weak Formulation

We denote the time step size by δt and introduce the time level $t^n = n\delta t$.

For the time discretization of the structure problem, we follow the strategy in [1], where the Newmark method with $\gamma = 2\beta = 1$ was proposed:

$$\int_{\Omega_0^{\mathrm{s}}} \rho_{\mathrm{s}} \frac{\partial^2 d^{\mathrm{s}}}{\partial t^2} \cdot v^{\mathrm{s}} dx_0 \approx \frac{2}{\delta t^2} \int_{\Omega_0^{\mathrm{s}}} \rho_{\mathrm{s}} d^{\mathrm{s},n+1} v^{\mathrm{s}} dx_0 - \frac{2}{\delta t^2} \int_{\Omega_0^{\mathrm{s}}} \rho_{\mathrm{s}} (d^{\mathrm{s},n} + \delta t w^{\mathrm{s},n}) v^{\mathrm{s}} dx_0.$$

Here $w^{\mathrm{s},n}$ is the structure domain velocity at time t^n. Using the calculated displacement $d^{\mathrm{s},n+1}$ at time t^{n+1}, we update the structure domain velocity $w^{\mathrm{s},n+1} = 2(d^{\mathrm{s},n+1} - d^{\mathrm{s},n})/\delta t - w^{\mathrm{s},n}$. This leads to the following variational problem, which must be solved in each time step:

Find $d^{\mathrm{s},n+1} = d^{\mathrm{s}}(t^{n+1}) \in V_g^{\mathrm{s}}(t^{n+1})$ such that for all $v^{\mathrm{s}} \in V_0^{\mathrm{s}}$:

$$\begin{aligned} &\frac{2}{\delta t^2} \int_{\Omega_0^{\mathrm{s}}} \rho_{\mathrm{s}} d^{\mathrm{s},n+1} v^{\mathrm{s}} dx_0 + \int_{\Omega_0^{\mathrm{s}}} [\lambda^l \mathrm{div} d^{\mathrm{s},n+1} \mathrm{div} v^{\mathrm{s}} + 2\mu^l \epsilon(d^{\mathrm{s},n+1}) : \epsilon(v^{\mathrm{s}})] dx_0 \\ &= \frac{2}{\delta t^2} \int_{\Omega_0^{\mathrm{s}}} \rho_{\mathrm{s}} (d^{\mathrm{s},n} + \delta t w^{\mathrm{s},n}) v^{\mathrm{s}} dx_0. \end{aligned}$$

Time Semi-discretized Fluid Weak Formulation

Firstly, we compute the harmonic extension of the fluid domain:

Find $d^{\mathrm{f},n+1} \in D_g^{\mathrm{f}}(t^{n+1})$ such that for all $\phi \in D_0^{\mathrm{f}}$:

$$\int_{\Omega_0^{\mathrm{f}}} \nabla d^{\mathrm{f},n+1} : \nabla \phi dx_0 = 0.$$

Then the computational fluid domain is given by $\Omega^{\mathrm{f}}(t^{n+1}) = d^{\mathrm{f},n+1} + \Omega_0^{\mathrm{f}}$, and we set $w^{\mathrm{f},n+1} = ((d^{\mathrm{f},n+1} - d^{\mathrm{f},n})/\delta t) \circ (\mathcal{A}_{t^{n+1}})^{-1}$ for the fluid domain velocity.

For the fluid problem an implicit Euler scheme is used:

$$\frac{\mathrm{d}}{\mathrm{d}t} \int_{\Omega^{\mathrm{f}}(t)} \rho_{\mathrm{f}} u \cdot v^{\mathrm{f}} dx \bigg|_{t^{n+1}} \approx \frac{\int_{\Omega^{\mathrm{f}}(t^{n+1})} \rho_{\mathrm{f}} u^{n+1} \cdot v^{\mathrm{f},n+1} dx - \int_{\Omega^{\mathrm{f}}(t^n)} \rho_{\mathrm{f}} u^n \cdot v^{\mathrm{f},n} dx}{\delta t},$$

where $v^{\mathrm{f}} = \hat{v}^{\mathrm{f}} \circ (\mathcal{A}_t)^{-1}$, in particular, $v^{\mathrm{f},k} = \hat{v}^{\mathrm{f}} \circ (\mathcal{A}_{t^k})^{-1}$. The non-linear convective term is treated in a semi-implicit way (see [2]). Then we obtain the following time semi-implicit fluid weak formulation:

