The Twenty First International Conference on Domain Decomposition Methods

INRIA Rennes-Bretagne Atlantique
Campus de Beaulieu, 35042 Rennes Cedex

June 25–29, 2012

Schedule
Author Index
Session Index

Version Date: June 27, 2012

DDXXI

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Session Index

Schedule

Time	Event	Location
	Monday, June 25	
7:45-8:45	Registration	INRIA Reception
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10:30-12:15	Parallel Sessions (M16P1, M10P1, M14, M6P1, M5, C1)	Amphi, Markov, Petri, Turing, I50, I51
12:15-14:00	Lunch	
14:00-14:45	Plenary (P2 - Géraldine Pichot)	Amphi
14:45-15:30	Plenary (P3 - Axel Klawonn)	Amphi
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16:00-18:10	Parallel Sessions (M16P2, M10P2, M13P1, M6P2, M8, C2)	Amphi, Markov, Petri, Turing, I50, I51
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9:15-10:00	Plenary (P5 - Jin-Fa Lee)	Amphi
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14:00-14:45	Plenary (P6 - Clemens Pechstein),	Amphi
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9:15-10:00	Plenary (P8 - Beatrice Riviere)	Amphi
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14:00-14:45	Plenary (P9 - Xiao-Chuan Cai)	Amphi
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	Thursday, June 28	
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9:15-10:00	Plenary (P12 - Chen-Song Zhang)	Amphi
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9:15-10:00	Plenary (P14 - Michael Holst)	Amphi
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12:15-14:00	Lunch	
14:00-15:45	Parallel Sessions (C15, C13, C16, C17)	Amphi, Markov, Petri, Turing
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M T W T F	Monday, June 25, 2012					
7:45-8:45	Registration					
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9:15-10:00	Plenary P1 (Chair: Ralf Kornhuber) Laurence Halpern					
10:00-10:30	Coffee Break					
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	Frédéric Hecht	Maksymilian Dryja	Ismael Herrera	Heiko Berninger	Martin J. Gander	Jaroslav Haslinger
	Alfio Quarteroni	Juan Galvis	Luis Miguel de la Cruz	Pablo Javier Blanco	Clark Dohrmann	Brahim Nouiri
	Caroline Japhet	Hyea Hyun Kim	Alberto Rosas	Eva Casoni	Jörg Willems	Ihor I. Prokopyshyn
	Zakaria Belhachmi	Chang-Ock Lee	Iván Contreras	Paola Gervasio	Kévin Santugini	Alexandros Markopoulos
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14:45-15:30	Plenary P3 (Chair: Ralf Kornhuber) Axel Klawonn					
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	Christian Waluga	Jungho Lee	Victorita Dolean	Simona Perotto	Eric Blayo	Daniel Choi
	Yvon Maday	L. Beirão da Veiga	Nicole Spillane	Franz Rammerstorfer	Bas van 't Hof	Vincent Visseq
	François-Xavier Roux	Xuemin Tu	Jinchao Xu	Human Rezaijafari	Mart Borsboom	Geoffrey Desmeure
	Oldřich Vlach	Olof Widlund	Juan Galvis	Anton Schiela	Fred Wubs	Julien Riton
	Todd Arbogast	Jun Zou	Clark Dohrmann			Philippe Karamian
19:00-20:00	Welcoming cocktail					

M T W T F	Tuesday, June 26, 2012					
8:30-9:15	Plenary P4 (Chair: Alfio Quarteroni) Marcus Sarkis					
9:15-10:00	Plenary P5 (Chair: Alfio Quarteroni) Jin-Fa Lee					
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	Rémi Abgrall Michel Bercovier Victor M. Calo Krishan P. S. Gahalaut	Yvon Maday Michael Minion Rim Guetat Felix Kwok	Petr Vanek Robert Scheichl James Brannick Marco Buck	Lea Conen Hui Zhang Erwin Veneros Bertrand Thierry	Patrick Le Tallec Mohammed Lemou Emmanuel Frénod Heiko Berninger	Marco Discacciati Marina Vidrascu Christian Engwer K. C. Park (C18)
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16:00-17:45	M7 P3 Amphi	M15 P2 Markov	M13 P3 Petri	M9 P2 Turing	M11 P2 I50	C7 I51
	Angela Kunoth Luca F. Pavarino Satyendra Tomar Rafael Vazquez	Stefan Güttel Martin J. Gander Jacques Laskar Julien Salomon	Florian Thomines Ivan Graham Jan Nordbotten Xiaozhe Hu	Olaf Steinbach Jin-Fa Lee Eric Darrigrand Yogi Erlangga	François Golse Giacomo Dimarco Sudarshan Tiwari Jérôme Michaud	Daniel Szyld Feng-Nan Hwang Santiago Badia Laurent Berenguer
18:00-22:00	Scientific committee meeting					

M T W T F	Wednesday, June 27, 2012					
8:30-9:15	Plenary P7 (Chair: Laurence Halpern) Hyea Hyun Kim					
9:15-10:00	Plenary P8 (Chair: Laurence Halpern) Beatrice Riviere					
10:00-10:30	Coffee Break					
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14:00-14:45	Plenary P9 (Chair: Petter Bjørstad) Xiao-Chuan Cai					
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15:30-16:00	Coffee Break					
16:00-17:45	M2 P2 Amphi	M18 P1 Markov	M20 Petri	M9 P4 Turing	C9 I50	C10 I51
	Bernd Flemisch Paul-Marie Berthe J.-B. Apoung Kamga Anthony Michel	Alexandre Pieri Kolja Brix Christoph Lehrenfeld Eun-Hee Park	Talal Rahman Juan Galvis Robert Scheichl Rui Du	Ronan Perrussel Jack Poulson Zhen Peng Stéphane Lanteri	Chao Yang Felix Kwok Martin Cermak D. Tromeur-Dervout	Jyri Leskinen M. Khaled Gdoura Thu Huyen Dao Guillaume Houzeaux

M T W T F	Thursday, June 28, 2012					
8:30-9:15	Plenary P11 (Chair: Olof Widlund) Blanca Ayuso de Dios					
9:15-10:00	Plenary P12 (Chair: Olof Widlund) Chen-Song Zhang					
10:00-10:30	Coffee Break					
10:30-12:15	M17 P1 Amphi	M18 P2 Markov	M19 P1 Petri	M3 Turing	C11/C18 I50	C14 I51
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12:15-13:30	Lunch					
13:30-22:00	Excursion and dinner					

M T W T F	Friday, June 29, 2012					
8:30-9:15	Plenary P13 (Chair: Susanne Brenner) Ralf Hiptmair					
9:15-10:00	Plenary P14 (Chair: Susanne Brenner) Michael Holst					
10:00-10:30	Coffee Break					
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	Soheil Hajian Ronald Haynes Joel Phillips Yingxiang Xu	Frédéric Hecht Pierre Jolivet Christophe Prud'homme Abdoulaye Samake	Martin Weiser Gernot Plank Ricardo Ruiz Baier Maxime Sermesant	Martin J. Gander Xuemin Tu Francois-Xavier Roux Olof Widlund	Oliver Rheinbach Simone Scacchi Nejib Zemzemi Gwenol Grandperrin	Andreas Langer Firmim Andzembe Okoubi Francisco Bernal Samia Riaz
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	Kab Seok Kang Pawan Kumar Lori Badea	Leszek Marcinkowski Ange Toulougoussou Hui Zhang Christian Rey	Patrick Le Tallec Thomas Dickopf Debasish Pradhan Frédéric Magoulès	Kirill Pichon Gostaf Ajit Patel Eliseo Chacón Vera Beatriz Eguzkitza		
15:45-16:15	Closing					

Plenary Lecture P1

Schedule Author Index Session Index

Date: Monday, June 25
Time: 9:15-10:00
Location: Amphi
Chairman: Ralf Kornhuber

9:15-10:00 : Laurence Halpern
Optimized Schwarz Waveform Relaxation and Applications to Semilinear Equations
Abstract

Optimized Schwarz Waveform Relaxation and Applications to Semilinear Equations

Schedule Author Index Session Index

Laurence Halpern
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Abstract

Optimized Schwarz waveform relaxation have been presented for the first time in DD11 in 1999 [2]. These algorithms permit to solve the equations in time windows,in different subdomains in space, exchanging informations at the end of the time interval on the interfaces. They use Robin or Ventcell transmission conditions, and can be used without overlap if necessary. A key issue in this process is to choose the coefficients in order to accelerate the convergence.

We concentrate in this presentation on parabolic equations and systems. The coefficients of the transmission operators can be characterized through a best approximation problem on a compact set whose dimensions are related to the parameters of the discretization. For the advection diffusion equation, a complete analysis in one dimension was performed in [1], some results were announced in DD19, but complete general formulas will only be available for DD21. The first part of my talk will deal with this important question.

As soon as this is solved, we can address the extension to nonlinear problems, with various questions in mind

1) How to use the optimized coefficients ?
2) How to prove convergence of the algorithm ?
3) How to relate the Schwarz waveform relaxation algorithm and the algorithm of resolution of nonlinear problem (like Newton's algorithm).

All these questions will be considered, and applications to the reactive transport system will be presented.

[1] D. Bennequin, M. Gander and L. Halpern. A Homographic Best Approximation Problem with Application to Optimized Schwarz Waveform Relaxation. Math. Comp. 78 (2009), no. 265, 185223.
[2] M. Gander, L. Halpern and F. Nataf. Optimal Convergence for Overlapping and Non-Overlapping Schwarz Waveform Relaxation. Eleventh International Conference on Domain Decomposition Methods (London, 1998), 2736 (electronic), DDM.org, Augsburg, 1999.

Plenary Lecture P2

Schedule Author Index Session Index

Date: Monday, June 25
Time: 14:00-14:45
Location: Amphi
Chairman: Ralf Kornhuber

14:00-14:45 : Géraldine Pichot
On Robust Numerical Methods for Solving Flow in Stochastic Fracture Networks
Abstract

On Robust Numerical Methods for Solving Flow in Stochastic Fracture Networks

Schedule Author Index Session Index

Géraldine Pichot
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Abstract

Working with random domains requires the development of specific and robust numerical methods to be able to solve physical phenomena whatever the generated geometries. Hydrogeology is a typical area of application where one has to face uncertainty about the geometry and the properties of the domain since the available information on the underground media is local, gathered through in-situ experiments with outcrops and wells. From measurements, statistical laws are derived that allow the generation of natural-like random media.

The focus of this talk will concern flow in discrete fracture networks. The parameters governing the fractures lengths, shapes, orientations, positions as well as their hydraulic conductivity are stochastic. Our objective is to design robust numerical methods to solve Poiseuille's flow in large and heterogeneous stochastic fracture networks.

The first part will deal with the meshing strategies required to obtain a good quality mesh for any generated networks. The second part will be devoted to numerical techniques to solve the flow equations. A Mortar-like method to deal with non-conforming meshes at the fracture intersections will be presented as well as a Schur complement approach to solve the linear system of interest in parallel.

This work is a joined work with Jocelyne Erhel, Baptiste Poirriez, Jean-Raynald de Dreuzy, Patrick Laug and Thomas Dufaud.

Plenary Lecture P3

Schedule Author Index Session Index

Date: Monday, June 25
Time: 14:45-15:30
Location: Amphi
Chairman: Ralf Kornhuber

14:45-15:30 : Axel Klawonn
Deflation, Projector Preconditioning and Robust Domain Decomposition Methods
Abstract

Deflation, Projector Preconditioning and Robust Domain Decomposition Methods

Schedule Author Index Session Index

Axel Klawonn
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Abstract

In this talk, projector preconditioning, also known as the deflation method, is applied to the FETI-DP and the BDDC method in order to create a second, independent coarse problem. It may as well be used to improve the robustness, e.g., for almost incompressible elasticity problems and second order elliptic partial differential equations with discontinuous coefficients. In addition, it will be shown that standard FETI-DP methods are robust for elasticity problems with respect to coefficient jumps within the subdomains. Here, the convergence of FETI-DP methods for problems in 3D with almost incompressible inclusions or compressible inclusions with different material parameters embedded in a compressible matrix material is analyzed. It can also be demonstrated that these FETI-DP algorithms are robust for challenging problems from nonlinear biomechanics with almost incompressible material properties. Finally, it will be reported on results for certain overlapping Schwarz methods for elliptic problems with discontinuous coefficents not aligned with the interface.
The results in this talk are based on different joint projects with Sabrina Gippert, Patrick Radtke, Oliver Rheinbach, and Olof Widlund.

Plenary Lecture P4

Schedule Author Index Session Index

Date: Thursday, June 28
Time: 8:30-9:15
Location: Amphi
Chairman: Alfio Quarteroni

8:30-9:15 : Marcus Sarkis
DDMs for DG Discretizations
Abstract

DDMs for DG Discretizations

Schedule Author Index Session Index

Marcus Sarkis
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Abstract

We consider a second order elliptic equation with discontinuous coefficients. The domain is defined as a geometrically (possibly nonconforming) decomposition of substructures. Inside each substructure, a conforming triangulation is introduced and a conforming or DG finite element method is considered. To handle nonmatching meshes and coefficient jumps across substructure interfaces, we consider proper DG discretizations. The first part of the talk we discuss a priori error estimates including the case where the coefficient is anisotropic. The second part of the talk we discuss solvers based on FETI-DP, BDDC, Neumann-Neumann and Average Schwarz methods which are robust with respect to coefficients jumps, number of subdomains, local mesh sizes and mesh sizes ratio across substructure interfaces. Cases where the coefficient varies inside the substructures are also discussed. Numerical results are presented.
The results were obtained in collaboration with Prof. Maksymilian Dryja, Dr. Juan Galvis and Dr. Piotr Krzyzanowski.

Plenary Lecture P5

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Date: Tuesday, June 26
Time: 9:15-10:00
Location: Amphi
Chairman: Alfio Quarteroni

9:15-10:00 : Jin-Fa Lee
An Expedition to Solving a Multiscale Electromagnetic Problem
Abstract

An Expedition to Solving a Multiscale Electromagnetic Problem

Schedule Author Index Session Index

Jin-Fa Lee
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Abstract

This talk centers on the full-wave solution of an electromagnetic wave scattering from a composite aircraft with multi-scale geometrical features. As in many engineering applications, the target/object considered herein is built by putting together many different parts and components, and each of them can be changed and modified due to the design and operational needs. For example, the mockup fighter jet can be carrying different pay loads for different missions, varying the thickness of the lossy thin coatings to study the effectiveness of radar absorption, and fine tuning the engine inlet to reduce EM echo area etc. As a consequence, a full-wave solution strategy which incorporates hierarchical geometrical partitioning/decomposition seamlessly would be highly desirable for many mission critical engineering studies. In solving the EM wave scattering from such a complicated and multiscale composite aircraft, we have encountered many unexpected surprises as well as expected technical difficulties. At the end, our pursuit of a rigorous full-wave solution proved to be prolific, thought-provoking, but most of them all, educational. In this talk, I shall elucidate a few major highlights of our journey:

- Non-conformal integral equation domain decomposition methods for multiscale EM problems.
- A generalized combined field integral equation method with multiple surface traces for modeling penetrable targets.
- An integral equation domain decomposition method for EM wave scattering from deep cavities such as engine inlets.
- A multiscale finite element method to incorporate honeycomb metamaterials structures on a dielectric radome.
- The use of the best local sub-domain computational electromagnetic (CEM) solver and the multi-solver domain decomposition method.

Plenary Lecture P6

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Date: Tuesday, June 26
Time: 14:00-14:45
Location: Amphi
Chairman: David Keyes

14:00-14:45 : Clemens Pechstein
Substructuring for Multiscale Problems
Abstract

Substructuring for Multiscale Problems

Schedule Author Index Session Index

Clemens Pechstein
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Abstract

FETI, FETI-DP, and the related balancing Neumann-Neumann and BDDC methods are the most widely used iterative substructuring methods for the solution of large sparse systems stemming from finite element discretizations of elliptic partial differential equations. For simplicity, consider the scalar elliptic equation $-\mathrm{div}(\alpha\nabla u) = f$. However, we let the diffusion coefficient α vary over many orders of magnitude in an unstructured way on the computational domain, which justifies to call this a *multiscale problem.* It is known that if α is *resolved* by the subdomain partitioning (i.e. constant in each subdomain), then the FETI and FETI-DP preconditioners (and their balancing counterparts) can be made robust with respect to the jumps of α across subdomain interfaces. However, for many highly varying coefficients, a straightforward application of the 'standard' delivers pessimistic condition number bounds. In the first part of this talk, I will discuss the application (and adaption) of FETI methods to the multiscale problem above. Using weighted Poincaré inequalities – a theoretical tool that is interesting in itself – robustness of FETI can be proved rigorously under certain monotonicity conditions on α. We will also investigate in how far these conditions are necessary. Furthermore, for piecewise constant coefficients α, the performance of FETI depends on the "geometry" of α (i.e. on the subregions where α is constant), and I will work out how to quantify this dependence. The second part of this talk is devoted to the more difficult case of FETI-DP, which requires weighted Poincaré inequalities with suitably weighted averages. Finally, for real-life problems with highly varying coefficients, the challenge is how to *adapt* the constraints in FETI-DP/BDDC in order to achieve robustness, and I will sketch some ideas into this direction. The research underlying this talk was done in joint collaboration with Rob Scheichl, Marcus Sarkis, and also with Clark Dohrmann.

Plenary Lecture P7

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 8:30-9:15
Location: Amphi
Chairman: Laurence Halpern

8:30-9:15 : Hyea Hyun Kim
Recent Advances in Domain Decomposition Methods for the Stokes Problem
Abstract

Recent Advances in Domain Decomposition Methods for the Stokes Problem

Schedule Author Index Session Index

Hyea Hyun Kim
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Abstract

Domain decomposition methods for the Stokes problem are developed under a more general framework, which allows both continuous and discontinuous pressure functions and more flexibility in the construction of the coarse problem. For the case of discontinuous pressure functions, a coarse problem related to only primal velocity unknowns is shown to give scalability in both dual and primal types of domain decomposition methods. The two formulations are shown to have the same extreme eigenvalues and the ratio of the two extreme eigenvalues weakly depends on the local problem size. This property results in a good scalability in both the primal and dual formulations for the case with discontinuous pressure functions. The primal formulation can also be applied to the case with continuous pressure functions and various numerical experiments are carried out to present promising features of our approach.

Plenary Lecture P8

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 9:15-10:00
Location: Amphi
Chairman: Laurence Halpern

9:15-10:00 : Beatrice Riviere
Discontinuous Galerkin Methods for Multiphysics Problems
Abstract

Discontinuous Galerkin Methods for Multiphysics Problems

Schedule Author Index Session Index

Beatrice Riviere
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Abstract

The numerical solution of coupled subdomains characterized by different types of flows is presented. Examples of such coupled flows include the environmental problem of groundwater contamination through rivers or the industrial manufacturing of filters. The coupling at the interfaces between subdomains is based on the Beavers-Joseph-Saffman conditions.

The proposed algorithms employ discontinuous Galerkin methods. These methods are well-suited to coupling different physics at different scales. Information is transmitted through fluxes defined on the interface. In addition, the use of adaptive mesh refinement and non-matching grids is facilitated by the lack of continuity constraints between the mesh elements. A monolithic approach is compared with a decoupled approach based on the two-grid technique. Finally, the coupling of discontinuous Galerkin methods with finite element methods or finite volume methods is formulated in a domain decomposition setting.

Plenary Lecture P9

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 14:00-14:45
Location: Amphi
Chairman: Petter Bjørstad

14:00-14:45 : Xiao-Chuan Cai
Monolithic Schwarz Algorithms for Simulation and Optimization of Blood Flows
Abstract

Monolithic Schwarz Algorithms for Simulation and Optimization of Blood Flows

Schedule Author Index Session Index

Xiao-Chuan Cai
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Abstract

The class of overlapping Schwarz algorithms has been well studied for elliptic problems. In this talk, we discuss the application of Schwarz algorithms for several more challenging problems including the implicit solution of coupled fluid-structure interaction problems arising in the simulation of blood flows in compliant arteries and the shape optimization of steady state incompressible flows. We show by numerical experiments that, after some proper modifications, multilevel Schwarz algorithms work quite well for these nonlinear systems of coupled multi-physics problems and good scalability results are obtained on parallel machines with thousands of processors. This is a joint work with Yuqi Wu and Rongliang Chen.

Plenary Lecture P10

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 14:45-15:30
Location: Amphi
Chairman: Petter Bjørstad

14:45-15:30 : Eberhard Bänsch
A Finite Element Method for Particulate Flow
Abstract

A Finite Element Method for Particulate Flow

Schedule Author Index Session Index

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Abstract

Particulate flow, i.e. flow of a (Newtonian) carrier liquid loaded with rigid particles, plays an important role in many technical applications. From a mathematical point of view, particulate flows give rise to an interesting and involved free boundary problem, where the flow field and the motion of the particles are coupled through the forces exerted by the flow and Newton's law for the particles' motion. In this talk a one-domain finite element method to solve this problem in 2d and 3d is presented. The main ingredients consist of a splitting scheme in time, a subspace projection method to account for the restriction of the flow field to a rigid body motion in those parts of the domain occupied by the particles and adaptivity to resolve the geometric problems.

Plenary Lecture P11

Schedule Author Index Session Index

Date: Thursday, June 28
Time: 8:30-9:15
Location: Amphi
Chairman: Olof Widlund

8:30-9:15 : Blanca Ayuso de Dios
Solvers for Discontinuous Galerkin Methods
Abstract

Solvers for Discontinuous Galerkin Methods

Schedule Author Index Session Index

Blanca Ayuso de Dios
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Abstract

The talk will discuss the use of old and the design of new Subspace Correction and Domain Decomposition techniques for developing and analyzing efficient solvers for DG methods (for some very simple model problems).

Plenary Lecture P12

Schedule Author Index Session Index

Date: Thursday, June 28
Time: 9:15-10:00
Location: Amphi
Chairman: Olof Widlund

9:15-10:00 : Chen-Song Zhang
Fast Auxiliary Space Preconditioning: Implementation and Applications in Complex Flows
Abstract

Fast Auxiliary Space Preconditioning: Implementation and Applications in Complex Flows

Schedule Author Index Session Index

Chen-Song Zhang
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Abstract

Over the last few decades, intensive research has been done on developing efficient and practical iterative solvers for discretized PDEs. One useful mathematical technique, that has drawn a lot of attention recently, is a general framework called Auxiliary Space Preconditioning. This framework represents a large class of methods that transform a complicated system, by using auxiliary spaces, into a sequence of simpler systems and construct efficient preconditioners with efficient solvers for these simpler systems. In this talk, we will discuss recent development of this method for simulating simple and complex multiphase fluids. We will also introduce a new software library, FASP, designed and implemented for auxiliary space preconditioners. In particular, we will demonstrate some industrial applications of the FASP package for enhanced oil recovery techniques.

Plenary Lecture P13

Schedule Author Index Session Index

Date: Friday, June 29
Time: 8:30-9:15
Location: Amphi
Chairman: Susanne Brenner

8:30-9:15 : Ralf Hiptmair
Novel Multi-Trace Boundary Element Methods for Scattering
Abstract

Novel Multi-Trace Boundary Element Methods for Scattering

Schedule Author Index Session Index

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Abstract

We consider the scattering of acoustic or electromagnetic waves at a penetrable object composed of different homogeneous materials, that is, the material coefficients are supposed to be piecewise constant in sub-domains. This makes possible to recast the problem into boundary integral equations posed on the interfaces. Those can be discretized by means of boundary elements (BEM). This approach is widely used in numerical simulations and often relies on so-called first-kind single-trace BIE, also known as PMCHWT scheme in electromagnetics. These integral equations directly arise from Calderón identities, but after BEM discretization give rise to poorly conditioned linear systems, for which no preconditioner seems to be available so far.

As a remedy we propose new multi-trace boundary integral equations; whereas the single-trace BIE feature unique Cauchy traces on sub-domain interfaces as unknowns, the multi-trace idea takes the cue from domain decomposition and tears the unknowns apart so that *local* Cauchy traces are recovered. Two of them live on each interface and thus we dub the methods “multi-trace”. The benefit of localization is the possibility of Calderón preconditioning.

Multi-trace formulations come in two flavors. A first variant, the *global multi-trace approach*, is obtained from the single-trace equations by taking a “vanishing gap limit”, see [X. Claeys and R. Hiptmair, *Boundary integral formulation of the first kind for acoustic scattering by composite structures*, Comm. Pure Applied Math., in press (2012)] and [X. Claeys and R. Hiptmair, *Electromagnetic scattering at composite objects: A novel multi-trace boundary integral formulation*, M2AN, in press (2012)]. The second variant is the *local multi-trace method* and is based on local coupling across sub-domain interfaces, see [R. Hiptmair and C. Jerez-Hanckes, *Multiple traces boundary integral formulation for Helmholtz transmission problems*, Adv. Appl. Math., (2011), doi: 10.1007/s10444-011-9194-3]. Both methods are amenable to Calderón preconditioning.

Plenary Lecture P14

Schedule Author Index Session Index

Date: Friday, June 29
Time: 9:15-10:00
Location: Amphi
Chairman: Susanne Brenner

9:15-10:00 : Michael Holst
Error Estimates and the Finite Element Exterior Calculus for Critical Exponent Problems in Geometric Analysis and General Relativity
Abstract

Error Estimates and the Finite Element Exterior Calculus for Critical Exponent Problems in Geometric Analysis and General Relativity

Schedule Author Index Session Index

Michael Holst
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Abstract

We consider adaptive methods for nonlinear critical exponent partial differential equations arising in geometric analysis and mathematical physics. After presenting some motivating examples, we describe an approach to establishing a priori Galerkin finite element error estimates without the need for angle conditions to first obtain discrete pointwise control of the nonlinearity. We then show how the a priori error estimates can themselves be used to establish pointwise control of discrete solutions, without the need for a discrete maximum principle, and hence again without the need for angle conditions. We then describe a new approach to analyzing the geometric error made if the domain is a Riemannian manifold rather than a polyhedral domain. The approach involves the development of variational crimes analysis in Hilbert complexes, and then using the abstract framework to develop analogues of the Strang Lemmas for the Finite Element Exterior Calculus (FEEC). We indicate how this variational crimes framework in FEEC recovers the classical a priori surface finite element estimates of Dziuk and Demlow, and allows for substantial generalizations, including hypersurfaces of arbitrary spatial dimension, the Hodge Laplacean, nonlinear problems, as well as semilinear parabolic and hyperbolic problems. This is joint work with a number of colleagues over the last three years.

Mini Symposium M1

Finite Element Packages with Domain Decomposition Solvers

Schedule Author Index Session Index

Organizers: Frédéric Hecht, Frédéric Nataf, Christophe Prud'homme

Abstract

Most linear solvers fall into one of these four categories: direct solvers, incomplete factorizations, multigrid methods or domain decomposition methods. Domain decomposition methods are naturally parallel. They are unique in the sense that they can be thought of in terms of partial differential equations, often in their variational forms. For this reason they are natural to implement and use in finite element packages such as Freefem++ or Feel++ (formerly known as Life). We present recent developments in this direction that enable these packages to address large scale problems on clusters or HPC platforms. We explain how to implement domain decomposition methods in these frameworks and give numerical examples.

M1 Schedule

Finite Element Packages with Domain Decomposition Solvers

Schedule Author Index Session Index

Date: Friday, June 29
Time: 10:30-12:15
Location: Markov
Chairman: Frédéric Hecht, Frédéric Nataf, Christophe Prud'homme

10:35-11:00 : Frédéric Hecht
Some Ways to Implement Domain Decomposition Methods in Freefem++
Abstract

11:00-11:25 : Pierre Jolivet
Multilevel Spectral Coarse Space Methods in Freefem++
Abstract

11:25-11:50 : Christophe Prud'homme
Domain Decomposition Methods in Feel++
Abstract

11:50-12:15 : Abdoulaye Samake
Substructuring Preconditioners for the Mortar Method in Feel++
Abstract

Some Ways to Implement Domain Decomposition Methods in Freefem++

Session Schedule Author Index Session Index M1

Frédéric Hecht
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Abstract

We present how to solve Poisson equation with Schwarz method, in FreeFem++ software. The Poisson problem on domain Ω with boundary Γ in $L^2(\Omega)$:is

$$-\Delta u = f, \text{ in } \Omega, \text{ and } u = g \text{ on } \Gamma,$$

where f and g are two given functions of $L^2(\Omega)$ and of $H^{\frac{1}{2}}(\Gamma)$, Let introduce $(\pi_i)_{i\in I}$ a regular positive partition of the unity of Ω in $N_p = \#I < +\infty$, and denote Ω_i the sub domain which is the support of π_i function and also denote Γ_i the boundary of Ω_i.

The parallel Schwarz method with overlapping is Let $\ell = 0$ the iterator and an initial guest u^0 respecting the boundary condition (i.e. $u^0_{|\Gamma} = g$).

$$\forall i \in I \quad -\Delta u_i^\ell = f, \text{ in } \Omega_i, \text{ and } u_i^\ell = u^\ell \text{ on } \Gamma_i \tag{1}$$

$$u^{\ell+1} = \sum_{i\in I} \pi_i u_i^\ell \tag{2}$$

After discretization with the Lagrange finite element method, with a compatible mesh $\mathcal{T}_{hi}$ of Ω_i, i. e., the exist a global mesh $\mathcal{T}_h$ such that $\mathcal{T}_{hi}$ is include in $\mathcal{T}_h$. The problem is find

$$\forall i \in I, \ \forall v_{hi} \in V_{0hi} \quad \int_{\Omega_i} \nabla u_{hi}^\ell . \nabla v_{hi} = \int_{\Omega_i} f v_{hi}, \text{ and } u_{hi}^\ell = u_h^\ell \text{ on } \Gamma_i \tag{3}$$

$$u_h^{\ell+1} = \sum_{i\in I} \pi_i u_{hi}^\ell \tag{4}$$

We show how to the solve the problem $(3-4)$ by four methods: a basic Schwarz algorithm (for teaching), a first acceleration with a GMRES algorithm to compute the solution of the engine problem : find u^ℓ such that it's equal to the next iterate $u^{\ell+1} = u^\ell$, and two classical Restricted Additive Schwarz methods with and without a coarse grid preconditioned.

Finally, we try these algorithms on Elasticity problem and Stokes problem and show some numerical result of this problem.

Multilevel Spectral Coarse Space Methods in `FreeFem++`

Session Schedule Author Index Session Index M1

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Abstract

Coarse space correction is essential to achieve algorithmic scalability in domain decomposition methods. Our goal here is to build robust coarse spaces for Schwarz-type preconditioners for elliptic problems with highly heterogeneous coefficients when the discontinuities are not just across but also along subdomain interfaces, using local spectral information based on an analysis of the underlying partial differential equations.

This construction is then implemented within the `C++` *domain specific language* `FreeFem++`, and the numerical efficiency of our method is assessed on large-scale computer architectures.

Domain Decomposition Methods in Feel++

Session Schedule Author Index Session Index M1

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Abstract

We present our advances in domain decomposition methods including shwartz, mortar and three fields methods by showing that the embedding language in C++ called FEEL++ goes little in the way of expressivity and closeness to the mathematical language . First we will focus on the overlapping and nonoverlapping schwartz methods with and without relaxation and the Aitken procedure to compute the relaxation parameter. And then we turn to the nonconforming formulations of domain decomposition methods: the mortar method where the main idea is to enforce the weak continuity between the solutions on each subdomain by introducing a Lagrange multiplier corresponding to this connection constraint and then the three fields method in which the disadvantage of introducing the third unknown field is compensated for by the fact that all subdomains are treated exactly in the same way, which results in an easier implementation and possibly, when considering the parallelization of the method, in an easier balancing of the load between processors. The numerical tests will support the above domain decomposition methods using the Feel++ framework. The numerical tests are also in parallel and we rely on PETSc.

Substructuring Preconditioners for the Mortar Method in Feel++

Session Schedule Author Index Session Index M1

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Abstract

We deal with the efficient solution of the linear system arising from the discretization by the mortar method, a nonconforming version of the domain decomposition methods, in two and three dimensions. This kind of preconditioners has already been applied to the mortar methods in two dimensions or the case of order one finite elements and in an abstract framework including high order finite elements and wavelets. We focus on the simple model problem: find $u : \Omega \to \mathbb{R}$, with Ω bounded polyhedral domain of $\mathbb{R}^3$, verifying $-\nabla \cdot \mathbf{a}\nabla u = f$ in $\Omega, u = 0$ on $\partial\Omega$. We split the discrete solution $u_h \in \prod X_h^\ell$ as the sum of three suitable constructed contributions $u_h = u_h^0 + u_h^F + u_h^W$, with $u_h^0 \in \prod X_h^\ell \cap H_0^1(\Omega_\ell)$ corresponding to nodes interior to the subdomains, u_h^F corresponding to nodes interior to the faces of the subdomains, and u_h^W corresponding to nodes on the wirebasket. We propose a block diagonal preconditioner $\hat{A}^{-1}$ for the mortar method. More precisely it is defined as the matrix corresponding to a bilinear form defined as $\hat{a}(u_h, v_h) := \hat{a}^0(u_h^0, v_h^0) + \hat{a}^F(u_h^F, v_h^F) + \hat{a}^W(u_h^W, v_h^W)$. The two bilinear forms $\hat{a}^0$ and $\hat{a}^F$ are themselves block diagonal, the blocks corresponding respectively to the subdomains and to the faces composing the interface. We shall show that the overall system enjoys $\mathrm{cond}(\hat{A}^{-1}A) \leq C\left(1 + \max_\ell \log \frac{H_\ell}{h_\ell}\right)^4$. where ℓ is a subdomain index. We will present an analysis of this preconditioner as well as numerical results obtained with the Feel++ framework.

Mini Symposium M2

Domain Decomposition for Porous Media Flow and Transport

Schedule Author Index Session Index

Organizers: Caroline Japhet and Michel Kern

Abstract

Porous media flow and transport have many applications such as far field simulations of underground nuclear waste disposal, geological storage of CO2, or reservoir engineering... A salient feature of subsurface flow and transport processes is the heterogeneity of the medium with physical properties ranging over several orders of magnitude. Other challenges presented by these models involve widely differing space-time scales. Accurately resolving these features requires fines meshes, and thus the solution of large systems. Domain decomposition methods are a very important part of a solution procedure. The aim of this minisymposium is to bring together scientists working in this field to report about recent developments. Work presented will range from space-time methods, to model coupling for multiphase flow, algebraic solution procedures, numerical zoom preconditioners, parallel simulators and non-linear extensions of the framework.

Part 1

Part 2

M2P1 Schedule

Domain Decomposition for Porous Media Flow and Transport

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 10:30-12:15
Location: Amphi
Chairman: Caroline Japhet and Michel Kern

10:35-11:00 : Oliver Sander
Discretizations for the Richards Equation Based on Kirchhoff Transformation
Abstract

11:00-11:25 : Thi Thao Phuong Hoang
Space-Time Domain Decomposition For Mixed Formulations of Transport Problems In Porous Media
Abstract

11:25-11:50 : Frédéric Nataf
Algebraic Domain Decomposition Methods for Highly Heterogeneous Problems
Abstract

11:50-12:15 : Zhangxin Chen
GPU-based Parallel Reservoir Simulators
Abstract

M2 Abstract

Part 2

Discretizations for the Richards Equation Based on Kirchhoff Transformation

Session Schedule Author Index Session Index M2

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Abstract

We consider a discretization of the Richards equation constructed by first applying the Kirchhoff transformation and then discretizing the transformed equation using first-order finite elements. If the permeability and saturation functions are independent of space only with respect to a partition of the domain, we transform separately on each subdomain and combine the transformed subproblems by nonlinear transmission conditions. We give various characterizations of the resulting discretizations, and demonstrate optimal a priori error bounds. The discretizations are solver-friendly in the sense that each subdomain problem becomes equivalent to a convex minimization problem, which can be solved efficiently using monotone multigrid methods. For the overall problem we use different substructuring methods, and show that they are robust with respect to large variations of the soil parameters.

Space Time Domain Decomposition Methods For Mixed Formulations of Transport Problems In Porous Media

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Abstract

The far field simulation of underground nuclear waste disposal site requires a high computational cost due to the widely varying properties of different materials, the different length and time scales, and the high accuracy requirements. Nonoverlapping domain decomposition methods allow local adaptation in both space and time and result in parallel algorithms. We have extended the optimized Schwarz waveform relaxation (OSWR) method, successfully used for finite elements and finite volumes, to the case of mixed finite elements with their local mass-conservation property. Another choice is the substructuring method, which has been shown to be efficient for steady state problems with strong heterogeneities. We study a time-dependent Schur complement method, which is the algebraic counterpart of the discrete Steklov Poincaré operator, and introduce the Neumann preconditioner as well as weight matrices (following work of De Roeck Le Tallec) designed to make the convergence speed independent of the heterogeneities. Both methods enable the use of local time steps when the subdomains have highly different physical properties. Their performance is illustrated on test cases suggested by nuclear waste disposal problems. This work is supported by ANDRA, the French Agency for Nuclear Waste Management.

Algebraic Domain Decomposition Methods for Highly Heterogeneous Problems

Session Schedule Author Index Session Index M2

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Abstract

We consider the solving of linear systems arising from porous media flow simulations with high heterogeneities. Using a Newton algorithm to handle the non-linearity leads to the solving of a sequence of linear systems with different but similar matrices and right hand sides. The parallel solver is a Schwarz domain decomposition method. The unknowns are partitioned with a criterion based on the entries of the input matrix. This leads to substantial gains compared to a partition based only on the adjacency graph of the matrix. From the information generated during the solving of the first linear system, it is possible to build a coarse space for a two-level domain decomposition algorithm that leads to an acceleration of the convergence of the subsequent linear systems. We compare two coarse spaces: a classical approach and a new one adapted to parallel implementation.

GPU-based Parallel Reservoir Simulators

Session Schedule Author Index Session Index M2

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Abstract

Large-scale reservoir simulation demands significant computational time so improving its computational efficiency becomes crucial. Graphics Processing Unit (GPU), a high-profile parallel processor with hundreds of microprocessors, offers great potential in parallel reservoir simulation because of its efficient power utilization and high computational efficiency. In addition, its cost is relatively low, making large-scale parallel reservoir simulation possible for most of desktop users. In this presentation several GPU-based parallel linear solvers and preconditoners will be discussed. They include the GMRES, BiCGSTAB and ORTHOMIN solvers and the incomplete LU (ILU) factorization, domain decomposition and algebraic multigrid preconditioners. These solvers and preconditoners have been coupled with an in-house black-oil simulator to speedup reservoir simulation. In the numerical experiments performed, the SPE 10 problem, a 3D heterogeneous benchmark model with over one million grid blocks, is selected to test the speedup of the resulting black-oil simulator. On the state-of-the-art CPU and GPU platforms, the new GPU implementation can achieve a speedup of over eight times in solving linear systems arising from this SPE 10 problem compared with the CPU implementation.

M2P2 Schedule

Domain Decomposition for Porous Media Flow and Transport

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 16:00-17:45
Location: Amphi
Chairman: Caroline Japhet and Michel Kern

16:05-16:30 : Bernd Flemisch
Model Coupling for Multiphase Flow in Porous Media
Abstract

16:30-16:55 : Paul-Marie Berthe
Space-Time Domain Decomposition with Finite volumes for Porous Media Applications
Abstract

16:55-17:20 : Jean-Baptiste Apoung Kamga
A Numerical Zoom Preconditioner for Discontinuous Galerkin Domain Decomposition Approximation of Darcy Flow
Abstract

17:20-17:45 : Anthony Michel
Time Space Domain Decomposition for Reactive Transport in Porous Media. Application to CO2 Geological Storage
Abstract

M2 Abstract

Part 1

Model Coupling for Multiphase Flow in Porous Media

Session Schedule Author Index Session Index M2

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Abstract

Numerical models for flow and transport in porous media are valid for a particular set of processes, scales, levels of simplification and abstraction, grids etc. The coupling of two or more specialised models is a method of increasing the overall range of validity while keeping the computational costs relatively low. Several coupling concepts are reviewed in this talk with a focus on the authors work in this field. The concepts are divided into temporal and spatial coupling concepts, of which the latter is subdivided into multi-process, multi-scale, multi-dimensional, and multi-compartment coupling strategies. Examples of applications for which these concepts can be relevant include groundwater protection and remediation, carbon dioxide storage, nuclear-waste disposal, soil dry-out and evaporation processes as well as fuel cells and technical filters. In particular, we focus on the coupling of single-phase compositional non-isothermal free flow and two-phase porous media flow.

Space-Time Domain Decomposition with Finite Volumes for Porous Media Applications

Session Schedule Author Index Session Index M2

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Abstract

In the context of porous media applications such as nuclear waste repositories, we consider the solving of time dependent advection-diffusion problems with possibly highly discontinuous coefficients, modelling radionucleides flow and transport in the underground. Because of the widely varying properties of the different materials, one need to use different time step and mesh size in different regions of the computational domain. The Optimized Schwarz Waveform Relaxation (OSWR) method, which has been developed over the last decade, is one possible strategy since it allows to use different space-time discretizations in subdomains, possibly nonconforming and needs a very small number of iterations to converge. This method has been analyzed with discontinous Galerkin method for the time discretization, so that rigorous analysis can be made for any degree of accuracy, and time steps can be adaptively controlled by a posteriori error analysis. On the other hand, using a Discrete Duality Finite Volume (DDFV) method allows to use any type of meshes in space, including highly non conforming meshes. Thanks to the discrete variational formulation of the scheme, local refinement of the meshes can be done with efficient a posteriori error estimators.

We design and study an extension of the DDFV scheme to time dependent advection-diffusion problems in the context of optimized Schwarz waveform relaxation, with a time discontinous Galerkin method. We propose a new discretization of the fluxes, which permit to prove the well-posedness of the nonconforming in time domain decomposition method and the convergence of the iterative solver to the global one domain scheme.