Find $(u^{n+1}, p^{n+1}) = (u(t^{n+1}), p(t^{n+1})) \in V_g^{\mathrm{f}}(t^{n+1}) \times Q^{\mathrm{f}}(t^{n+1})$ such that for all $(v^{\mathrm{f},n+1}, q^{\mathrm{f},n+1}) \in V_0^{\mathrm{f}}(t^{n+1}) \times Q^{\mathrm{f}}(t^{n+1})$:

$$\begin{aligned}
&\frac{1}{\delta t} \int_{\Omega^{\mathrm{f}}(t^{n+1})} \rho_{\mathrm{f}} u^{n+1} \cdot v^{\mathrm{f},n+1} dx - \int_{\Omega^{\mathrm{f}}(t^{n+1})} \rho_{\mathrm{f}} \left(\mathrm{div} w^{\mathrm{f},n+1}\right) u^{n+1} \cdot v^{\mathrm{f},n+1} dx \\
&\quad + \int_{\Omega^{\mathrm{f}}(t^{n+1})} \rho_{\mathrm{f}} \left(\left(\hat{u}^n - w^{\mathrm{f},n+1}\right) \cdot \nabla\right) u^{n+1} \cdot v^{\mathrm{f},n+1} dx \\
&\quad + 2\mu \int_{\Omega^{\mathrm{f}}(t^{n+1})} \varepsilon(u^{n+1}) : \varepsilon(v^{\mathrm{f},n+1}) dx - \int_{\Omega^{\mathrm{f}}(t^{n+1})} p^{n+1} \, \mathrm{div} v^{\mathrm{f},n+1} dx \\
&\quad = \frac{1}{\delta t} \int_{\Omega^{\mathrm{f}}(t^n)} \rho_{\mathrm{f}} u^n \cdot v^{\mathrm{f},n} dx + \int_{\Gamma_{\mathrm{in}}(t^{n+1})} g_{\mathrm{in}} \cdot v^{\mathrm{f},n+1} ds, \\
&- \int_{\Omega^{\mathrm{f}}(t^{n+1})} q^{n+1} \, \mathrm{div} u^{\mathrm{f},n+1} dx = 0,
\end{aligned}$$

where $\hat{u}^n = u^n \circ \mathcal{A}_{t^n} \circ (\mathcal{A}_{t^{n+1}})^{-1}$.

The Variational Form of the Interface Equation

In the weak form, the previously introduced Steklov–Poincaré operators become operators from the Sobolev space $H^{1/2}(\Gamma_0)$ (which is the space of traces of H^1-functions on Γ_0) to its dual $H^{-1/2}(\Gamma_0)$:

$$S_{\mathrm{s}} : H^{1/2}(\Gamma_0) \to H^{-1/2}(\Gamma_0), \quad S_{\mathrm{f}} : H^{1/2}(\Gamma_0) \to H^{-1/2}(\Gamma_0).$$

Then we end up with the following problem:

Find $\lambda \in H^{1/2}(\Gamma_0)$ such that for all $\mu \in H^{1/2}(\Gamma_0)$:

$$\langle S_{\mathrm{f}}(\lambda), \mu \rangle_{\Gamma_0} + \langle S_{\mathrm{s}}(\lambda), \mu \rangle_{\Gamma_0} = 0. \tag{10}$$

2 Newton's Method for the Interface Equation

The problem (10) has to be solved at each time level $t = t^{n+1} = t^n + \delta t$. For simplicity, we will drop the time variables in the following.