A Numerical Zoom Preconditioner for Discontinuous Galerkin Domain Decomposition Approximation of Darcy Flow

Session Schedule Author Index Session Index M2

Jean-Baptiste Apoung Kamga
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Abstract

Mixed discontinous Galerkin methods are well suited for the approximation of Darcy Flow when accurate approximation of the velocity is required. These methods are unfortunately too much memory consuming and can thus benefit from parallel programming in a distributed memory environment. But because of their modal approximation nature, their substructuration in the framework of non overlapping domain decomposition method is not straightforwards. In the present, by introducing the notion of *interface skeleton*, a substructure procedure in designed and the acceleration of the solution of the substructured problem is preformed with the help of the numerical zoom techniques. Two and three dimensional numerical tests are furnished for illustration.

Time Space Domain Decomposition for Reactive Transport in Porous Media. Application to CO2 Geological Storage

Session Schedule Author Index Session Index M2

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Abstract

Numerical modeling is the main way to reduce uncertainties about the long term evolution of CO2 storage in the underground. When super-critical CO2 is injected in an aquifer, it dissolves in brine, modifying its chemical properties. The resulting aqueous solution may react with the host rocks in the reservoir, the cements around the wells or the shale into the cap-rock. In order to predict the location and amount of rock modifications, we have developed coupled reactive transport and multiphase flow models. Up to now, these simulations are always restricted to coarse grids because they are too expensive in term of CPU-Time. However, we know that the main numerical difficulties are usually localized in space and time. As a consequence, these difficulties may be strongly reduced if we could isolate the reactive zones and solve them separately. SWR time space domain decomposition techniques have already proved their efficiency in solving such a problem for scalar linear convection-diffusion equations and recently for semi-linear scalar problems. However, there remains to prove it for system of nonlinear equations like realistic multi-species reactive transport problems.

With the support of the ANR-SHPCO2 project, we have implemented a SWR domain decomposition framework and a global reactive transport module in the CEA-IFPEN parallel platform Arcane. By allowing the domain decomposition in space to be dynamic and the time step to be adapted by sub-domain, we have been able to track the reactive zones. Even if these results are already impressive, we haven't been able to measure the effective gain of performance of this strategy. As a consequence, a new performance study is necessary.

In this paper we will present the last performance results obtained for a few study test cases and discuss their sensitivity to the mesh, the size of the overlap and the optimized parameters.

Mini Symposium M3

Finite Elements for First-Order System Formulations of Interface Problems

Schedule Author Index Session Index

Organizers: Pavel Bochev and Gerhard Starke

Abstract

Least squares finite element methods for first-order system formulations of models in fluid and solid mechanics have become increasingly popular in recent years. Such methods allow simultaneous approximation of all process variables by finite element spaces that are not subject to joint stability (inf-sup) conditions. Norm-equivalence of least-squares functionals leads to symmetric and positive definite linear systems and optimal error estimates with respect to suitable error norms. These advantages lead to simpler formulations of the coupling conditions arising in connection to interface problems. The talks in this minisymposium are concerned with different aspects of the treatment of coupling conditions for interface problems in the context of first-order system formulations. This includes the treatment of moving interfaces modeled by phase-field or level-set methods. An important issue for numerical approximations of flow models is the accuracy of mass conservation. This will also be a central topic in these contributions. The session will also address iterative methods based on domain decomposition ideas for such interface models.

M3 Schedule

Finite Elements for First-Order System Formulations of Interface Problems

Schedule Author Index Session Index

Date: Thursday, June 28
Time: 10:30-12:15
Location: Turing
Chairman: Pavel Bochev and Gerhard Starke

10:35-11:00 : James Adler
Constrained First-Order System Least Squares for Improved Mass Conservation
Abstract

11:00-11:25 : Pavel Bochev
Least-Squares Methods for Mesh-Tying
Abstract

11:25-11:50 : Fleurianne Bertrand
Least Squares Methods with Interface Approximation for Two Phase Stokes Flow
Abstract

11:50-12:15 : Steffen Münzenmaier
Least Squares Finite Element Methods for Coupled Generalized Newtonian Stokes-Darcy Flow
Abstract

Constrained First-Order System Least Squares for Improved Mass Conservation

Session Schedule Author Index Session Index M3

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Abstract

In complex fluid flow simulations, there is a tradeoff between obtaining solutions that are accurate with a reasonable amount of computational work and satisfying certain conservation laws exactly. For instance, in incompressible fluid flow, conservation of mass takes the form of making sure the fluid velocities are divergence-free. In magnetohydrodynamics, one must satisfy conservation of mass as well as the solenoidal constraint that the magnetic field is divergence-free (i.e. there are no magnetic monopoles). Many methods have been applied to such systems, some being conservative at the cost of accuracy of the momentum equations and others at the cost of efficiency in the solver. First-order system least-squares approaches have also been applied and yield efficient methods for approximating solutions to coupled fluid mechanics problems. However, without proper care, the auxiliary conservation equations may not be solved to a sufficient accuracy. In this talk, we propose a constrained least-squares approach, where we augment the first-order system and minimize the least-squares functional subject to some constraint. Here, we only look at a simple diffusion equation, but present the main ideas, including what types of finite-element spaces to use and the solution algorithm. A domain decomposition or multilevel approach is employed to solve the constrained problem on local subdomains and coarse grids and used to update the unconstrained solution as needed. Thus, we approximate the solution accurately and efficiently using the least-squares method, while still conserving the appropriate quantity.

Least-Squares Methods for Mesh-Tying

Session Schedule Author Index Session Index M3

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Abstract

Mesh tying refers to the finite element solution of Partial Differential Equations (PDEs) on a union of independently meshed subdomains. This task arises in computational modeling of scientific and engineering problems posed on domains with complex geometries. Efficient grid generation for such domains often requires separate meshing of their parts. If the interfaces between the parts are curved, their independent meshing generally leads to adjoining surfaces that do not coincide spatially. A minimal requirement for any mesh-tying method is a consistency condition called *patch test*. A method passes a patch test of order k if it can recover solutions of the governing equations that are global polynomials of degree k. Mesh-tying formulations based on standard Galerkin methods experience difficulties passing such tests because the presence of gaps and overlaps between the domain parts causes physical energy to be undercounted or overcounted. As a result, most state of the art methods only pass patch tests of order 1, i.e., they preserve at most globally linear solutions. In this talk we present an alternative approach which utilizes least-squares variational principles. A least-squares functional is a sum of the residuals of the PDEs measured in Sobolev space norms. As a result, such a functional always vanishes at the exact solution. By exploiting this property, we formulate a least-squares method for mesh-tying, which automatically passes a patch test of the same order as the finite element space employed in its definition. Specifically, because in mesh-tying applications the non-coincident interfaces are close, small interface perturbations eliminate the voids and create overlapping domains. Then, by measuring residual energy and not physical energy, a least-squares functional may measure energy redundantly in those subdomain intersections. Numerical results demonstrate the potential of this idea.

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[1] Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energys National Nuclear Security Administration under contract DE-AC04-94AL85000.

Least Squares Methods with Interface Approximation for Two Phase Stokes Flow

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Abstract

We consider the coupled problem with Stokes flow in two subdomains separated by an interface. At the interface, continuity of the velocity and the momentum balance condition for the stress tensor need to be imposed. The interface is characterized by a level set function which satisfies an appropriate transport equation and the problem can be written as a domain decomposition problem.

In this talk we first present how the stationary Stokes problem can be written as a first order system. For numerical results a combination of $H(\mathrm{div})$-conforming Raviart-Thomas and standard H^1-conforming elements is used.

After that we analyze the effect of approximated flux boundary conditions on Raviart-Thomas finite elements in order to get the effect of the approximated interface on the momentum balance condition. In particular, we present an estimate for the normal flux on interpolated boundaries.

Least Squares Finite Element Methods for Coupled Generalized Newtonian Stokes-Darcy Flow

Session Schedule Author Index Session Index M3

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Abstract

In this talk we consider a coupled Stokes-Darcy flow problem. The domain is decomposed into a fluid and a porous media region. In the fluid region the problem is given by the Stokes equations for generalized Newtonian flow whereas in the porous media a Darcy equation for generalized Newtonian flow has to be solved. Both flows are treated as a first order system in a pseudostress-velocity formulation for the Stokes problem and a volumetric flux-hydraulic potential formulation for the Darcy problem. The coupling along an interface is done by using the well known Beavers-Joseph-Saffman interface condition. A least squares finite element method is used for the numerical approximation of the solution. It will be shown that the least squares functional corresponding to the nonlinear first order system is an efficient and reliable error estimator which allows for adaptive refinement.

Mini Symposium M4

On the Origins of Domain Decomposition Methods

Schedule Author Index Session Index

Organizers: Martin J. Gander

Abstract

Domain decomposition methods have been developed in various contexts, and with very different goals in mind. This minisymposium has the purpose to show in detail where some of the most successfull domain decomposition methods come from, and why they have been invented. It will trace the history of the alternating Schwarz method, which fixed an important gap in Riemann's proof of the famous Riemann mapping theorem, the development of substructuring in the engineering community at Boeing by Przemieniecki around the same time of the re-invention of the finite element method, the formulation of the FETI (Finite element Tearing and Interconnect) method in Paris and its evolution, and the revolution brought by the invention of the additive Schwarz method and the abstract Schwarz framework.

M4 Schedule

On the Origins of Domain Decomposition Methods

Schedule Author Index Session Index

Date: Friday, June 29
Time: 10:30-12:15
Location: I50
Chairman: Martin J. Gander

10:35-11:00 : Martin J. Gander
On the Origins of the Alternating Schwarz Method
Abstract

11:00-11:25 : Xuemin Tu
Origin of Iterative Substructuring Methods
Abstract

11:25-11:50 : Francois-Xavier Roux
FETI: Finite Element Tearing and Interconnecting
Abstract

11:50-12:15 : Olof Widlund
Early Work on Two-Level Schwarz Algorithms and where it had led us
Abstract

On the Origins of the Alternating Schwarz Method

Session Schedule Author Index Session Index M4

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Abstract

Many people know that Hermann Amandus Schwarz invented his alternating method in order to show that solutions to Laplace's equation exist on domains with quite complicated shapes; but why was this so important in 1869? We answer this question by going back to Riemann's PhD thesis from 1851, where on the last pages, one can find the famous Riemann Mapping Theorem. In the proof, which is constructive, there is an important gap, and when Riemann was challenged, he simply replied that to close the gap, it suffices to use the Dirichlet principle, which Dirichlet was teaching in his lectures to undergraduate students during the same time. Unfortunately, Weierstrass quickly found a counterexample, in which the Dirichlet principle led to an incorrect conclusion. While Riemann replied to Weierstrass that his theorem nevertheless is true, an international competition was launched to close the gap in the proof of the Riemann mapping theorem. This theorem can be considered to be the foundation of Riemann's theory of analytic functions, and it was Schwarz who managed to close this gap with his alternating method.

Origin of Iterative Substructuring Methods

Session Schedule Author Index Session Index M4

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Abstract

Iterative substructuring methods (Schur complement methods) are non-overlapping domain decomposition methods. The name is borrowed from the structural engineering community back to 1970s. These methods are preconditioned iterative methods for the subdomain boundary values (primal methods) or the normal derivatives (dual methods). The active development of these methods was started from the early 1990s. The most popular methods among this class are BDDC and FETI–DP algorithms which originate from the balancing NN and one-level FETI methods. In this talk, we will trace these methods back to 1963 by Przemieniecki and give a review of the development.

FETI: Finite Element Tearing and Interconnecting

Session Schedule Author Index Session Index M4

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Abstract

In this paper we recall the origin of the FETI method, initially derived from mixed finite element methods with discontinuous solutions and Lagrange multipliers for enforcing the continuity condition. The fundamental ingredients of the original FETI method, the use of discrete, eventually redundant Lagrange multipliers, the issue with ill-posed local Neumann problems that leads to the natural coarse grid projector of FETI, are presented. The two-level FETI method ideas with enrichment of the coarse space is introduced with its main drawback due to the fact that the coarse operator is not sparse. Then the best known extensions of the method that lead to a sparse coarse preconditioner, the FETI-DP and the FETI-2LM methods are presented.

Early Work on Two-Level Schwarz Algorithms and where it had led us

Session Schedule Author Index Session Index M4

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Abstract

Around the time of DD1, it was realized that the classical Schwarz algorithm could be improved by adding an additional coarse level solver. In addition, an alternative additive variant was introduced to simplify the parallelization of these iterative methods; the additive variant had in fact been introduced earlier in Novosibirsk by Matsokin and Nepomnayaschikh. The additive algorithms inspired the development of an abstract Schwarz frame work, which has been important to the development of many domain decomposition algorithms. The original algorithms required that the fine mesh is a refinement of a coarse conventional finite element model. Progress in removing this assumption will be discussed including the introduction of coarse methods based on quite irregular coarse basis elements and the successful use of these algorithms for almost incompressible elasticity. The family of abstract Schwarz methods has also been extended and several hybrid methods of this kind will be presented.

Mini Symposium M5

Exotic Coarse Spaces for Domain Decomposition Methods

Schedule Author Index Session Index

Organizers: Martin J. Gander, Laurence Halpern, Kévin Santugini

Abstract

In the absence of a coarse grid, domain decomposition methods can only exchange information between adjacent subdomains, and hence such methods can never converge in less iterations than the diameter of the connectivity graph between subdomains. Domain decomposition methods without a coarse grid can therefore not be scalable. By adding a coarse grid, it becomes possible to transfer information globally at each iteration, and one can then hope to obtain a scalable method, i.e. a method where the number of iteration does not (or only weakly) depend on the number of subdomains. But how should such a coarse grid be constructed ? Is it really necessary to base the long range information exchange on a grid, or is it also possible to construct a mechanism of long range interaction by other coarse space components?

This minisymposium gives you an overview of developments in this direction over the last few years. Coupling coarse grids with elaborate domain decomposition methods has proved to be difficult, and many other techniques can be found in the literature to add coarse space components to domain decomposition algorithms. Some of these attempts are based on algebraic considerations like deflation, some work directly on discrete subsets of the unknowns, like wirebasket techniques, and again others try to reason at the continuous level. Designing efficient domain decomposition algorithms with a coarse space requires great care and insight, and is a major area of current research in domain decomposition methods.

M5 Schedule

Exotic Coarse Spaces for Domain Decomposition Methods

Schedule Author Index Session Index

Date: Monday, June 25
Time: 10:30-12:15
Location: I50
Chairman: Martin J. Gander, Laurence Halpern, Kévin Santugini

10:35-11:00 : Martin J. Gander
A new Coarse Grid Correction for RAS
Abstract

11:00-11:25 : Clark Dohrmann
Lower Dimension Coarse Spaces for Overlapping Schwarz Algorithms
Abstract

11:25-11:50 : Jörg Willems
Spectral Coarse Space Construction in Robust Multilevel Methods
Abstract

11:50-12:15 : Kévin Santugini
Discontinuous Coarse Space Corrections (DCS) for Optimized Schwarz Methods
Abstract

A new Coarse Grid Correction for RAS

Session Schedule Author Index Session Index M5

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Abstract

Restricted Additive Schwarz (RAS) has become one of the most popular Schwarz method, especially for non-symmetric problems. In contrast to Additive Schwarz, there is however no comprehensive convergence theory available for RAS: for the one level variant, there is a proof of equivalence to the classical parallel Schwarz method introduced by Lions. We are interested in the precise interplay between the RAS iterates and the coarse grid correction. We show that for a classical coarse grid correction with one (or a few) nodes within each subdomain, the interaction with the RAS iterates can lead to oscillations which are very unfavorable for the convergence of the method. Placing the coarse grid correction nodes into the overlap removes these oscillations and leads to an overall much faster two level method. For one dimensional problems, one can even obtain a method which converges in one iteration, i.e. one parallel subdomain solve, and one coarse grid correction, provided the coarse grid shape functions have certain properties.

Lower Dimension Coarse Spaces for Overlapping Schwarz Algorithms

Session Schedule Author Index Session Index M5

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Abstract

We present some new coarse spaces for overlapping Schwarz algorithms of lower dimension than earlier ones. The first one is a vertex-based coarse space for scalar problems in the plane which is significantly smaller than a previous one based on both subdomain vertices and edges. In addition, the condition number estimate associated with the vertex-based coarse space is improved by a factor of $\log(H/h)$ in comparison to the estimate for the richer coarse space. Lower dimension coarse spaces and some theoretical results are also presented for three dimensional problems. In comparison to iterative substructuring methods such as FETI-DP or BDDC, a vertex only coarse space does not suffer from a linear factor of H/h in condition number estimates. Numerical examples are presented to confirm the theory and to demonstrate the utility of the coarse spaces.

Spectral Coarse Space Construction in Robust Multilevel Methods

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Abstract

In the design of robust two-level domain decomposition methods the choice of the coarse spaces is crucial. Here the term “robust” refers to convergence rates which are independent of problem and mesh parameters. Important instances of such problem parameters are in particular (highly) varying coefficients. For symmetric positive definite (SPD) systems the objective of achieving robustness with respect to mesh parameters has been successfully addressed by e.g. two-level alternating Schwarz methods employing standard coarse spaces. For arbitrarily general configurations robustness with respect to (large) coefficient variations has proved to be a more complicated goal.

For the stationary heat equation Efendiev and Galvis introduced a coarse space construction based on local generalized eigenvalue problems. Using only those eigenmodes in the coarse space construction corresponding to eigenvalues below a predefined threshold resulted in an overlapping additive Schwarz method yielding a condition number independent of variations in the coefficients. The approach of using local spectral problems for constructing coarse spaces was then further generalized to abstract SPD operators in a paper to which the author contributed.

In the present contribution we consider the generalization of the two-level approach to a multilevel method. This extension, which is carried out in the framework of (nonlinear) algebraic multilevel iterations (AMLI), is mainly motivated by the high numerical cost for solving generalized eigenvalue problems, which limits the applicability of the two-level method, since either the local generalized eigenvalue problems or the global coarse problem become too big. Our approach is analyzed in a rather general setting, which is shown to be applicable to the stationary heat equation, the equations of linear elasticity, and equations arising in the solution of Maxwell’s equations.

Analogous to the two-level method the crucial ingredient in our approach is the construction of a hierarchy of spaces $V_L \subset V_{L-1} \subset \ldots \subset V_l \subset V_{l-1} \subset \ldots \subset V_0$ based on generalized eigenvalue problems. Our analysis essentially relies on an inexact stable decomposition property for two consecutive spaces $V_{l+1} \subset V_l$ with a constant that is independent of problem and mesh parameters. Based on this we define a nonlinear AMLI whose convergence rate is independent of problem and mesh parameters. We present some numerical results for the scalar elliptic equation for high contrast multiscale geometries verifying our analytical findings.

Discontinuous Coarse Space Corrections (DCS) for Optimized Schwarz Methods

Session Schedule Author Index Session Index M5

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Abstract

In this presentation, we explain why continuous coarse spaces are a suboptimal choice when combined with domain decomposition methods that have discontinuous iterates, like Optimized Schwarz Methods (OSM), or Restricted Additive Schwarz methods (RAS). As an alternative, we propose discontinuous coarse spaces for such domain decomposition methods. For linear problems, we explain the design of one particular discontinuous coarse space and present an algorithm that computes an efficient discontinuous coarse space corrector for the special case of an OSM. While the algorithm is suitable for higher dimensions, it has the special property of converging in a single coarse iteration for one-dimensional linear problems. We expect Discontinuous Coarse Spaces (DCS) to become standard practice for methods with discontinous iterates in the coming decade.

Mini Symposium M6

Heterogeneous Domain Decomposition Methods

Schedule Author Index Session Index

Organizers: Oliver Sander and Marco Discacciati

Abstract

For the modeling of most application problems different type of equations may be required. This may be due either to the fact that such problems consist of more than one single phenomenon, or that the same phenomenon is described by equations of different type in different regions of the computational domain with the aim of reducing the overall computational cost.

If the domains of definition of these equations are separated, heterogeneous domain decomposition methods come into play. Coupling conditions between the different models need to be formulated and investigated analytically. Solution algorithms must be constructed and their convergence properties must be studied. The sheer number of combinations make this field abound with interesting mathematical and numerical problems.

This minisymposium aims at informing about recent developments in this field.

Part 1

Part 2

M6P1 Schedule

Heterogeneous Domain Decomposition Methods

Schedule Author Index Session Index

Date: Monday, June 25
Time: 10:30-12:15
Location: Turing
Chairman: Oliver Sander and Marco Discacciati

10:35-11:00 : Heiko Berninger
Strategies for the Coupling of Ground and Surface Water
Abstract

11:00-11:25 : Pablo Javier Blanco
Coupling Dimensionally-Heterogeneous Models in Hemodynamics Simulations
Abstract

11:25-11:50 : Eva Casoni
Zonal Modeling Approach in Aerodynamic Simulation
Abstract

11:50-12:15 : Paola Gervasio
Virtual Control Method for Heterogeneous Problems
Abstract

M6 Abstract

Part 2

Strategies for the Coupling of Ground and Surface Water

Session Schedule Author Index Session Index M6

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Abstract

We discuss four different situations in which surface water is coupled to ground water where the latter is considered partially saturated and modelled by the Richards equation. The strategies for the coupling that we want to discuss involve modelling aspects as well as discretization and solution techniques. The modelling of the coupling will be provided by mass conservation, i.e., continuity of the water flux, and either pressure continuity or clogging effects which lead to pressure discontinuities across the interface. Models for the surface water that we use are given by a water reservoir modelled by a single ODE and including seepage faces, shallow water equations and, finally, the ponding of water modelled by various ODE's (one for each point on the interface) and influenced by rainfall. The treatment of Signorini-type outflow conditions is achieved by a special discretization of the Kirchhoff-transformed Richards equation that leads to convex minimization problems. The coupling is treated either by time-explicit or time-implicit discretization where the latter is solved by heterogeneous domain decomposition methods like Dirichlet-Neumann- or Robin-Neumann-type iterations. We illustrate the different coupling strategies and models by several numerical examples.

Coupling Dimensionally-Heterogeneous Models in Hemodynamics Simulations

Session Schedule Author Index Session Index M6

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Abstract

We will present a generic black-box approach for the strong coupling of heterogeneous flow models. The strategy is envisaged for problems arising in computational hemodynamics, but extensions to other field of physics and engineering are straightforward. Specifically, we will quickly revisit existing methodologies to carry out the coupling of dimensionally-heterogeneous models, and then we will focus on transient non-linear problems as those encountered when modeling the blood flow in the cardiovascular system. The proposed methodology is employed to split a coupled 3D-1D-0D closed-loop model of the cardiovascular system into the corresponding black-boxes standing for the 3D (specific vessels), 1D (systemic arteries/peripheral vessels) and 0D (venous/cardiac/pulmonary circulation) models. In addition, the acceleration of convergence in transient simulations will be discussed. We present several examples of application showing the robustness and suitability of this approach.

Zonal Modeling Approach in Aerodynamic Simulation

Session Schedule Author Index Session Index M6

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Abstract

Zonal solvers are used to simulate CFD processes by physically partitioning a flow domain into several regions with the aim of improving the computational overhead while maintaining accuracy. The main idea is to use a simpler model in particular regions in order to speed up the solver and use the full model, which is computationally expensive, only where it is essential to capture the appropriate physics. In aerodynamic simulations, ignoring the viscous effects far from sharp layers leads to the coupling of Navier-Stokes and Euler equations or Euler and full potential equations for irrotational isentropic flows.

In this talk we will focus on the analysis of two forms of the interface conditions for the scalar non-linear equation, also including the development of an efficient parallel strategy. Finally, their application to aerodynamic model problems that involve the coupling between Euler and Navier-Stokes will be shown.

Virtual Control Method for Heterogeneous Problems

Session Schedule Author Index Session Index M6

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Abstract

In this communication we present the Virtual Control Method (VCM) to address heterogeneous and multiphysics problems by overlapping subdomain splitting. VCM was proposed by Glowinski et al. in the Eighties, reconsidered by J.L. Lions and O. Pironneau at the end of the Nineties for homogeneous couplings and then by the authors of this talk for heterogeneous problems. The basic idea of VCM consists in introducing suitable functions called virtual controls which play the role of unknown boundary data on the interfaces of the decomposition and in minimizing in a suitable norm (defined on either the overlap or the interfaces) the difference between the two solutions defined on the same overlap region. Thus the convergence rate improves noticeably with respect to that of Schwarz method, especially when thin overlap between subdomains is considered. Depending on the norm used in the minimization process, the VCM can be considered to solve differential problems either characterized by high regularity of the solution (as in homogeneous problems) or, on the contrary, by a low global regularity, as it happens in heterogeneous problems associated to multi-physics models. In this talk we consider the coupling between advection and advection-diffusion equations featuring boundary layers, as well as the Stokes/Darcy coupling, with the aim of discussing both theoretical and computational aspects in applying VCM.

M6P2 Schedule

Heterogeneous Domain Decomposition Methods

Schedule Author Index Session Index

Date: Monday, June 25
Time: 16:00-17:45
Location: Turing
Chairman: Oliver Sander and Marco Discacciati

16:05-16:30 : Simona Perotto
Hierarchical Model Reduction: a Domain Decomposition Approach
Abstract

16:30-16:55 : Franz Rammerstorfer
Mortar FEM/BEM Coupling for Poroelastodynamics
Abstract

16:55-17:20 : Human Rezaijafari
A Stabilized Hybrid Discontinuous Galerkin Scheme for the Nonisothermal Coupling of Stokes and Darcy Flow
Abstract

17:20-17:45 : Anton Schiela
Energy Minimizers of the Coupling of a Cosserat Rod to an Elastic Continuum
Abstract

M6 Abstract

Part 1

Hierarchical Model Reduction: a Domain Decomposition Approach

Session Schedule Author Index Session Index M6

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Abstract

In this presentation we focus on a model reduction strategy, known as hierarchical model (Hi-Mod) reduction, suited to deal with physical phenomena characterized by a dominant dynamics. As a paradigm, we can mention, for instance, flows through porous media, flows in tubular domains (as in haemodynamics) or in a channel network (as in hydrodynamics).

The idea is to reduce the reference 2D or 3D (full) problem to a 1D (reduced) model associated with the leading direction, by suitably lumping the information along the less significant transverse directions. In particular, the reduced model is locally characterized by a different level of detail in describing the phenomenon at hand according to the local relevance of the transverse dynamics. For this purpose, we resort to different discretization schemes in correspondence with the leading and the transverse directions. The leading direction is spanned by a classical finite element scheme, while the transverse ones are expanded into a modal basis. Thus the level of detail of the reduced model locally varies by employing a different number of modal functions in different areas of the domain. A domain decomposition approach is used to enforce suitable matching conditions among the regions characterized by a different number of modes ([1, 2]).

In particular, the actual goal is to devise a model-adaptive procedure, to automatically detect the areas as well as the number of modal functions to be locally employed.

References

[1] A. Ern, S. Perotto and A. Veneziani, Hierarchical model reduction for advection-diffusion-reaction problems. In *Numerical Mathematics and Advanced Applications*, Springer-Verlag, Berlin Heidelberg, K. Kunisch, G. Of, O. Steinbach Eds. (2008), 703-710. Proceedings of the 7th European Conference on Numerical Mathematics and Advanced Applications.

[2] S. Perotto, A. Ern and A. Veneziani. Hierarchical local model reduction for elliptic problems: a domain decomposition approach. Multiscale Model. Simul., **8** (2010), no. 4, 1102–1127.

Mortar FEM/BEM Coupling for Poroelastodynamics

Session Schedule Author Index Session Index M6

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Abstract

In many engineering applications the numerical simulation of wave propagation phenomena in porous media is of great interest, e.g., in soil mechanics. In this work, aspects of time-dependent surface-coupled problems using different discretization techniques in different subdomains are studied. Based on Biot's theory the governing equations for the linear poroelastic continua are formulated. The coupling is done within the framework of Tearing and Interconnecting methods with finite elements (FETI) and accordingly boundary elements (BETI). This are special non-overlapping domain decomposition methods, based on the realization of Dirichlet-to-Neumann-maps (DtN-maps) for each subdomain separately. Their representation is independent of the local discretization scheme. By means of a classical FEM discretization we use a Newmark scheme in time. The discrete DtN-map for a single subdomain problem can be realized by eliminating the inner unknowns via Schur complement. In case of boundary elements, symmetric Galerkin as well as collocation schemes are applicable. For time discretization the Convolution Quadrature Method is applied. Again, using the Schur complement gives the discrete DtN-map. The continuity of the field across the interfaces is ensured by Lagrange Multipliers. Instead of the conventional node-by-node coupling this formulation is based on the Mortar Method, i.e. the weak coupling is performed in terms of Lagrange multiplier spaces. Therewith, we gain the flexibility of choosing different triangulations for each subdomain, which do not have coincident nodes. At the end, some numerical results in 3D are given to verify the presented method and to illustrate the flexibility of the algorithm.

A Stabilized Hybrid Discontinuous Galerkin Scheme for the Nonisothermal Coupling of Stokes and Darcy Flow

Session Schedule Author Index Session Index M6

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Abstract

The simulation of coupled free flow and porous media flow is of special interest in many fields of application, e.g. groundwater contamination and filtration problems. In this work, we consider an instationary, nonisothermal model to simulate transport processes where the (Navier-)Stokes and Darcy equations are used to describe the motion of the free and the porous media flow. The model is based on a two-domain approach and on nonisothermal compositional submodels where the employed coupling conditions for mass, momentum and energy are based on flux continuity and thermodynamic equilibrium (see [1]). Following the work of [2] we make use of divergence-conforming Finite Elements to discretize the coupled system. We apply a mixed Discontinuous Galerkin and Finite Element method to the (Navier-)Stokes and Darcy equations, respectively. Based on the idea of having the same error order within both subdomains we allow the use of Finite Elements of possibly different order as well as nonmatching meshes at the interface. The transport and energy-balance equations are treated with a Flux-Corrected-Transport Finite Element method (FEM-FCT, see [3]), to supress unphysical oscillations due to the convection dominated case. We present numerical examples to verify the functionality of the given coupling concept with special focus on the transfer processes across the interface.

References

[1] K. Mosthaf, K. Baber, B. Flemisch, R. Helmig, A. Leijnse, I. Rybak and B. Wohlmuth, *A new coupling concept for two-phase compositional porous media and single-phase compositional free flow*, Water Resour. Res. **47** (2011), 1-19.
[2] G. Kanschat and B. Rivière, *A strongly conservative finite element method for the coupling of Stokes and Darcy flow*, J. Comput. Phys. **229** (2010) 5933-5943
[3] D. Kuzmin, M. Möller and S. Turek, Multidimensional FEM-FCT schemes for arbitrary time-stepping, Int. J. Numer. Meth. **42** (2003), no 3, 265-295

Energy Minimizers of the Coupling of a Cosserat Rod to an Elastic Continuum

Session Schedule Author Index Session Index M6

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Abstract

We formulate the static mechanical coupling of a geometrically exact Cosserat rod to an elastic continuum. The coupling conditions accommodate for the difference in dimension between the two models. Also, the Cosserat rod model incorporates director variables, which are not present in the elastic continuum model. Two alternative coupling conditions are proposed, which correspond to two different configuration trace spaces. For both we show existence of solutions of the coupled problems. We also derive the corresponding conditions for the dual variables and interpret them in mechanical terms.

Mini Symposium M7

Domain Decomposition, Preconditioning and Solvers in Isogeometric Analysis

Schedule Author Index Session Index

Organizers: Lourenço Beirão da Veiga, Michel Bercovier, Simone Scacchi

Abstract

Isogeometric Analysis (IGA) is a novel and extremely promising numerical methodology for the analysis of PDE problems, that integrates Computer Aided Design (CAD) geometric parametrization and Finite Element analysis. Isogeometric Analysis, introduced in 2005 by T.J.R. Hughes and co-workers, is having a strong impact on the engineering community, with a large amount of publications and computer codes being developed in a few years. Very recently, the research community of this quickly growing field has started to tackle the design of efficient solvers for IGA discrete systems, and in particular of Domain Decomposition methods yielding parallel and scalable IGA preconditioners. The high (global) regularity of the NURBS spaces employed in IGA discretizations introduces both new difficulties and opportunities for the construction and analysis of novel solution techniques. The aim of the minisymposium is to bring together researchers in both fields of Isogeometric Analysis and Domain Decomposition, focusing on the latest developments and fostering new research.

Part 1

Part 2

Part 3

M7P1 Schedule

Domain Decomposition, Preconditioning and Solvers in Isogeometric Analysis

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 10:30-12:15
Location: Amphi
Chairman: Lourenço Beirão da Veiga, Michel Bercovier, Simone Scacchi

10:35-11:00 : Remi Abgrall
Isogeometric Analysis for Compressible Fluid Dynamics
Abstract

11:00-11:25 : Michel Bercovier
Isogeometric Analysis and Schwarz Non-Matching Overlapping Domain Decomposition Methods
Abstract

11:25-11:50 : Victor M. Calo
Solver Performance for Higher-Continuous Basis
Abstract

11:50-12:15 : Krishan P. S. Gahalaut
Multigrid Solver for Isogeometric Discretization
Abstract

M7 Abstract

Part 2

Part 3

Isogeometric Analysis for Compressible Fluid Dynamics

Session Schedule Author Index Session Index M7

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Abstract

During high order simulations, the subparametric discretization used for geo-metry's representation (usually piecewise-linear) may lead to errors dominating errors related to the variable field discretization. For instance, solving the conservative form of the Euler equations generate a spurious entropy that spoil the solution.
We look at an isogeometric analysis aproach to solve this problem: in such methods, we use the same basis fonctions to represent the variables field and to discretize the geometry, what ensure the same order of errors for the geometric approximation and the variables discretization.
Widely used in CAO, NURBS (Non Uniform Rational B-Spline) basis functions allow an exact representation of complicated geometries that we can meet in fluid mechanic and form an ideal family of basis functions for isogeometric ana-lysis. A lot of works coupling NURBS and finite elements method have already shown that the exact representation of geometric model improve significantly the numerical results, both on quadrangular and triangular meshes. In our work, we focus on a scheme of Residual Distribution Scheme-type (RDS) which give an interesting alternative to classical finite volume schemes: they are more accurate and their stencil is more compact.
In this talk, we will first present the adaptation to isogeometric analysis of a Lax-Friedrichs-type RDS by the implementing of NURBS basis function in it. Then, we will explain how to generate NURBS meshes and how to use mesh adaptation with NURBS meshes to reduce approximation error. At least, to illustrate the work, we will show some isogeometric numerical results for compressible fluid dynamics and we will compared them with the results provided by the same scheme on a piecewise-linear mesh.

Isogeometric Analysis and Schwarz Non-Matching Overlapping Domain Decomposition Methods

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Abstract

Isogeometric Analysis (IGA) is a recent technique for the discretization of Partial Differential Equations (PDEs). One uses basis functions developed for Computer Aided Geometry for Design (CAGD) such as Non Uniform B-Splines (NURBS) to define the same global isoparametric transformation for the ”exact” computational domain and for the basis functions for the PDE solution [1]. In practice objects (i.e. domains) are made up of collections of trimmed patches of CAGD transformations defined through boolean operations such as union, difference or intersection (Constructive Solid Geometry:CSG). Hence it is natural to consider Domain Decomposition methods as candidate solvers for ”real life' domains. Each atom of the CSG construct can be considered as a simple domain, where IGA is directly implemented. A different application of DD to IGA is analyzed in [2].

In the present work we study the simplest Schwarz Additive Domain Decomposition Method (SADDM) [3] We suppose that our primitive patches are overlapping at least pair wise and that the respective isoparametric transformations are *non matching*: the pair of reference grid and knots defining each physical domain are not related. Since at intersection the boundary of one domain is a trimming line (or surface) for its neighbor , we developed an efficient method to interpolate on a trimming line. The implementation is done then using a standard open source IGA code for each domain, GeoPDEs [4] and illustrated on several domains in 2 and 3D. As we have shown elsewhere there is no maximum principle for IGA approximations in 2D or 3D. Hence one needs some preconditioning to ensure convergence as well as for large number of domains. We will discuss several types of preconditioning: such as lower degree approximations, multi-grid in relation to IGA.

[1] J.A. Cottrell,T.J.R. Hugues,Y. Bazilevs: Isogeometric Analysis ,Wiley,UK, 2009.

[2] L. Beirao da Veiga, D. Cho, L. Pavarino, S. Scacchi: Overlapping Schwarz Methods for Isogeometric Analysis, preprint IMATI-CNR 8PV11/5/0, 2011.

[3] A. Toselli, O. Widlund: Domain Decomposition Methods-Algorithms and Theory, Springer, 2004.

[4] C. de Falco, A. Reali, R. Vazquez. GeoPDEs: a research tool for Isogeometric Analysis of PDEs. *Advances in Software Engineering*,40 (2011), 1020-1034.

Solver Performance for Higher-Continuous Basis

Session Schedule Author Index Session Index M7

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Abstract

Many applications take advantage of the higher-order continuity provided by isogeometric analysis. The approximability of higher continuous spaces per degree of freedom is superior to that of traditional finite element spaces. This suggests that isogeometric analysis links geometry to analysis and is a more efficient method. However, the connection between the number of degrees of freedom and the order of approximation is not a measure of efficiency. We also need to consider the cost associated with solving the linear system generated by the discrete weak form. The system formed by higher continuous basis has a denser structure than that of standard finite element spaces. The support of more continuous functions extends beyond element boundaries resulting in a more connected matrix. The more connected matrix degrades the performance of direct solver. Similarly for iterative solvers, the matrices resulting from higher-continuous basis functions contain more non-zero entries. The number of non-zero entries directly relates to the cost of the matrix-vector multiplication, a major contribution to the cost of the iterative method. We will present estimates for the increase in cost. However, developing cost estimates for iterative solvers is far more complex than in direct solvers. While the cost per iteration increases, the number of iterations is also an important component. Thus, our analysis includes estimates for the iteration count required for convergence using conjugate gradients with some popular preconditioners. Our numerical results indicate how to construct optimal preconditioners for the Laplace problem.

Multigrid Solver for Isogeometric Discretization

Session Schedule Author Index Session Index M7

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Abstract

We present the (geometric) multigrid methods for the isogeometric discretization of Poisson equation. The smoothing property of the relaxation method, and the approximation property of the intergrid transfer operators are analyzed for two-grid and multi-grid cycles. It is shown that the convergence of the multigrid solver is independent of the discretization parameter h, and that the overall solver is of optimal complexity. Supporting numerical results are provided for the smoothing property, the approximation property, convergence factor and iterations count for V-, W- and F- cycles, and the linear dependence of V-cycle convergence on the smoothing steps. The numerical results are complete up to polynomial degree $p = 4$, and for minimum smoothness C^0 and maximum smoothness C^{p-1}.

M7P2 Schedule

Domain Decomposition, Preconditioning and Solvers in Isogeometric Analysis

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 14:45-15:35
Location: Amphi
Chairman: Lourenço Beirão da Veiga, Michel Bercovier, Simone Scacchi

14:45-15:10 : Christian Hesch
Mortar Based Domain Decomposition for Isogeometric Analysis
Abstract

15:10-15h35 : Stefan Kleiss
IETI - Isogeometric Tearing and Interconnecting
Abstract

M7 Abstract

Part 1

Part 3

Mortar Based Domain Decomposition for Isogeometric Analysis

Session Schedule Author Index Session Index M7

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Abstract

In the present talk we present a nonconform domain decomposition method based on mortar methods in the context of isogeometric analysis. The use of NURBS as basis for Galerkin-based discretization schemes within the field of nonlinear elasticity necessitates the development of a versatile interface between sub-domains which are independently h-, p- or k-refined. Additionally it is of interest to provide an adequate coupling mechanism for sub-domains, discretized either with Lagrangian or NURBS based shape functions. In particular, we apply a variant of the mortar method using Lagrangian based shape functions for the interpolation of the Lagrange multiplier field, evaluated at the discrete surface of the master side. Since the control points of the NURBS may, but do not have to be part of the geometry, we can not use them directly for the tying of the dissimilar meshes, although these control points are our primarily variables. Here, we take advantage of the fact that NURBS of arbitrary order are always partitioned by knot-vectors, which gives us a similar finite element structure as for Lagrangian meshes, and apply the linear interpolation of the Lagrange multiplier field directly to the elements of the surface of the master side. At last, well-known concepts to enhance numerical stability for the discretization in time can also been applied to both, the mechanical field as well as to the interface constraints. Those energy-momentum conserving schemes facilitate algorithmic conservation of total energy as well as both momentum maps and can be applied without modification to isogeometric problems.

IETI – Isogeometric Tearing and Interconnecting

Session Schedule Author Index Session Index M7

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Abstract

Finite Element Tearing and Interconnecting (FETI) methods are a powerful approach to designing solvers for large-scale problems in computational mechanics. The numerical simulation problem is subdivided into a number of independent subproblems, which are then coupled in appropriate ways. NURBS- (Non-Uniform Rational B-spline) based isogeometric analysis (IGA) applied to complex geometries requires to represent the computational domain as a collection of several NURBS geometries. Since there is a natural decomposition of the computational domain into several subdomains, NURBS-based IGA is particularly well suited for using FETI methods.

We propose the new IsogEometric Tearing and Interconnecting (IETI) method, which combines the advanced solver design of FETI with the exact geometry representation of IGA. We describe the IETI framework for two classes of simple model problems (Poisson and linearized elasticity) and discuss the coupling of the subdomains along interfaces (both for matching interfaces and for interfaces with T-joints, i.e. hanging knots). Attention is also paid to the construction of a suitable preconditioner for the iterative linear solver used for the interface problem. We report several computational experiments to demonstrate the performance of the proposed IETI method.