Newton's method applied to the interface equation is given by

$$\lambda^{k+1} = \lambda^k + \delta\lambda^k$$

with

$$\left(S_{\mathrm{s}}^{'}(\lambda^k) + S_{\mathrm{f}}^{'}(\lambda^k)\right)\delta\lambda^k = -\left(S_{\mathrm{s}}(\lambda^k) + S_{\mathrm{f}}(\lambda^k)\right).$$

After spatial discretization this linear problem is solved by the GMRES method.

The method requires the evaluation of $S_{\mathrm{s}}(\lambda)$, $S_{\mathrm{f}}(\lambda)$, $S_{\mathrm{s}}'(\lambda)\delta\lambda$ and $S_{\mathrm{f}}'(\lambda)\delta\lambda$, see [9] for details how to compute these quantities.

3 Finite Element Discretization on Hybrid Meshes

The spatial discretization was done by a finite element method. Let $\mathcal{M}_h$ be the original subdivision of the domain $\Omega \subset \mathbb{R}^3$ into tetrahedra, hexahedra, prisms and pyramids, which is assumed to be admissible, i.e. any two elements from $\mathcal{M}_h$ either have no intersection, or have a common face, or have a common edge, or have a common vertex. Let $\mathcal{T}_h$ be the admissible subdivision into tetrahedra, obtained in the following way: we add points at the centers of quadrilateral faces and subdivide each of them into four triangles, then add a point at the center of the element, and finally connect this center point with all the original vertices and the face center points.

As our finite element space on the hybrid mesh $\mathcal{M}_h$ we first take the standard P_1 finite element space on the underlying tetrahedral mesh $\mathcal{T}_h$ and then replace the degrees of freedom associated to the added points by averaging over neighboring vertices of the original mesh.

This extended P_1 finite element is used for discretizing the structure problem and the interface problem. For the fluid problem we also used the same finite element complemented by a pressure stabilization Petrov-Galerkin (PSPG) and a streamline upwind Petrov-Galerkin (SUPG) technique.

4 AMG for the Structure and the Fluid Sub-problems

After discretization in time and space, linear systems of the form

$$A_{\mathrm{s}h}\underline{d}_{\mathrm{s}h} = \underline{f}_{\mathrm{s}h} \quad \text{and} \quad \begin{pmatrix} A_{\mathrm{f}h} & B_{1,\mathrm{f}h}^T \\ B_{2,\mathrm{f}h} & -C_{\mathrm{f}h} \end{pmatrix} \begin{pmatrix} \underline{u}_h \\ \underline{p}_h \end{pmatrix} = \begin{pmatrix} \underline{f}_{\mathrm{f}h} \\ \underline{g}_{\mathrm{f}h} \end{pmatrix} \tag{11}$$

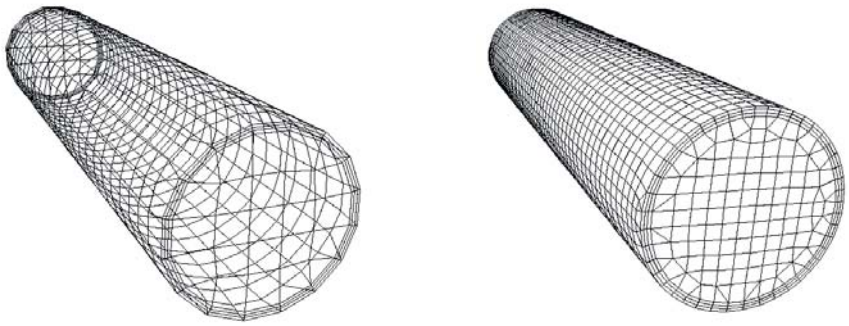
arise at each time step for the structure and the fluid sub-problems, respectively.

The first problem in (11) is symmetric positive and definite, for which the AMG solvers were studied in [3, 4], where a generalization of the classical AMG approach (see [5]) for scalar problems to systems of partial differential equations is discussed.