M7P3 Schedule

Domain Decomposition, Preconditioning and Solvers in Isogeometric Analysis

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 16:00-17:45
Location: Amphi
Chairman: Lourenço Beirão da Veiga, Michel Bercovier, Simone Scacchi

16:05-16:30 : Angela Kunoth
Multilevel Preconditioning for Isogeometric Analysis
Abstract

16:30-16:55 : Luca F. Pavarino
Overlapping Schwarz Methods for Isogeometric Analysis
Abstract

16:55-17:20 : Satyendra Tomar
Algebraic Multilevel Iteration Method for Isogeometric Discretization of Elliptic Problems
Abstract

17:20-17:45 : Rafael Vazquez
Multilevel Preconditioning for Isogeometric Analysis Based on Hierarchical Splines
Abstract

M7 Abstract

Part 1

Part 2

Multilevel Preconditioning for Isogeometric Analysis

Session Schedule Author Index Session Index M7

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Abstract

We consider elliptic PDEs (partial differential equations) in the framework of isogeometric analysis, i.e., we treat the physical domain by means of a B-spline or NURBS mapping which we assume to be regular. The numerical solution of the PDE is based on tensor products of B-splines of degree p on uniform grids of grid spacing h. We construct additive multilevel preconditioners and show that they are asymptotically optimal, i.e., the spectral condition number of the resulting stiffness matrix is independent of h. Together with a nested iteration scheme, this enables an iterative solution scheme of optimal linear complexity. The theoretical results are substantiated by numerical examples in up to three space dimensions for different degrees p. This is joint work with Annalisa Buffa, Helmut Harbrecht, Giancarlo Sangalli and others.

Overlapping Schwarz Methods for Isogeometric Analysis

Session Schedule Author Index Session Index M7

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Abstract

We construct and analyze an overlapping Schwarz preconditioner for elliptic problems discretized with NURBS-based isogeometric analysis. We consider both scalar problems and the system of linear elasticity, in two and three dimensions. The preconditioner is based on partitioning the domain of the problem into overlapping subdomains, solving local isogeometric problems on these subdomains and solving an additional coarse isogeometric problem associated with the subdomain mesh. We develop an h-analysis of the preconditioner, showing in particular that the resulting algorithm is scalable and its convergence rate depends linearly on the ratio between subdomain and "overlap sizes", for fixed polynomial degree p and regularity k of the basis functions. Numerical results in 2D and 3D tests show the good convergence properties of the preconditioner with respect to the isogeometric discretization parameters h, p, k, number of subdomains N, overlap size and also with respect to jumps in the coefficients of the elliptic operator.

References:

L. Beirão da Veiga, D. Cho, L. F. Pavarino, S. Scacchi. *Overlapping Schwarz methods for Isogeometric Analysis*. SIAM J. Numer. Anal., 2012

Algebraic Multilevel Iteration Method for Isogeometric Discretization of Elliptic Problems

Session Schedule Author Index Session Index M7

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Abstract

In this talk we shall present an algebraic multilevel iteration method for solving linear systems arising from the isogeometric discretization of elliptic boundary value problems. Theoretical bounds for the constant γ in the strengthened Cauchy-Bunyakowski-Schwarz inequality will be discussed. Some numerical results, supporting the theoretical estimates, will be presented. A comparison with recently introduced multigrid methods for isogeometric discretization [1] will also be drawn.

[1] K.P.S. Gahalaut, J.K. Kraus and S.K. Tomar: Multigrid Methods for Isogeometric Discretization. Submitted for publication. Also available as RICAM Report 2012-08.

Multilevel Preconditioning for Isogeometric Analysis Based on Hierarchical Splines

Session Schedule Author Index Session Index M7

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Abstract

Isogeometric analysis (IGA) is a non-standard technique for the discretization of PDEs, which basically consists on approximating the solution of the equations with the same functions that describe the geometry in CAD, such as NURBS or B-splines. The drawback of these functions is their tensor product structure, that forces the refinement to propagate to all the domain. Some alternative functions that allow for local refinement, such as T-splines and LR-splines, have been already proposed as a tool for IGA. Also hierarchical splines, that are already well known in the CAD community, were studied in the isogeometric context. Like T-splines, hierarchical splines can be refined locally maintaining high interelement continuity, but with a much easier construction and implementation. The advantage of this is that they can be easily defined also in the three-dimensional case and for arbitrary degrees.

In this work we present a construction of hierarchical splines that defines the set of basis functions in a truly hierarchical way. This approach gives a canonical construction of hierarchical splines, avoiding some of the difficulties present in previous works, such as the necessity of removing linearly dependent functions. At the same time, it automatically provides all the ingredients for the construction of multilevel preconditioners. We will show preliminary numerical results of the performance of the BPX preconditioner using hierarchical splines.

Mini Symposium M8

Domain Decomposition Techniques in Practical Flow Applications

Schedule Author Index Session Index

Organizers: Menno Genseberger, Mart Borsboom, Martin J. Gander

Abstract

The last decades domain decomposition techniques have been incorporated in large computer codes for real life applications. This minisymposium brings together some of them, here the aim is twofold. On one hand the organizers want to illustrate the importance of domain decomposition (for instance for modelling flexibility or parallel performance) in the application field. On the other hand the intention is to highlight the applied domain decomposition techniques, to discuss these approaches and -if needed- reconsider or further improve them. The application area is restricted to hydrodynamics, as this will yield a good basis for further discussion. The presentations consider domain decomposition techniques in large computer codes that are being used world wide for shallow water flow in coastal areas, lakes, rivers, ocean flow and climate modelling.

M8 Schedule

Domain Decomposition Techniques in Practical Flow Applications

Schedule Author Index Session Index

Date: Monday, June 25
Time: 16:00-17:45
Location: I50
Chairman: Menno Genseberger, Mart Borsboom, Martin J. Gander

16:05-16:30 : Eric Blayo
Interface Conditions and Domain Decomposition Methods for Ocean-Atmosphere Coupling
Abstract

16:30-16:55 : Bas van ’t Hof
Water Level Predictions with WAQUA: Domain Decomposition on a Daily Basis
Abstract

16:55-17:20 : Mart Borsboom
Analysis and Optimization of the Coupling Between Non-Overlapping Subdomains in 1D
Abstract

17:20-17:45 : Fred Wubs
HYMLS: a Robust Parallel Preconditioner for Fluid Flow Computations
Abstract

Interface Conditions and Domain Decomposition Methods for Ocean-Atmosphere Coupling

Session Schedule Author Index Session Index M8

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Abstract

The interactions between atmosphere and ocean play a major role in many geophysical phenomena, distributed over a wide range of temporal scales (e.g. breeze diurnal cycle, tropical cyclones, global climate dynamics...). Therefore the numerical simulation of such phenomena require atmospheric and oceanic coupled models, which properly represent the behavior of the boundary layers encompassing the air-sea interface and their two-way interactions. Early studies making use of such coupled models often have difficulties in capturing themesoscale air-sea interaction, thus leading to a very weak coupling generally inconsistent with available observations. Deficiencies appear both in the formulation of the physical parameterizations and in the algorithmic approach used for thecoupling. In this talk, we will address this problem from the point of view of domain decomposition methods. We will show that present coupling methods used for ocean-atmosphere coupled models can be written in the formalism of Schwarz iterative algorithms, and correspond to methods which are not pushed to convergence, and may lead to quite imperfect coupling. In particular, atmosphere and ocean coupled solutions may exhibit a strong sensitivity to model parameters and can be inherently uncertain. We will show that using improved coupling algorithms (like Schwarz methods) can reduce this sensitivity quite significantly, and we will discuss the overall objective of achieving a mathematically andphysically consistent ocean-atmosphere coupling.

Water Level Predictions with WAQUA: Domain Decomposition on a Daily Basis

Session Schedule Author Index Session Index M8

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Abstract

For over fifteen years, the Dutch Ministry for Infrastructure and the Environment has been working on domain decomposition techniques for an application called WAQUA. My presentation will cover the following aspects of domain decomposition in WAQUA.

WAQUA is an application for 2D and 3D simulation of flow and transport in estuaries, coastal waters and rivers. It is a consultancy tool, used for predicting the effects of (infrastructural) changes, and is also used in operational systems which guard the country's safety from floods. Especially in operational systems, it is a unique product, which predicts water levels along the Dutch coast very accurately. WAQUA is used by the Dutch Meteorological Institute (KNMI), Storm Surge Warning Service (SVSD) and Hydro Meteo Centra (HMCs) and others.

The domain decomposition approach for WAQUA, developed by VORtech Computing, has evolved from a method for parallel computing into a tool for increased modelling flexibility, because every domain in the simulation may have its own grid and physical model.

WAQUA's use for consultancy and in vital operational systems creates strong requirements on usability, robustness and computational efficiency of the software and the domain decomposition functionality. Usability requirements have inspired (graphical) tools for coupling the domains, as well as pre- and postprocessing facilities which allow the user to interpret the input and output of different domains in as simple and intuitive a way as possible.

The computational requirements (robustness and efficiency) are met using a powerful system for information exchange between different domains, allowing programmers of numerical algorithms to specify very exactly and very compactly which information to exchange at which times. This allows numerical aspects of distributed calculations to be separated from the technical aspects of data exchange.

This powerful data-exchange support is illustrated by the distributed algorithm used in the convection-diffusion equation: a simple red-black Jacobi iteration. An interesting case is the solution of the coupled momentum and continuity equations, a large number of distributed, one-dimensional, nonlinear systems. The solution of these systems requires a generalization of the efficient *Block Jacobi Two Way Gaussian Elimination* for cases with grid refinements.

Analysis and Optimization of the Coupling Between Non-Overlapping Subdomains in 1D

Session Schedule Author Index Session Index M8

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Abstract

Besides using domain decomposition techniques to partition problems for parallel computation, some of these techniques can also be used to partition problem areas in subdomains that each may be modeled differently. It would be useful to have a multi-domain simulation environment where each subdomain could have its own numerical approach, i.e., its own computational grid, time step, discretization method, and iterative solver. Allowing different model equations per subdomain (describing the same physical process of course) would further enhance modeling flexibility. For example, an elaborate flow model, a high-order accurate discretization, a fine grid, and a small time step may only be required in areas where the detailed LES simulation of turbulent flow is important, e.g., near a structure for the prediction of forces. Elsewhere a simpler and cheaper modeling approach may suffice.

A multi-domain modeling framework where computational models can be set up independently per subdomain requires couplings at the subdomain interfaces that are generally applicable and do not introduce any significant errors. Suitable methods and techniques need to be developed to realize such general-purpose interfaces at the physical level, at the numerical level, as well as at the software/hardware level. This talk will address some of the numerical issues associated with subdomain interfaces. By considering the 1D linear shallow-water equations, an analysis can be performed on the space discretizations applied inside the subdomains and at the interface between them, on the spurious effects that this may cause, and on the minimization of these errors by optimization of the schemes applied at the interface. As for the integration in time, general applicability (and parallelization as well) requires the coupling between subdomains to be explicit. This poses no problem if an explicit time integration scheme is applied. However, if the scheme is (partially) implicit, iterations across the subdomains are generally required. We will show that by applying high-order extrapolations in time or other explicit techniques at the interface, highly improved yet stable initial solution estimates can be obtained that will strongly accelerate that iteration process.

An optimized general-purpose DD coupling based on 1D linear theory is essential and a good start, but nonlinear and multi-dimensional directional effects need to be included as well. We will comment on these issues at the presentation.

HYMLS: a Robust Parallel Preconditioner for Fluid Flow Computations

Session Schedule Author Index Session Index M8

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Abstract

In this contribution we discuss a new hybrid direct/iterative approach to the solution of a special class of saddle point matrices arising from the discretization of the steady (Navier-) Stokes equations on an Arakawa C-grid, F-matrices. In general such matrices come about from a finite volume discretization (Marker and Cell) on a structured grid as is common for incompressible flows. The two-level method introduced here [2] is derived from a direct method for the same type of problems [1] and has the following properties: (i) it is very robust, even close to the point where the solution becomes unstable; (ii) a single parameter controls fill and convergence, making the method straightforward to use; (iii) the convergence rate is independent of the number of unknowns; (iv) it can be implemented on distributed memory machines in a natural way [3]; (v) the matrix on the second level has the same structure and numerical properties as the original problem, so the method can be applied recursively; (vi) the iteration takes place in the divergence-free space, so the method qualifies as a 'constraint preconditioner'; (vii) the approach can also be applied to a number of simpler problems like the Poisson and Darcy equations. In the talk we will give an outline of the method and show its parallel performance on the incompressible Navier-Stokes equations.

[1] A.C. de Niet and F.W. Wubs. Numerically stable LDL^T-factorization of F-type saddle point matrices. *IMA J. Numer. Anal.*, 29:208234, 2009.

[2] Fred W. Wubs and Jonas Thies. A Robust Two-Level Incomplete Factorization for (Navier)Stokes Saddle Point Matrices. *SIAM. J. Matrix Anal. and Appl.* 3 2:1475–1499, 2011.

[3] Jonas Thies and Fred Wubs. Design of a Parallel Hybrid Direct/Iterative Solver for CFD Problems. *Proceedings of the 2011 IEEE 7th International Conference on E-Science*, pp. 387–394, 2011.

Mini Symposium M9

Fast Solvers for Helmholtz and Maxwell equations

Schedule Author Index Session Index

Organizers: Victorita Dolean, Ronan Perrussel, Hui Zhang, Peng Zhen

Abstract

Wave propagation problems are intrinsically very challenging from mathematical point of view especially in the time-harmonic regime. The major difficulties stem from the mathematical models (Helmholtz or Maxwell equations) which are indefinite Partial Differential Equations treating phenomena of ondulatory nature. During the last decade, motivated by a large panel of applications, approximation and solution methods for these equations, arose the interest of a largePof scientific community. In this mini-symposium we propose an overview of different techniques which lead to fast solutions of Helhmholtz and Maxwell equations: optimized transmission conditions, discrete frameworks of domain decomposition methods, coupling of different discretizations in the domain decomposition spirit or the treatement of heterogeneous problems. The mini-symposium is organized in four sessions as follows: in the first session, chaired by Hui Zhang, after giving an overview of different solvers (speaker 1), different ways of improving the convergence of domain decomposition algorithms applied to Helmholtz equations by analythically inspired methods are shown (speakers 2-4). The second session chaired by Peng Zhen (speakers 5-8) is about coupling discretizations or alternative solvers/preconditioners for Helmholtz and Maxwell's equations. The third one, chaired by Victorita Dolean (speakers 9-12), treats the topic of discrete frameworks leading to efficient solvers. The last session, chaired by Ronan Perrussel (speakers 13-16) mainly oriented on Maxwell's equations, treats about efficient computations and implementations of domain decomposition methods.

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M9P1 Schedule

Fast Solvers for Helmholtz and Maxwell equations

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 10:30-12:15
Location: Turing
Chairman: Victorita Dolean, Ronan Perrussel, Hui Zhang, Peng Zhen

10:35-11:00 : Lea Conen
An Overview of Multigrid and Domain Decomposition Methods for the Helmholtz Equation
Abstract

11:00-11:25 : Hui Zhang
Optimized Schwarz Methods with Overlap for Helmholtz Equation
Abstract

11:25-11:50 : Erwin Veneros
Optimized Schwarz Methods for Maxwell Equations with Discontinous Coefficients
Abstract

11:50-12:15 : Bertrand Thierry
Improved Domain Decomposition Method for the Helmholtz Equation
Abstract

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Part 3

Part 4

An Overview of Multigrid and Domain Decomposition Methods for the Helmholtz Equation

Session Schedule Author Index Session Index M9

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Abstract

The Helmholtz equation describes linear, time-harmonic wave problems that arise in many engineering fields such as aeronautics and electromagnetic applications. The wave-like behavior of the solution and the indefinite, severely ill-conditioned linear systems resulting from its discretization by means of finite elements make its numerical solution difficult. Special care is needed to cope with these peculiarities as standard iterative methods such as multigrid are not robust for Helmholtz problems. Therefore, during the last decades, a large variety of suitably adapted iterative methods has been proposed aiming at the development of efficient and fast solution strategies.

In this talk, we give an overview of multigrid and domain decomposition methods for the solution of the systems stemming from finite element discretizations of the Helmholtz equation. In particular, the definition of a coarse space capturing the characteristics of the problem and the definition of suitable transmission conditions – between subdomains in the domain decomposition and between grid levels in the multigrid context – are crucial. We present techniques that have been developed and successfully applied by different authors in the course of the last years and discuss differences to the positive definite, elliptic case.

Optimized Schwarz Methods with Overlap for the Helmholtz Equation

Session Schedule Author Index Session Index M9

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Abstract

Optimized Schwarz methods *without* overlap for the Helmholtz equation were proposed in *Gander, Magoules, Nataf, 2002* and further developed in *Gander, Halpern, Magoules, 2007*. We will present here optimized Schwarz methods with overlap for the Helmholtz equation. By scaling the mesh size and the wave-number, and using asymptotic analysis, we are able to give easy to use formulas for calculating the optimized parameters of the method. The convergence rates are also derived from the corresponding asymptotic analysis, and we can show an important improvement, due to the overlap. We also illustrate our theoretical results with numerical experiments.

Optimized Schwarz Methods for Maxwell Equations with Discontinous Coefficients

Session Schedule Author Index Session Index M9

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Abstract

We study non-overlapping Schwarz Methods for solving time-harmonic Maxwell's equations in heterogeneous media. For this paper we consider Maxwell's equations in two dimensions in both the transverse electric and transverse magnetic mode formulations. We first present the classical Schwarz Method for the problem, which uses characteristic transmission conditions and can therefore be used without overlap. Choosing the interfaces between the subdomains aligned with the discontinuities in the coefficients, we prove convergence of the method for a model problem. We then define several optimized transmission conditions dependent on the discontinuities of the magnetic permeability and the electric permittivity. These conditions are determined by solving the corresponding min-max problems. We prove asymptotically that the resulting methods converge in certain cases independently of the mesh parameter, even though the methods are non-overlapping. We illustrate our theoretical results with numerical experiments.

Improved Domain Decomposition Method for the Helmholtz Equation

Session Schedule Author Index Session Index M9

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Abstract

In this talk we will present recent improvements to the quasi-optimal domain decomposition method for the Helmholtz equation presented in [1]. The key point of the method is the construction of an accurate local approximation of the exact Dirichlet-to-Neumann operator which leads to a new transmission operator between sub-domains. We will show that this local approximation, based on complex Pad approximants, is well-suited for large scale parallel finite element simulations of high frequency scattering problems, with either manual or automatic mesh partitioning. In particular, we will show that our algorithm is quasi-optimal in the sense that the convergence rate of the iterative solver depends only slightly on both the frequency and the mesh refinement.

[1] Y. Boubendir, X. Antoine and C. Geuzaine, *A Quasi-Optimal Non-Overlapping Domain Decomposition Algorithm for the Helmholtz Equation.* Journal of Computational Physics 231 (2), (2012), pp.262-280

M9P2 Schedule

Fast Solvers for Helmholtz and Maxwell equations

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 16:00-17:45
Location: Turing
Chairman: Victorita Dolean, Ronan Perrussel, Hui Zhang, Peng Zhen

16:05-16:30 : Olaf Steinbach
Coupled Finite and Boundary Element Methods for Vibro-Acoustic Interface Problems
Abstract

16:30-16:55 : Jin-Fa Lee
Integral Equation Domain Decomposition Method for Solving Electromagnetic Wave Scattering from Deep Cavities
Abstract

16:55-17:20 : Eric Darrigrand
OSRC Preconditioner and Fast Multipole Method for 3D Helmholtz Equation: a Spectral Analysis
Abstract

17:20-17:45 : Yogi Erlangga
Shift-Operator-Based Domain Decomposition Method for the Helmholtz Equation
Abstract

M9 Abstract

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Coupled Finite and Boundary Element Methods for Vibro-Acoustic Interface Problems

Session Schedule Author Index Session Index M9

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Abstract

For the vibro–acoustic simulation of completely immersed bodies such as submarines we discuss coupled finite and boundary element formulations in the frequency domain. While the vibro–elastic material in a bounded domain is handled by a standard finite element approach, the acoustic fluid in the unbounded exterior domain is incorporated via boundary integral equations. As well known for boundary integral equations related to the Helmholtz equation, special care is required to avoid spurious modes. We discuss several formulations which all result in stable discretization schemes, and we discuss appropriate solution methods for the direct problem. To determine critical frequencies for the coupled problem we discuss related eigenvalue problems. When considering a simplified Laplace model in the exterior domain, the eigenvalue problem is linear in the wave number, while for the Helmholtz model we have to solve an eigenvalue problem which depends nonlinear on the wave number. We present both analytic and numerical results for such classes of coupled problems. The talk is based on joint work with G. Unger and A. Kimeswenger.

Integral Equation Domain Decomposition Method for Solving Electromagnetic Wave Scattering from Deep Cavities

Session Schedule Author Index Session Index M9

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Abstract

We introduce a novel boundary integral equation domain decomposition method for solving time harmonic electromagnetic wave scattering from a large and deep inlet with lossy thin coating, which is embedded in an arbitrarily shaped host body. The proposed method follows a hierarchical domain partitioning strategy. First, in order to decompose the entire problem domain into the interior cavity and the exterior host body regions, an artificial surface is placed over the opening of the inlet, as a transmission interface. Subsequently, a new generalized combined field integral equation (G-CFIE), whose unknowns include surface traces of both electric and magnetic fields, is employed for both the interior and the exterior sub-domains. Finally, the required couplings between the interior cavity and the exterior host body are then completely channeled through the artificial interface surface, via the application of a Robin transmission condition. Additionally, the interior cavity region can be further partitioned into many sub-domains, again through artificial surfaces across the inlet aperture. In doing so, the dense couplings in traditionally integral equations are effectively transformed into sparse couplings as in finite methods such as finite element methods. Our treatments of the interior cavity region indeed bear many similarities to the popular boundary element tearing and interconnecting (BETI) methodology. The strength and flexibility of the proposed method will be distinctly illustrated by means of several representative numerical examples.

OSRC Preconditioner and Fast Multipole Method for 3-D Helmholtz Equation: a Spectral Analysis

Session Schedule Author Index Session Index M9

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Abstract

In acoustic scattering, integral equation methods are widely used to study wave propagation outside a bounded 3-D domain. Thanks to these techniques, the governing boundary-value problem is reduced to an integral equation on the surface Γ of the scatterer ([Colton-Kress, 83]). We consider here impenetrable bodies with smooth boundaries and incident time-harmonic plane waves. The Combined Field Integral Equation (CFIE) is uniquely solvable in $H^{1/2}(\Gamma)$ for all frequency $k > 0$, and implies the first and second traces of the double-layer potential respectively denoted M and D. In terms of numerical iterative resolution, this equation does not provide a good spectral behavior due to the strongly singular and non-compact operator D. A strategy consists in preconditioning the operator D by introducing an efficient approximation $\widetilde{V}$ of the exterior Neumann-to-Dirichlet (NtD) map, using On-Surface Radiation Condition (OSRC) methods ([Antoine-Darbas, 07]). The preconditioned equation is uniquely solvable for all wavenumbers and exhibits very interesting spectral properties. The OSRC technique only involves local operators, so that the numerical implementation of the preconditioning operator requires only the use of a sparse direct solver and does not really affect the cost of the iterative resolution of the CFIE equation. The most expensive part of the resolution is still consequent to the integral operators. The Fast Multipole Method (FMM) ([Coifman-Rokhlin-Wandzura, 93]) is then considered to deal with the operators M and D. A thorough study of the eigenvalues behavior is realized in order to illustrate the impact of the OSRC-preconditioning technique on the spectrum of the CFIE operator. The resolution scheme is applied to several numerical test-cases (sphere, cube, trapping domain). The convergence of the GMRES corroborates the spectral analysis. Only a few GMRES iterations are required for both high frequencies and refined meshes. The computation cost follows the FMM behavior. Combining the OSRC preconditioner and the FMM proves to be a very efficient approach to solve the CFIE at high frequencies.

Shift Operator-Based Domain Decomposition Method for the Helmholtz Equation

Session Schedule Author Index Session Index M9

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Abstract

The publication of [1], has revived research on fast iterative solvers for the Helmholtz equation. While it is believed that the Helmholtz equation still poses problems for classical iterative solvers [3], the role of complex (shifted) Laplacian as preconditioner has been recognized very well in handling difficulties due to indefiniteness of the Helmholtz matrix. In general, the convergence of a Krylov method, preconditioned by the shifted Laplacian, behaves linearly with respect to the wavenumber, with a small constant. In a rather obscure paper [2], the authors incorporate a second-level preconditioner to tackle the small eigenvalues based on a modified deflation (shift) operator, which successfully shifts small eigenvalues towards the upper bound. Numerical tests revealed an almost wavenumber-independent convergence, the first time ever observed.

The shift operator mimics both domain-decomposition and multigrid. While it is easier to implement the operator in a multigrid fashion, it is interesting to see if this operator can be cast into a domain-decomposition method, event though not in the classical sense. This will somehow enlarge the deflation subspace and provide a more efficient means to handle the shift operator via subdomain solves. While for the Poisson problem, this is already the case, for the Helmholtz equation, this is feasible but the implementation is not straightforward.

In this talk, we will discuss and show what has been achieved in this direction, based on 1D and 2D Helmholtz problems.

[1] Y.A. Erlangga, C.W. Oosterlee, and C. Vuik, A novel multigrid-based preconditioner for the heterogeneous Helmholtz equation, SIAM Journal on Scientific Computing, 27 (2006), pp. 1471–1492.

[2] Y.A. Erlangga and R. Nabben, On a multilevel Krylov method for the Helmholtz equation preconditioned by shifted Laplacian, Electronic Transaction on Numerical Analysis, 2008 (31) pp. 203–234.

[3] O.G. Ernst and M.J. Gander, Why it is difficult to solve Helmholtz problems with classical iterative methods, in Numerical Analysis of Multiscale Problems, I. Graham, T. Hou, O. Lakkis and R. Scheichl, Editors, Springer Verlag, 2011.

M9P3 Schedule

Fast Solvers for Helmholtz and Maxwell equations

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 10:30-12:15
Location: Turing
Chairman: Victorita Dolean, Ronan Perrussel, Hui Zhang, Peng Zhen

10:35-11:00 : Rosalie Belanger-Rioux
A Fast and Accurate Absorbing Boundary Condition for the Helmholtz Equation
Abstract

11:00-11:25 : Achim Schadle
Curl-Conforming Hardy Space Infinite Elements for Exterior Maxwell Problems
Abstract

11:25-11:50 : Ana Alonso Rodriguez
Finite Element Construction of Discrete Harmonic Fields
Abstract

11:50-12:15 : Martin Huber
Hybrid Domain Decomposition Solvers for the Helmholtz Equation
Abstract

M9 Abstract

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A Fast and Accurate Absorbing Boundary Condition for the Helmholtz Equation

Session Schedule Author Index Session Index M9

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Abstract

Constructing accurate absorbing, radiating or non-reflecting boundary conditions (hereafter ABCs) for the Helmholtz equation

$$\Delta u(x) + \frac{\omega^2}{c^2(x)} u(x) = f(x)$$

in heterogeneous media (i.e. c not constant) is difficult and costly. In particular, there are a number of applications, such as imaging, where is it necessary to solve this equation numerous times with various right hand sides f. We propose here a general framework for rapidly constructing and evaluating good ABCs by compressing the Dirichlet to Neumann (DtN) map D,

$$D : u(s) \rightarrow \frac{\partial u}{\partial \nu}(x)$$

where u is the desired solution, ν the outward pointing normal to the boundary of interest Γ, and $s, x \in \Gamma$. This is based on our previous work on the DtN map for the Helmholtz equation, which showed that this map is separable and low-rank for the constant media half-space case when x and s are sufficiently well separated. Once we have obtained a compressed and accurate approximation $\hat{D}$ to the DtN map D, it can repeatedly be used in any Helmholtz solver for a bounded domain in order to simulate an unbounded domain. Namely, one uses the following boundary condition

$$\frac{\partial u}{\partial \nu}(s) - \hat{D}u(s) = 0, \;\; s \in \Gamma$$

on any boundary Γ for which one wishes to have an absorbing boundary condition. We use matrix probing in order to rapidly solve for the DtN map, and this conveniently gives us a compressed map. The computational complexity of this step is equivalent to solving the original problem, with the desired ABC and a zero right hand side, a few times only. Since matrix probing fits the DtN map to a few well-chosen matrices, it has greater potential for accuracy and flexibility in variable media. Hence we may use matrix probing on the DtN map as a precomputation for solving the wave equation multiple times on unbounded domains in variable media.

Curl-Conforming Hardy Space Infinite Elements for Exterior Maxwell Problems

Session Schedule Author Index Session Index M9

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Abstract

The construction of a new kind of prismatic infinite elements for electromagnetic scattering and resonance problems will be described. Transparent boundary conditions are realized by the pole condition. The pole condition, as radiation condition, states that a function is outgoing if and only if a certain transformation of this function belongs to a Hardy space. We use tensor products of cochain complexes to obtain four different infinite element spaces which form an exact sequence corresponding to the deRham complex in the exterior domain. Numerical tests indicate super-algebraic convergence in the number of additional unknows per degree of freedom on the coupling boundary.

Finite Element Construction of Discrete Harmonic Fields

Session Schedule Author Index Session Index M9

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Abstract

We study an efficient algorithm for the finite element construction of discrete harmonic fields in a not simply-connected bounded three-dimensional domain Ω. The construction of discrete harmonic fields is essential in the numerical approximation of magnetostatic and **H**-based formulations of the eddy-current model. It is well known that the dimension of the space of harmonic fields is equal to the first Betti number of Ω, n_Ω. The fundamental point in the classical way for determining a basis of the space of harmonic fields is the fact that there exists n_Ω connected orientable Lipschitz surfaces Σ_n, with $\partial\Sigma_n \subset \partial\Omega$, such that every curl-free vector field in Ω has a global potential in $\Omega \setminus \cup_n \Sigma_n$. These surfaces are called Seifert surfaces: each one of them "cuts" a non-bounding cycle in Ω. From the computational point of view, in general topological situations it can be difficult to explicitly determine the Seifert surfaces (for instance, in the case of "knotted" domains). We propose an alternative method that works for general topological domains and does not need the determination of cutting surfaces.

Hybrid Domain Decomposition Solvers for the Helmholtz Equation

Session Schedule Author Index Session Index M9

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Abstract

In this talk, an hybrid finite element methods (FEM) for the scalar and vectorial wave equation, which is equivalent to a discontinuous Gelerkin Method, based on the Ultra Weak Variational Formulation is investigated.

Motivated by hybrid FEMs for the Laplace equation, the tangential continuity of the flux is broken across element interfaces. In order to reinforce continuity again, Lagrange multipliers supported only on the element facets are introduced. These multipliers can be interpreted as the tangential component of the unknown field. By adding a second set of multipliers, representing the tangential component of the flux field, it is possible to eliminate the volume degrees of freedom cheaply element by element. This approach allows to reduce the original system of equations to a much smaller system for the Lagrange multipliers.

A very challenging point is to solve the resulting system of equations. Different preconditioners for Krylov space solvers are discussed in the talk. Apart from multiplicative and additive Schwarz block preconditioners with blocks containing degrees of freedom related to one facet or element, respectively, or an elementwise BDDC preconditioner, a domain decomposition preconditioner is constructed. This preconditioner solves in each iteration step local problems on subdomains by directly inverting the system matrix, and it can be therefore implemented efficiently in parallel.

It is shown by numerical experiments, that the iterative solvers have good convergence properties for large scale computations. Our approach allows the solution of three dimensional problems up to 50 wavelength per domain for the scalar Helmholtz equation, and up to 30 wavelength per domain for the vectorial wave equation.

M9P4 Schedule

Fast Solvers for Helmholtz and Maxwell equations

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 16:00-17:45
Location: Turing
Chairman: Victorita Dolean, Ronan Perrussel, Hui Zhang, Peng Zhen

16:05-16:30 : Ronan Perrussel
Schwarz Methods for Time-Harmonic Maxwell's Equations Discretized by a Hybridized Discontinuous Galerkin Method
Abstract

16:30-16:55 : Jack Poulson
A Parallel Sweeping Preconditioner for High-Frequency Heterogeneous 3d Helmholtz Equations
Abstract

16:55-17:20 : Zhen Peng
Speed up Non-conformal DDM Convergence using an Asymmetric Optimal Transmission Condition
Abstract

17:20-17:45 : Stéphane Lanteri
Discretization of Optimized Schwarz Methods for Maxwell's Equations
Abstract

M9 Abstract

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Schwarz Methods for Time-Harmonic Maxwell's Equations Discretized by a Hybridized Discontinuous Galerkin Method

Session Schedule Author Index Session Index M9

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Abstract

Schwarz-type domain decomposition methods are presented for the solution of 3d time-harmonic Maxwell's equations. They are coupled with a hybridizable discontinuous Galerkin (HDG) method for the discretization of the problem. The formulation and the implementation of the HDG method associated with Schwarz algorithms are detailed. Numerical results show that the HDG method has an optimal convergence rate and can save both CPU time and memory cost compared to more classical DG methods.

A Parallel Sweeping Preconditioner for High-Frequency Heterogeneous 3d Helmholtz Equations

Session Schedule Author Index Session Index M9

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Abstract

A parallelization of a recently introduced 'sweeping' preconditioner for high-frequency heterogeneous Helmholtz equations is presented along with experimental results for the full SEG/EAGE Overthrust seismic model at 30 Hz. While the setup and application costs of the sweeping preconditioner are trivially $O(N^{4/3})$ and $O(N \log N)$, this study provides strong empirical evidence for Engquist and Ying's observation that the number of iterations required for the convergence of GMRES with the sweeping preconditioner is essentially independent of the frequency of the problem. Generalizations to more complicated time-harmonic wave equations are also briefly discussed since the techniques behind our parallelization are not specific to the Helmholtz equation.

Speed up Non-conformal DDM Convergence using an Asymmetric Optimal Transmission Condition

Session Schedule Author Index Session Index M9

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Abstract

It has been recognized that the convergence of non-overlapping DDMs depends heavily upon the transmission conditions employed to enforce the continuity of tangential fields on the domain interfaces. Recently, a second order transmission condition and an optimal transmission condition, which involve two second-order transverse derivatives, are proposed to enable convergence for both propagating and evanescent electromagnetic waves across domain interfaces. Additionally, by solving the second-order vector wave equation, we have placed more emphasis on one vector field than the other. For example, in the so-called E-field formulation, which is the formulation adopted herein, we have observed the accuracy in the electric field computed is better than the accuracy obtained in the magnetic field. This discrepancy in accuracy leads to different effective frequency spectrum for both E and H fields for finite discretization. To exploit this observation, we further adjust the parameters in the optimal transmission condition to result in an asymmetric transmission condition which has different spectral convergences for both transverse electric (TE) and transverse magnetic (TM) fields, respectively. Furthermore, through an analysis of the DDM with the higher order TCs, we show that there still exists a weakly converging region centered on the cutoff modes at each sub-domain interfaces. A global plane wave deflation technique is then proposed as an effective coarse grid preconditioner.

Discretization of Optimized Schwarz Methods for Maxwell's Equations

Session Schedule Author Index Session Index M9

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Abstract

We study here optimized Schwarz domain decomposition methods for solving the time-harmonic Maxwell equations discretized by a discontinuous Galerkin (DG) method. Due to the particularity of the latter, a discretization of a more sophisticated Schwarz method is not straightforward. A strategy of discretization is shown in the framework of a DG weak formulation, and the equivalence between multi-domain and single-domain solutions is proved. The proposed discrete framework is then illustrated by some numerical results through the simulation of two-dimensional propagation problems in homogeneous and heterogeneous media.

Mini Symposium M10

New Developments of FETI, BDDC, and Related Domain Decomposition Methods

Schedule Author Index Session Index

Organizers: **Xuemin Tu and Olof Widlund**

Abstract

Finite Element Tearing and Interconnecting (FETI) and Balancing Domain Decomposition by Constraints (BDDC) methods are among the most powerful domain decomposition methods. They are also closely related in that they use the same or similar coarse components in their preconditioners. Lately, coarse solvers borrowed from these and other iterative substructuring algorithms have also been combined with local solvers defined on a set of overlapping subdomains. In this mini symposium, some of this new work will be discussed. Among the applications are Stokes' equations, elasticity, some problems posed in H(curl), discontinuous Galerkin methods, and isogeometric analysis.

Part 1

Part 2

M10P1 Schedule

New Developments of FETI, BDDC, and Related Domain Decomposition Methods

Schedule Author Index Session Index

Date: Monday, June 25
Time: 10:30-12:15
Location: Markov
Chairman: Xuemin Tu and Olof Widlund

10:35-11:00 : Maksymilian Dryja
ASM for DG Discretization of Anisotropic Elliptic Problems
Abstract

11:00-11:25 : Juan Galvis
Domain Decomposition Preconditioners for High-Contrast Multiscale Problems
Abstract

11:25-11:50 : Hyea Hyun Kim
Two-Level Overlapping Schwarz Algorithms for a Staggered Discontinuous Galerkin Method
Abstract

11:50-12:15 : Chang-Ock Lee
A Two-Level Nonoverlapping Schwarz Algorithm for the Stokes Problem
Abstract

M10 Abstract

Part 2

ASMs for DG Discretization of Anisotropic Elliptic Problems

Session Schedule Author Index Session Index M10

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Abstract

In the talk a Discontinuous Galerkin (DG) discretization of an elliptic second order equation with discontinuous anisotropic coefficients is considered. The DG discretization is applied elementwise, on a matching triangulation.

First, an analysis of the discrete problem will be presented, including the error bound. Next, Additive Schwarz Methods (ASMs) will be designed and analyzed, with rates of the convergence independent of the jumps of the coefficients.

The preliminary ASM for the discussed problem is determined by local problems defined on each triangle and a coarse problem defined in the space of piecewise constant functions on the fine mesh. This preconditioner is optimal and independent of te jumps of the coefficients.

To get a fully parallel algorithm, the coarse problem of the above method can be solved by known ASMs. This leads to various new ASMs. One of them, which convergence rate is independent of the coefficient jumps is discussed in detail.

Domain Decomposition Preconditioners for High-Contrast Multiscale Problems

Session Schedule Author Index Session Index M10

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Abstract

We give an overview of our results from the point of view of coarse-grid multiscale model reduction by highlighting some common issues in coarse-scale approximations and two-level preconditioners. Reduced models discussed in this paper rely on coarse-grid spaces computed by solving local spectral problems. We define local spectral problems with a weight function computed with a choice of initial multiscale basis functions. We emphasize the importance of this initial choice of multiscale basis functions for both coarse-scale approximation and for preconditioners. In particular, we discuss various choices of initial basis functions and use some of them in our simulations. We show that a naive choice of initial basis functions, e.g., piecewise linear functions, can lead to a large dimensional spaces that are needed to achieve (1) a reasonable accuracy in the coarse-scale approximation or (2) contrast-independent condition number of preconditioned matrix within two-level additive Schwarz methods. While using a careful choice of initial spaces, we can achieve (1) and (2) with smaller dimensional coarse spaces.

Two-Level Overlapping Schwarz Algorithms for a Staggered Discontinuous Galerkin Method

Session Schedule Author Index Session Index M10

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Abstract

Two overlapping Schwarz algorithms are developed for a discontinuous Galerkin (DG) finite element approximation of second order scalar elliptic problems in both two and three dimensions. The discontinuous Galerkin formulation is based on a staggered discretization introduced by Chung and Engquist for the acoustic wave equation. Two types of coarse problems are introduced for the two-level Schwarz algorithms. The first is built on a nonoverlapping subdomain partition, which allows quite general subdomain partitions, and the second on introducing an additional coarse triangulation that can also be quite independent of the fine triangulation. Condition number bounds are established and numerical results are presented.

A Two-Level Nonoverlapping Schwarz Algorithm for the Stokes Problem

Session Schedule Author Index Session Index M10

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Abstract

A general framework of a two-level nonoverlapping Schwarz algorithm for the Stokes problem is developed. This framework allows both discontinuous and continuous pressure finite element spaces. The coarse problem is built by algebraic manipulation after selecting appropriate primal unknowns just like in BDDC algorithms. Performance of the suggested algorithm is presented depending on the selection of finite elements and primal unknowns. Under the same set of primal unknowns, the algorithm for the case with discontinuous pressure functions outperforms one with continuous pressure functions. For the two-dimensional Stokes problem, the algorithm with a set of primal unknowns consisting of velocity unknowns at corners, averages of velocity components over common edges, and pressure unknowns at corners presents good scalability when continuous pressure test functions are used. In both two- and three-dimensional Stokes problems, an improvement can be made for the case with continuous pressure test functions by applying the suggested algorithm to the interface problem, which is obtained by eliminating velocity unknowns and pressure unknowns interior to each subdomains.

M10P2 Schedule

New Developments of FETI, BDDC, and Related Domain Decomposition Methods

Schedule Author Index Session Index

Date: Monday, June 25
Time: 16:00-18:10
Location: Markov
Chairman: Xuemin Tu and Olof Widlund

16:05-16:30 : Jungho Lee
Large-Scale Differential Variational Inequalities for Phase-Field Modeling
Abstract

16:30-16:55 : Lourenco Beirão da Veiga
BDDC Preconditioners for Isogeometric Analysis
Abstract

16:55-17:20 : Xuemin Tu
FETI-DP Domain Decomposition Methods for Incompressible Stokes Equation
Abstract

17:20-17:45 : Olof Widlund
BDDC for some problems posed in $H(curl)$
Abstract

17:45-18:10 : Jun Zou
An Overlapping Domain Decomposition Algorithm for Parameter Identifications
Abstract

M10 Abstract

Part 1

Large-Scale Differential Variational Inequalities for Phase-Field Modeling

Session Schedule Author Index Session Index M10

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Abstract

Recent progress on the development of scalable differential variational inequality multigrid-based solvers for the phase-field approach to mesoscale materials modeling is described. We have developed an active-set method for variational inequalities in PETSc, leveraging experience by the optimization community in TAO. A geometric multigrid solver in PETSc is used to solve the resulting linear systems. We present strong and weak scaling results for 2D coupled Allen-Cahn/Cahn-Hilliard systems.