The system matrix of the second problem in (11) is a saddle point matrix. The AMG approach applied to this type of problem, in particular, to the Stokes or the linearized Navier-Stokes (*Oseen*) problem, stems from previous contributions in [6, 7, 8]. We extended these results to the system arising from the stabilized finite element discretization for the *Oseen* problem on hybrid meshes. In particular, we constructed a stabilized P_1–P_1 hierarchy for the AMG solver on these hybrid meshes, see [9].

5 Numerical Results

We simulate a pressure wave in a cylinder of length 5 cm and radius 5 mm at rest. The thickness of the structure is 0.5 mm. The structure is considered linear and clamped at both the inlet and outlet. The fluid viscosity is set to $\mu = 0.035$, the Lamé constants to $\mu^l = 1.15 \times 10^6$ and $\lambda^l = 1.73 \times 10^6$, the density to $\rho^{\mathrm{f}} = 1.0$ and $\rho^s = 1.2$. The fluid and structure are initially at rest and a pressure of 1.332×10^4 dyn/cm^2 is set on the inlet for a time period of 3 ms. Two meshes[1] (see surface meshes in Fig. 2 as an illustration) are used for simulations:



(a) coarse mesh (b) fine mesh

Fig. 2. Fine and coarse meshes for simulations.

For all simulations, we use the same time step size $\delta t = 1$ ms and run the simulation until the same end time $t = 20$ ms as in [1].

A relative error reduction by a factor of 10^{-5} is achieved in 2–3 outer iterations. Each of these iterations requires 6–8 GMRES iterations for a relative error reduction

[1] All meshes in our test examples were provided by Dipl.- Ing. Ferdinand Kickiger, CAE Software Solutions Wolfkersbühelstr. 23, A-3730 Eggenburg, Austria. See webpage: www.meshing.org.

by a factor of 10^{-5}. For solving the structure problem, about 10 preconditioned conjugate gradient iterations with AMG preconditioning are needed for a relative error reduction by a factor of 10^{-8}, for the fluid problem about 5 AMG iterations for a relative error reduction by a factor of 10^{-8}. Almost the same numbers of iterations were observed for the coarse and the fine mesh.

References

1. S. Deparis, M. Discacciati, G. Fourestey, and A. Quarteroni. Fluid-structure algorithms based on Steklov-poincaré operators. *Comput. Methods Appl. Mech. Eng.*, 195:5797–5812, 2006.
2. M.A. Fernández and M. Moubachir. A Newton method using exact Jacobians for solving fluid-structure coupling. *Comput. Struct.*, 83(2–3):127–142, 2005.
3. M. Griebel, D. Oeltz, and M.A. Schweitzer. An algebraic multigrid method for linear elasticity. *SIAM. J. Sci. Comput.*, 25(2):385–407, 2003.
4. S. Reitzinger. *Algebraic Multigrid Methods for Large Scale Finite Element Equations*. PhD thesis, Johannes Kepler University, Linz, 2001.
5. J.W. Ruge and K. Stüben. Algebraic multigrid (AMG). In *Multigrid Methods*, volume 5 of *Fontiers in Applied Mathematics*, pp. 73–130, SIAM, Philadelphia, PA, 1986.
6. M. Wabro. *Algebraic Multigrid Methods for the Numerical Solution of the Incompressible Navier-Stokes Equations*. PhD thesis, Johannes Kepler University, Linz, 2003.
7. M. Wabro. Coupled algebraic multigrid methods for the oseen problem. *Comput. Vis. Sci.*, 7(3–4):141–151, 2004.
8. M. Wabro. AMGe – Coarsening strategies and application to the Oseen equations. *SIAM J. Sci. Comput.*, 27:2077–2097, 2006.
9. H. Yang. *Numerical Simulations of Fluid-Structure Interaction Problems on Hybrid Meshes with Algebraic Multigrid Methods*. PhD thesis, Johannes Kepler University, Linz, 2010.