BDDC Preconditioners for Isogeometric Analysis

Session Schedule Author Index Session Index M10

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Abstract

We study Balancing Domain Decomposition by Constraints (BDDC) preconditioners for NURBS-based Isogeometric Analysis of scalar elliptic and linear elasticity problems. Our construction is based on a generalized Schur complement system for NURBS functions and the analysis is based on appropriate discrete norms and scaling functions. The proposed isogeometric BDDC preconditioner with different choices of primal constraints is proven to be scalable in the number of subdomains and quasi-optimal in the ratio of subdomain and element sizes. Several numerical experiments in 2D and 3D confirm the theoretical convergence rate estimates obtained and also illustrate the preconditioner performance with respect to the polynomial degree and regularity of the NURBS basis functions, as well as its robustness with respect to discontinuities of the elliptic coefficients across subdomain boundaries.

References:

L. Beirão da Veiga, D. Cho, L.F. Pavarino, S. Scacchi, *BDDC preconditioners for Isogeometric Analysis.* Technical report IMATI-CNR, 2012

FETI-DP Domain Decomposition Methods for Incompressible Stokes Equation

Session Schedule Author Index Session Index M10

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Abstract

In this talk, a unified framework of FETI-DP algorithms will be discussed for solving the system of linear equations arising from the mixed finite element approximation of incompressible Stokes equations. Several previously developed FETI-DP algorithms can be represented under this framework. Their condition number estimates are also simplified using this framework. A distinctive feature of this framework is that both continuous and discontinuous pressures can be used in the finite element space, while previous FETI-DP algorithms are valid only for the case of using discontinuous pressures. Both lumped and Dirichlet type preconditioners will be discussed and numerical experiments of solving a two-dimensional incompressible Stokes problem will be provided.

BDDC for some problems posed in H(curl)

Session Schedule Author Index Session Index M10

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Abstract

The considerable challenge in developing domain decomposition algorithms for problems posed in H(curl) and discretized using Nédeléc edge elements is reflected in a quite sparse literature. This talk will discuss some of these challenges and new results recently developed for BDDC algorithms. These results improve and generalize findings by Toselli on the closely related FETI–DP algorithms and they also simplify some of his arguments.

An Overlapping Domain Decomposition Algorithm for Parameter Identifications

Session Schedule Author Index Session Index M10

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Abstract

In this talk we shall address an overlapping domain decomposition method for solving some inverse problems of identifying parameters in second order elliptic and parabolic systems, including the reconstruction of fluxes, heat sources and initial data. Some convergence analysis and numerical experiments of the proposed method will be presented and discussed. The work of Jun Zou was substantially supported by Hong Kong RGC grants (Projects 405110 and 404611)

Mini Symposium M11

Decomposition Strategies for Boltzmann's Equation

Schedule Author Index Session Index

Organizers: Heiko Berninger and Jérôme Michaud

Abstract

Solving Boltzmann's equation for non-uniform gases or for neutrino transport is a challenging task if large variations of the mean free path have to be considered. In particular, computation times become excessively large in high density regimes, i.e., if the mean free path is small. As a remedy, one uses suitable macroscopic limit equations such as Euler, Navier–Stokes or diffusion equations in regimes where microscopic kinetic effects may be neglected or considered in average. Then the question arises of how to couple these equations and Boltzmann's equation properly and how to solve the coupled system.

With regard to this problem of solving Boltzmann's equation in its different regimes, various decomposition and domain decomposition strategies have been established and investigated. This minisymposium intends to bring experts together who represent a variety of different approaches. Our aim is to obtain an overview on the achievements in this field and to discuss main challenges and open problems.

Part 1

Part 2

M11P1 Schedule

Decomposition Strategies for Boltzmann's Equation

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 10:30-12:15
Location: I50
Chairman: Heiko Berninger and Jérôme Michaud

10:35-11:00 : Patrick Le Tallec
Half Fluxes Coupling of Boltzmann and Navier Stokes Equations
Abstract

11:00-11:25 : Mohammed Lemou
On Micro-Macro Numerical Schemes for Multiscale Kinetic Equations
Abstract

11:25-11:50 : Emmanuel Frénod
Two-Scale Convergence and Kinetic Equations
Abstract

11:50-12:15 : Heiko Berninger
Neutrino Transport in Core Collapse Supernovae by Asymptotic Expansions of Boltzmann's Equation
Abstract

M11 Abstract

Part 2

Half Fluxes Coupling of Boltzmann and Navier Stokes Equations

Session Schedule Author Index Session Index M11

Patrick Le Tallec
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Abstract

Many practical problems require the simultaneous use of different physical models inside a given computational domain:
- local kinetic models must be used in shock or boundary layers when simulating rarefied flows or radiative effects,
- refined meshes or boundary layer models are needed in recirculation regions or next to solid boundaries.

Such situations are nicely treated by a general purpose domain decomposition strategy largely developed and applied by Quarteroni and coauthors, and based on the following steps:
- introduction of a unique underlying system of differential equations modelling all components of the system under study ;
- consistent multidomain splitting of the underlying Partial Differential Equations. The coupling conditions are deduced from the mathematical structure of the underlying mathematical model. The subdomains can be either constructed apriori by geometric or physical arguments, or adaptively updated during the solution process;
- introduction of local approximation strategies independently on each subdomain.

The proposed talk will review the application of this general strategy to kinetic models coupled to hydrodynamics simulations. The underlying physical model will be based on a full Boltzmann eaquation or on a fourteen moment asymptotic expansion of the Boltzmann equation proposed by D. Levermore. The Navier-Stokes equations can then be obtained by a Hilbert asymptotic expansion of these kinetic equations. Following the above general strategy lead then to coupling strategies which acts as if we were solving a kinetic model everywhere, the Navier-Stokes equations being simply obtained locally by replacing the kinetic model by their specific forms derived by the appropriate asymptotic expansion process.

On Micro-Macro Numerical Schemes for Multiscale Kinetic Equations

Session Schedule Author Index Session Index M11

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Abstract

The development of numerical methods to solve multi-scale kinetic equations has been the subject of active research in the past years, with applications in various fields: plasma physics, rarefied gas dynamics, aerospace engineering, semiconductors, radiative transfer, ... The general problem is to construct numerical schemes that are able to capture the properties of the various scales in the considered system, while the numerical parameters remain as independent as possible of the stiffness character of these scales. In systems of particles, on could have to deal for instance with various regimes in different regions of the physical space: Microscopic regime (kinetic) or macroscopic regimes (fluid, diffusion,etc...). In contrast with a domain decomposition method (with respect to these different regimes), which would require a delicate handling of the interfaces, we have developed (in 2008) a numerical method which is based on a decomposition of the distribution function in the whole space. The original model is then decomposed into a system of two equations: an equation on a macro part (equilibrium part) whose evolution is coupled to an equation on the remaining micro (kinetic) part. Suitable numerical schemes are then constructed on this formulation. This strategy is quite robust in the sense that it can be easily adapted to a large class of kinetic equations (Vlasov-BGK, Vlasov-Boltzmann, Vlasov-Landau, etc) and to various scales (kinetic/diffusion, kinetic/fluid kinetic/high fields, two-scales asymptotics, etc). The numerical schemes which are constructed from such formulations have the following so-called asymptotic preserving (AP) property: they are consistent with the model at the kinetic regime and they degenerate into a scheme which is consistent with the macro model in the desired asymptotic limit, the numerical parameters being fixed. In this talk, we shall first present the main lines of this strategy for different asymptotics. Then, we will show how to modify the approach in order to deal with the space boundary conditions and to provide good approximations of boundary layers as well. This last point was the subject of a recent work on which a main part of the talk will be focused.

Two-Scale Convergence and Kinetic Equations

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Abstract

In this talk, we will explain how Numerical Methods mixing Two-Scale Convergence and Macro-Micro decomposition can be set out for Kinetic Equation with a singular term generating large amplitude and high frequency oscillations in some regions of the domain where the Equation is defined, but not everywhere.

The presented work follows [3] where a Two-Scale Numerical method was built for the 2D-Vlasov-Poisson system. This method is well adapted to Kinetic Equation with a singular term which is uniform over the domain.

In [1] we proposed a strategy to develop Two-Scale Macro-Micro decomposition that is a path to the desired Numerical Methods. A simplified version or the desired Numerical Methods was built in [2].

[1] N. Crouseilles, E. Frénod, S. Hirstoaga, and A. Mouton. Two-Scale Macro-Micro decomposition of the Vlasov equation with a strong magnetic field. *Submitted.*

[2] E. Frénod, M. Gutnic, and S. Hirstoaga. First order Two-Scale Particle-in-Cell numerical method for Vlasov equation. *Submitted to: ESAIM Proceedings of CEMRACS 2011*, January 2012.

[3] E. Frénod, F. Salvarani, and E. Sonnendrücker. Long time simulation of a beam in a periodic focusing channel via a two-scale PIC-method. *Mathematical Models and Methods in Applied Sciences*, 19(2):175–197, 2009.

Neutrino Transport in Core Collapse Supernovae by Asymptotic Expansions of Boltzmann's Equation

Session Schedule Author Index Session Index M11

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Abstract

Simulations of core collapse supernovae require to model the radiative transfer of neutrinos. If the full Boltzmann equation is chosen as the model, computation times increase dramatically in regimes in which high density of neutrinos, i.e., small mean free paths prevail. In core collapse supernovae, neutrinos are trapped by the matter in high density regimes, where they are in diffusion, whereas they are practically freely streaming in low density regimes further away from the core. In both cases, the Boltzmann equation can be reduced considerably. This leads to the idea of a decomposition of the neutrino distribution function into trapped and streaming particle components used in the Isotropic Diffusion Source Approximation (IDSA) of Boltzmann's equation [**?**]. In this talk, we will give an introduction into the IDSA in spherical symmetry, both from a physical and a mathematical point of view. The main purpose of the talk will be to present a derivation of the IDSA by asymptotic analysis applying Chapman–Enskog and Hilbert expansions. Computational aspects and numerical solution techniques for the IDSA and the Boltzmann equation accompanied by numerical results will be subject of the talk by J. Michaud in the same minisymposium.

[1] M. Liebendörfer, S.C. Whitehouse, and T. Fischer. The Isotropic Diffusion Source Approximation for Supernova Neutrino Transport. *ApJ*, 698:1174–1190, 2009.

M11P2 Schedule

Decomposition Strategies for Boltzmann's Equation

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 16:00-17:45
Location: I50
Chairman: Heiko Berninger and Jérôme Michaud

16:05-16:30 : François Golse
A Coupling Method for Transport/Diffusion Problems
Abstract

16:30-16:55 : Giacomo Dimarco
Fluid Simulations with Localized Boltzmann Upscaling by Direct Monte Carlo
Abstract

16:55-17:20 : Sudarshan Tiwari
Simulation of the Boltzmann and the Navier-Stokes Equations with Particle Methods based on Domain Decomposition for Steady and Unsteady Flows
Abstract

17:20-17:45 : Jérôme Michaud
The IDSA and Boltzmann's Equation: Discretization, Comparison and Modeling Error
Abstract

M11 Abstract

Part 1

A Coupling Method for Transport/Diffusion Problems

Session Schedule Author Index Session Index M11

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Abstract

This talk presents a domain decomposition method on an interface problem for the linear transport equation between a diffusive and a non-diffusive region. To leading order, i.e. up to an error of the order of the particle mean free path in the diffusive region, the solution in the non-diffusive region is decoupled from the particle density in the diffusive region. The diffusive and non-diffusive regions are coupled at the interface at the next order of approximation in the mean-free path. Our analysis is based on an accurate description of the boundary layer at the interface, in terms of a half-space problem for the linear transport equation. Indeed, such half-space problems for the linear transport equation can be reduced to a linear integral equation of Wiener-Hopf type and can be solved explicitly. We take advantage of a very simple formulation of this solution due to Chandrasekhar.

Fluid Simulations with Localized Boltzmann Upscaling by Direct Monte Carlo

Session Schedule Author Index Session Index M11

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Abstract

In the present talk, we present a novel numerical algorithm to couple the Direct Simulation Monte Carlo method (DSMC) for the solution of the Boltzmann equation with a finite volume like method for the solution of the Euler equations. The method relies on the introduction of buffer zones which realize a smooth transition between the kinetic and the fluid regions. To facilitate the coupling and avoid the onset of spurious oscillations in the fluid regions which are consequences of the coupling with a stochastic numerical scheme, we use a new technique which permits to reduce the variance of the particle methods. In addition, the use of this method permits to obtain estimations of the breakdowns of the fluid models less affected by fluctuations and consequently to reduce the kinetic regions and optimize the coupling. In the last part of the talk several numerical examples are presented to validate the method and measure its computational performances.

Simulation of the Boltzmann and the Navier-Stokes Equations with Particle Methods based on Domain Decomposition for Steady and Unsteady Flows

Session Schedule Author Index Session Index M11

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Abstract

We present a coupling of the Boltzmann and the Navier-Stokes equations based on the domain decomposition strategy. We monitor the breakdown criterion of the continuum regime in time and decompose the domains. The Boltzmann equation is solved in the rarefied domain and the Euler/Navier-Stokes equations are solved in the continuum domain. We use a DSMC type of particle method to solve the Boltzmann equation. Earlier we have solved the compressible Euler equations by a kinetic particle method in the continuum regime. This method is a natural choice for coupling since the two schemes defer only in the collision processes and it is easier to handle the interface boundary conditions. However, kinetic particle methods are not the optimal choice. In recent years we have extended our earlier work in small scale geometries for unsteady as well as steady problems. In the continuum regime we solve the Navier-Stokes equations by a meshfree particle method. In this scenario meshfree methods are suitable since the interface between the continuum and rarefied domains is quite irregular. Meshfree methods are capable of handling such irregular boundaries easily. The classical Sod's shock tube problem is solved as a $1D$ test case, whereas as $2D$ test case a stationary driven cavity flow is solved for a large range of Knudsen numbers. For small Knudsen numbers all solutions obtained from pure Boltzmann, pure Navier-Stokes and the couplied solvers match perfectly. We further show that for larger Knudsen numbers, where the Navier-Stokes equations fail to predict the correct flow behavior, its solutions are still a good candidate to initialize a Boltzmann solver. Finally, we show the simulation of a moving droplet inside a rarefied regime.

The IDSA and Boltzmann's Equation: Discretization, Comparison and Modeling Error

Session Schedule Author Index Session Index M11

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Abstract

In this talk we recall the Boltzmann equation for neutrino transport used in core collapse supernovae as well as the Isotropic Diffusion Source Approximation (IDSA) of it [1]. The latter is presented and analysed in more detail in the talk of H. Berninger in this minisymposium. The purpose of this talk is to present a numerical treatment of a reduced Boltzmann model problem based on time splitting and finite volumes and revise the discretization of the IDSA for this problem [2]. Discretization error studies carried out on the reduced Boltzmann model problem and on the IDSA reveal errors of order one in both cases. By means of a numerical example, a detailed comparison of the reduced model and the IDSA is performed and interpreted. For this example, the IDSA modeling error with respect to the reduced Boltzmann model is numerically determined and localized.

[1] M. Liebendörfer, S.C. Whitehouse, and T. Fischer. The Isotropic Diffusion Source Approximation for Supernova Neutrino Transport. *ApJ*, 698:1174–1190, 2009.

[2] H. Berninger, E. Frénod, M.J. Gander, M. Liebendörfer, J. Michaud, and N. Vasset. A Mathematical Description of the IDSA for Supernova Neutrino Transport, its Discretization and a Comparison with a Finite Volume Scheme for Boltzmann's Equation. *Submitted to: ESAIM Proceedings of CEMRACS 2011.*

Mini Symposium M12

Domain Decomposition Techniques in Life Science Modeling and Simulation

Schedule Author Index Session Index

Organizers: Luca Gerardo-Giorda and Victorita Dolean

Abstract

In the recent decades, the availability of powerful computers led scientists to face problems coming from biomedical applications, and computational science has become a strong partner for medical doctors. Biomedical problems are very challenging in terms of both computational and mathematical complexity. Advanced geometry reconstruction techniques from medical imaging made very detailed anatomies available. If on the one hand *in silico* experiments based on real domains have become quite a standard in the community, on the other hand such a detailed description of the computational domain easily results in millions of degrees of freedom. At the same time, most problems in Life Science modeling aim at describing the coupling of different physiological and mechanical models. In both cases, domain decomposition methods are the most natural environment to both formulate and solve such problems. The minisymposium aims at gathering researchers who brought important scientific contributions in this field.

M12 Schedule

Domain Decomposition Techniques in Life Science Modeling and Simulation

Schedule Author Index Session Index

Date: Friday, June 29
Time: 10:30-12:15
Location: I50
Chairman: Luca Gerardo-Giorda and Victorita Dolean

10:35-11:00 : Oliver Rheinbach
Advances of FETI Methods in Biomechanics
Abstract

11:00-11:25 : Simone Scacchi
Parallel Bidomain Solvers for Cardiac Excitation
Abstract

11:25-11:50 : Nejib Zemzemi
Decoupled Time-Marching Schemes in Computational Cardiac Electrophysiology and ECG Numerical Simulation
Abstract

11:50-12:15 : Gwenol Grandperrin
Parallel Preconditioners for Solving Fluid-Structure Interactions Problems in Hemodynamics
Abstract

Advances of FETI Methods in Biomechanics

Session Schedule Author Index Session Index M12

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Abstract

In this talk we will present recent results on parallel simulations of soft biological tissue obtained by using a FETI-DP method within a fully parallel software environment built around FEAP. We will present new parallel scalability results on a Cray XT6m and will investigate well known alternatives to the load stepping schemes traditionally used in nonlinear structural mechanics within this context.

Parallel Bidomain Solvers for Cardiac Excitation

Session Schedule Author Index Session Index M12

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Abstract

The Bidomain model of electrocardiology describes the bioelectric activity of the cardiac muscle and consists of a parabolic degenerate system of non-linear partial differential equation (PDE). The PDEs are coupled with a system of ordinary differential equations (ODEs), modeling the cellular membrane ionic currents. The discretization of the Bidomain model in three-dimensional (3D) cardiac geometries yields the solution of large scale and ill-conditioned linear systems at each time step. The aim of this work is to develop parallel multilevel and block preconditioners, in order to reduce the high computational costs required by the solution of the Bidomain model in 3D domains of realistic size. We analyze the scalability of multilevel Schwarz block-diagonal and block-factorized preconditioners for the Bidomain model and compare them with multilevel Schwarz coupled preconditioners. 3D parallel numerical tests show that block preconditioners are scalable, but less efficient than the coupled preconditioners.

Decoupled Time-Marching Schemes in Computational Cardiac Electrophysiology and ECG Numerical Simulation

Session Schedule Author Index Session Index M12

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Abstract

This work considers the approximation of the cardiac bidomain equations, either isolated or coupled with the torso, via first order semi-implicit time-marching schemes involving a fully decoupled computation of the unknown fields (ionic state, transmembrane potential, extracellular and torso potentials). For the isolated bidomain system, we show that the Gauss-Seidel and Jacobi like splittings do not compromise energy stability; they simply alter the energy norm. Time-step constraints are only due to the semi-implicit treatment of the non-linear reaction terms. Within the framework of the numerical simulation of electrocardiograms (ECG), these bidomain splittings are combined with an explicit Robin-Robin treatment of the heart-torso coupling conditions. We show that the resulting schemes allow a fully decoupled (energy) stable computation of the heart and torso fields, under an additional mild CFL like condition. Numerical simulations, based on anatomical heart and torso geometries, illustrate the stability and accuracy of the proposed schemes

Parallel Preconditioners for Solving Fluid-Structure Interactions Problems in Hemodynamics

Session Schedule Author Index Session Index M12

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Abstract

Modeling Fluid-Structure Interaction (FSI) in the vascular system is mandatory to reliably compute flow indicators when the vessels undergo large deformations. The resolution of the fully 3D FSI problem is very expensive; in order to lower the time to solution and to address complex problems, a parallel framework is necessary. To achieve good performances on large scale parallel architectures, we have developed preconditioners for the fully coupled FSI system.

The important factors to measure parallel performances of a preconditioner are the independence on the number of iterations on the cpu count (scalability of the preconditioner), on the mesh size (optimality), and on the physical parameters (robustness), as well as the strong and weak scalability. We aim at devising specific preconditioners for High Performance Computing (HPC). In particular we take advantage of state of the art preconditioners for Navier-Stokes problems such as the Pressure-Convection-Diffusion (PCD) preconditioner introduced by Elman, Sylvester, and Wathen to solve efficiently the fluid part of the FSI model.

We compare the evolution of the number of iterations to solve the full system with classical methods on a physiological geometry. We also investigate the strong scalability of our FSI solver. All the computations are carried out using the open source finite element library LifeV (www.lifev.org) based on Trilinos (http://trilinos.sandia.gov).

Mini Symposium M13

Robust Multilevel Methods for Multiscale Problems

Schedule Author Index Session Index

Organizers: Thomas Dufaud, Johannes Kraus, Clemens Pechstein, Robert Scheichl, Jörg Willems

Abstract

Typical multiscale problems are PDEs with large variation in the coefficients (in particular high contrast). The probably "simplest" example is the scalar elliptic equation $-\mathrm{div}(\alpha\nabla u) = f$ with a uniformly elliptic coefficient α that can vary over several orders of magnitude throughout the domain. Other important examples are flows in heterogeneous porous media (used in oil reservoir or groundwater flow simulation) or problems in structural mechanics with heterogeneous, possibly anisotropic materials. Iterative solvers for large-scale problems of this type need not only be robust with respect to the discretization parameters, but also with respect to the coefficient heterogeneities, which is in general a difficult task. While there are several approaches yielding robust–or almost robust–performance in practical computations, the rigorous verification of this robustness is still an open problem in many situations. The goal of this minisymposium is to collect the state of the art in two- and multilevel solvers for multiscale problems and to explore the links to numerical upscaling. Special emphasis shall be put on how preconditioners, smoothers, coarse spaces, and upscaled equations are constructed (and adapted to the problem of interest) in order to gain robustness – both theoretically and numerically.

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M13P1 Schedule

Robust Multilevel Methods for Multiscale Problems

Schedule Author Index Session Index

Date: Monday, June 25
Time: 16:00-18:10
Location: Petri
Chairman: Thomas Dufaud, Johannes Kraus, Clemens Pechstein, Robert Scheichl, Jörg Willems

16:05-16:30 : Victorita Dolean
Analysis of Two-Level Method for Heterogeneous Darcy Equation based on Local Dirichlet to Neumann Maps
Abstract

16:30-16:55 : Nicole Spillane
GenEO: A Coarse Space based on Generalized Eigenvalue Problems in the Overlaps
Abstract

16:55-17:20 : Jinchao Xu
Single-Grid Multilevel Method
Abstract

17:20-17:45 : Juan Galvis
Multiscale Spectral AMGe Solvers for High-Contrast Flow Problems
Abstract

17:45-18:10 : Clark Dohrmann
Constraint and Weight Selection Algorithms for BDDC
Abstract

M13 Abstract

Part 2

Part 3

Part 4

Analysis of Two-Level Method for Heterogeneous Darcy Equation based on Local Dirichlet to Neumann Maps

Session Schedule Author Index Session Index M13

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Abstract

Coarse grid correction is a key ingredient in order to have scalable domain decomposition methods. For smooth problems, the theory and practice of such two-level methods is well established, but this is not the case for problems with complicated variation and high contrasts in the coefficients. We present here a rigorous analysis of a two-level overlapping additive Schwarz method (ASM) with a coarse space based on low frequency modes of local subdomains Dirichlet-to-Neumann (DtN) maps. We also provide an automatic criterion for the number of modes that need to be added per subdomain to obtain a convergence rate of the order of the constant coefficient case. Our method is suitable for parallel implementation and its efficiency is demonstrated by numerical examples on some challenging problems with high heterogeneities for automatic partitionings.

GenEO: A Coarse Space based on Generalized Eigenvalue Problems in the Overlaps

Session Schedule Author Index Session Index M13

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Abstract

We take advantage of the pre-existing framework for the study of two-level additive Schwarz preconditioners to build a coarse space which is robust with regard to heterogeneities in any of the coefficients in the PDEs. We do this regardless of the way the domain is split into subdomains.In order to identify which components of the solution should be part of the coarse space we solve local generalized eigenproblems in each subdomain. In fact, the eigenproblems are defined only on the part of each subdomain which is overlapped by neighbouring subdomains. This allows us to identify a family of low-frequency modes which slow down convergence. Our method is suitable for parallel implementation and its implementation only requires the knowledge of the element matrices. We give a rigorous theoretical result for the condition number of the two-level overlapping additive Schwarz method with this coarse space and demonstrate its efficiency through numerical examples on some challenging three dimensional problems with high heterogeneities for automatic partitions.

Single-Grid Multilevel Method

Session Schedule Author Index Session Index M13

Jinchao Xu
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Abstract

In this talk, I will present a new approach to designing algebraic multigrid methods for discretized PDEs discretized on general unstructured grids. The main issue to address is parallelization.

Multiscale Spectral AMGe Solvers for High-Contrast Flow Problems

Session Schedule Author Index Session Index M13

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Abstract

We construct and analyze multigrid methods with nested coarse spaces for second-order elliptic problems with high-contrast multiscale coefficients. The design of the methods utilizes stable multilevel decompositions with a bound that generally grows with the number of levels. To stabilize this growth, in our theory, we use AMLI-cycle multigrid which leads to an overall optimal cost algorithm. The robustness, with respect to the contrast, is guaranteed due to the combined effect of the Schwarz smoothers used and the spectral construction of the coarse bases. More specifically, in order to obtain an optimal multilevel decomposition, we combine multigrid ideas in the recent two-level methods in [Multiscale Model. Simul. 8(5), 1621-1644], and earlier, in the element-based algebraic multigrid methods (or AMGe), that use local spectral problems to enrich the coarse space. In general, the intermediate coarse spaces need to be enriched in order to get contrast-independent convergence. The general techniques presented here allow us to study the problem of an optimal enrichment in the sense of enriching with a minimal number of extra coarse degrees of freedom. Thus, the methods we develop are optimal, with respect to both the contrast and the number of levels used. Moreover, we have the potential to achieve this goal with a minimal number of coarse degrees of freedom. We present numerical results that illustrate our theoretical findings. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

Constraint and Weight Selection Algorithms for BDDC

Session Schedule Author Index Session Index M13

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Abstract

We present constraint and weight selections algorithms for Balancing Domain Decomposition by Constraints (BDDC) which can be used to design preconditioners of known quality. Both algorithms are motivated by the goal to minimize a certain condition number estimate that only requires local subdomain information. Numerical examples are presented for a variety of different problems to confirm the theory and to demonstrate the utility of the algorithms. These problems include subdomains with two or more materials, almost incompressible elasticity, and subdomains with irregular boundaries.

M13P2 Schedule

Robust Multilevel Methods for Multiscale Problems

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 10:30-12:15
Location: Petri
Chairman: Thomas Dufaud, Johannes Kraus, Clemens Pechstein, Robert Scheichl, Jörg Willems

10:35-11:00 : Petr Vanek
An Alternative to Domain Decomposition Methods based on Polynomial Smoothing
Abstract

11:00-11:25 : Robert Scheichl
Energy Minimizing Coarse Space Construction
Abstract

11:25-11:50 : James Brannick
Recent Advances in Algebraic Multigrid
Abstract

11:50-12:15 : Marco Buck
Domain Decomposition Preconditioners for the Multiscale Analysis of Linear Elastic Composites
Abstract

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An Alternative to Domain Decomposition Methods based on Polynomial Smoothing

Session Schedule Author Index Session Index M13

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Abstract

The domain decomposition methods are efficient tools for solving large–scale linear systems that originate from discretizing elliptic partial differential equations. In general, the computational domain is decomposed into subdomains and the global problem is solved by means of subdomain solvers. The typical domain decomposition method is a variational two–level method with a coarse–space of a resolution following closely the subdomain size and a massive smoother based on local solvers. This global frame is suited well for both distributed memory and shared memory architectures. The majority of a computational work is done by subdomain solvers; each subdomain solver is assigned to a single processor. This computational organization results in parallelism that uses the number of processors equal to the number of subdomains. Recently, it became popular to use massively parallel graphic cards in high–performance computing. Contemporary graphic cards are basically vector processors. Hence, it is desirable to design domain decomposition type methods that are based on the action $y = Ax$ as a key operation. Indeed, this action allows to use up to $n = ord(A)$ processors and can be performed very efficiently on a vector–type architecture. We propose a class of methods based on polynomial smoothing rather than subdomain solvers. The key operation is the action of a massive polynomial smoother with an error propagation operator $E = (I - \alpha_1 A). \dots .(I - \alpha_N A)$. Those methods are generally variational multigrid methods with aggressive coarsening and massive polynomial smoothing. For those methods, we prove an optimal convergence result (independent of both fine and coarse–level resolution) provided the degree of our polynomial smoother (number of smoothing Richardson sweeps) is about $1/2H/h$ (h is the fine and H is the coarse–level resolution). The resulting algorithms are (even in serial case) asymptotically much cheaper than domain decomposition methods based on subdomain solvers and allow for massive parallelism on vector machines.

Energy Minimizing Coarse Space Construction

Session Schedule Author Index Session Index M13

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Abstract

Energy minimizing coarse spaces are key to robust multilevel iterative methods for multiscale elliptic problems. They are at the heart of algebraic multigrid methods and have been extensively analysed over the last 5 years and extended also to systems of partial differential equations. How exactly to minimise the energy in an effective way, while still maintaining sparsity and scalability with a rigorous underlying analysis, is still an ongoing international quest. In this talk we would like to return to a simple algebraic method that we presented a few years ago, in the context of two-level additive Schwarz. Given a set of supports, this method finds such a minimum energy basis subject to a partition of unity constraint using one local solve per coarse space basis function and one global solve to enforce the partition of unity constraint. Although this global solve may seem prohibitively expensive, we argued then that a one-level overlapping Schwarz method is an effective and scalable preconditioner and showed that such a preconditioner can be implemented efficiently using the Sherman-Morrison-Woodbury formula. The result was an elegant, scalable, algebraic method for constructing a robust coarse space given only the supports of the coarse space basis functions. Numerical experiments confirmed this. We now present an analysis of this coarse space construction that rigorously justifies its optimal complexity. Numerical experiments also show that, when used in a two-level preconditioner, the energy minimizing coarse space gives better results than some other coarse space constructions. We finish with some comments on how to extend the ideas to energy minimizing coarse spaces with more than one constraint.

Recent Advances in Algebraic Multigrid

Session Schedule Author Index Session Index M13

James Brannick
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Abstract

In this talk, I will highlight several recent advances in the development and analysis of AMG coarsening algorithms. I will discuss various strategies for selecting the coarse variables and defining interpolation, in both the classical AMG and matching (aggregation) AMG settings. Numerical experiments of the proposed techniques applied to various challenging linear systems will also be provided.

Domain Decomposition Preconditioners for the Multiscale Analysis of Linear Elastic Composites

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Abstract

We analyse two-level overlapping Schwarz domain decomposition methods for a finite element discretization of the PDE system of linear elasticity. The focus in our study lies in the application to compressible, particle-reinforced composites in 3D with large jumps in their material coefficients. We present coefficient-explicit bounds for the condition number of the two-level Additive Schwarz preconditioned linear system. Thereby, we do not require that the coefficients are resolved by the coarse mesh. The bounds show a dependence of the condition number on the energy of the coarse basis functions, the coarse mesh and the overlap parameters. Similar estimates have been developed for scalar elliptic PDEs in the work of Graham, Lechner and Scheichl ("Domain decomposition for multiscale PDEs"). The coarse spaces to which they apply here are assumed to contain the six rigid body modes and can be considered as generalizations of the space of piecewise linear vector valued functions on a coarse triangulation. The developed estimates provide a concept for the construction of coarse spaces which can lead to preconditioners which are robust w.r.t. discontinuities in the Young's modulus and the Poisson ratio of the underlying composite.

To confirm the theoretical results numerically, we first extend the linear multiscale finite element method as formulated by T. Hou and X. Wu to the system of linear elasticity. E.g., using a multiscale coarse space and assuming that inclusions of high contrast are isolated in the interior of coarse elements, we observe condition number bounds independent of variations in the Young's modulus and the Poisson ratio. Further on, linear and energy minimizing coarse spaces are discussed.

M13P3 Schedule

Robust Multilevel Methods for Multiscale Problems

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 16:00-17:45
Location: Petri
Chairman: Thomas Dufaud, Johannes Kraus, Clemens Pechstein, Robert Scheichl, Jörg Willems

16:05-16:30 : Florian Thomines
A Systematic Coarse-Scale Model Reduction Technique for Parameter-Dependent Flows in Highly Heterogeneous Media
Abstract

16:30-16:55 : Ivan Graham
Multiscale Finite Elements for High-Contrast Elliptic Problems
Abstract

16:55-17:20 : Jan Nordbotten
Approximate Multilevel Solvers for Flow and Transport in Porous Media
Abstract

17:20-17:45 : Xiaozhe Hu
Parallel AMG Method on GPU
Abstract

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A Systematic Coarse-Scale Model Reduction Technique for Parameter-Dependent Flows in Highly Heterogeneous Media

Session Schedule Author Index Session Index M13

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Abstract

The talk will discuss a multiscale approach for solving the parameter-dependent elliptic equation with highly heterogeneous coefficients. In particular, we assume that the coefficients have both small scales and high contrast (where the high contrast refers to the large variations in the coefficients). The main idea of our approach is to construct local basis functions that encode the local features present in the coefficient to approximate the solution of parameter-dependent flow equation. Constructing local basis functions involves (1) finding initial multiscale basis functions and (2) constructing local spectral problems for complementing the initial coarse space. We use the Reduced Basis (RB) approach to construct a reduced dimensional local approximation that allows quickly computing the local spectral problem. This is done following the RB concept by constructing a low dimensional approximation offline. For any online parameter value, we use a reduced dimensional approximation of the local problem to construct multiscale basis functions. These local computations are fast and are used to solve the coarse-scale dimensional problem. The coarse problem is used to construct robust iterative methods of the domain decomposition type. Our numerical results show that one can achieve a substantial dimension reduction when solving the local spectral problems. We discuss convergence of the method and the computational cost of the proposed method.

Multiscale Finite Elements for High-Contrast Elliptic Problems

Session Schedule Author Index Session Index M13

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Abstract

We first discuss multiscale finite element methods for elliptic interface problems with high contrast coefficients. These are approximated on coarse quasiuniform meshes, which do not need to resolve the interfaces. The methods are H^1- conforming, and require the solution of subgrid problems for the basis functions on elements which straddle the coefficient interface, but use standard linear approximation otherwise. The methods have (optimal) convergence rate in the energy and L_2 norms, independent of the "contrast" (i.e. ratio of largest to smallest value) of the PDE coefficient. A key point is the introduction of coefficient-dependent interior boundary conditions for the subgrid problems. Since these boundary conditions are rather technical and delicate, we also investigate more generally applicable adaptive methods which aim to find appropriate boundary conditions automatically.

Approximate Multilevel Solvers for Flow and Transport in Porous Media

Session Schedule Author Index Session Index M13

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Abstract

The governing equations for flow and transport in porous media are characterized by non-linear constitutive relationships that are heterogeneous at almost every spatial scale. Furthermore, for geological applications, the heterogeneous coefficients of these constitutive relationships may be highly uncertain.

Thus for practical problems, the parameters of the governing equations are at best known statistically, and the simulation results must correspondingly be interperted to have a significant degree of uncertainty. Compounding this uncertainty is significant spatial extent of the domains of interest, forcing engineers to apply discrete spatial and temporal resolutions that cannot be considered converged.

Despite the large uncertainty in the model problem and the approximations introduced through discretization, it is common practice to solve the resulting linear and non-linear equations to a high degree of accuracy. This is necessary, as most numerical algorithms for solving the discrete system of non-linear equations are sensitive to approximation errors in the solvers.

Our goal is to provide inexact solvers that are in a sense *structure preserving*, while retaining efficiency. The exact structures to be preserved are dependent on both the underlying problem and overall numerical algorithm. Examples include monotonicity or divergence of the approximate solution. The structure preservation thus allows us to apply inexact linear and non-linear solvers without losing the robustness of the numerical algorithm.

In this talk, we present the framework a framework for structure-preserving inexact solvers in the setting on preconditioners for linear systems. From a theoretical viewpoint, we discuss how structure preservation allows for *a priori* and *a posteriori* estimates. From a practical viewpoint, we highlight the realization of the framework for time-dependent flows in fractured porous media. For this application, the framework allows us to extract coarse-level discretizations, which we use to construct an adaptive coarsening algorithm.

Parallel AMG Method on GPU

Session Schedule Author Index Session Index M13

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Abstract

Developing parallel algorithms for solving large sparse linear systems is an important and challenging task in scientific computing and practical applications. In this work, we develop a new parallel algebraic multigrid (AMG) method for GPU. The coarsening and smoothing procedures in our new algorithm are based on a region quadtree (octree in 3D) generated from an auxiliary grid. This provides (nearly) optimal load balance and predictable communication patterns — factors that make our new algorithm suitable for parallel computing, especially on GPU. Numerical results show that our new method can speed up the existing GPU code (CUSP from NVIDIA) by a factor of 4 on a quasi-uniform grid and by a factor of 2 on a shape-regular grid for certain model problems. This work is co-authored by J. Cohen, L. Wang, and J. Xu.

M13P4 Schedule

Robust Multilevel Methods for Multiscale Problems

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 10:30-12:15
Location: Petri
Chairman: Thomas Dufaud, Johannes Kraus, Clemens Pechstein, Robert Scheichl, Jörg Willems

10:35-11:00 : Baptiste Poirriez
Deflation and Neumann-Neumann Preconditioner for Schur Domain Decomposition Method
Abstract

11:00-11:25 : Thomas Dufaud
An Algebraic Multilevel Preconditioning Framework based on Information of a Richardson Process
Abstract

11:25-11:50 : Svetozar Margenov
Multilevel Preconditioning of Strongly Anisotropic Elliptic Problems
Abstract

11:50-12:15 : Johannes Kraus
Robust Domain Decomposition Multigrid Methods using Additive Schur Complement Approximation
Abstract

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Deflation and Neumann-Neumann Preconditionner for Schur Domain Decomposition Method

Session Schedule Author Index Session Index M13

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Abstract

Numerical simulations in fractured media are an essential tool for studying hydraulic properties. Discrete Fracture Networks are composed of many multiscale plane fractures intersecting each other, leading to complex geometries. We assume that the rock surrounding the fractures is impervious and we aim at simulating the flow in the fractures. Governing equations are Darcy's law and mass continuity, with continuity conditions at the intersections. Mesh generation is rather difficult in this context, because of the geometry, and requires a specific method. We apply a Mixed Hybrid Finite element method and get a large sparse symmetric positive definite (spd) linear system to solve.

We study a domain decomposition method, which takes advantages from both the direct method and the Preconditioned Conjugate Gradient (PCG). This Schur method reduces the global problem to an interface problem, with a natural domain decomposition based on fractures or fracture packs. We propose an original approach for optimizing the algorithm and a global preconditioning of deflation type. Since the Schur complement S is spd, we apply PCG to solve the linear system $Sx = b$. We use the classical Neumann-Neumann (NN) preconditioner. To gain in efficiency, we use only one Cholesky factorization of the subdomain matrices for the preconditionning and the conjugate gradient steps. We also define a coarse space, based on the subdomain definition, to apply a deflation preconditioner.

We do a theoretical complexity study of our algorithm. We use this study, with the numerical data, to compute experimental complexity. We compare the results between several combination for the preconditioner. Then, we confront our results with existing solvers.

An Algebraic Multilevel Preconditioning Framework based on Information of a Richardson Process

Session Schedule Author Index Session Index M13

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Abstract

A fully algebraic framework for constructing coarse spaces for multilevel preconditioning techniques is proposed. Multilevel techniques are known to be robust for scalar elliptic Partial Differential Equations with standard discretization and to enhance the scalability of domain decomposition method such as RAS preconditioning techniques. An issue is their application to linear system encountered in industrial applications which can be derived from non-elliptic PDEs. Moreover, the building of coarse levels algebraically becomes an issue since the only known information is contained in the operator to inverse. Considering that a coarse space can be seen as a space to represent an approximated solution of a smaller dimension than the leading dimension of the system, it is possible to build a coarse level based on a coarse representation of the solution. Drawing our inspiration from the Aitken-SVD methodology, dedicated to Schwarz methods, we proposed to construct an approximation space by computing the Singular Value Decomposition of a set of iterated solutions of the Richardson process associated to a given preconditioner. This technique does not involve the knowledge of the underlying equations and can be applied to build coarse levels for several preconditioners. Numerical results are provided on both academic and industrial problems, using two-level additive preconditioners built with this methodology.

Multilevel Preconditioning of Strongly Anisotropic Elliptic Problems

Session Schedule Author Index Session Index M13

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Maria Lymbery
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Abstract

The talk commences by giving an overview of the second order elliptic problem discretized by linear conforming or nonconforming Crouzeix-Raviart finite elements (FE). In order to obtain a prescribed accuracy a uniform recursive refinement of the initially introduced triangulation $\mathcal{T}_0$ is performed, and the nested meshes $\mathcal{T}_0 \subset \mathcal{T}_1 \subset \ldots \subset \mathcal{T}_\ell = \mathcal{T}_h$ are constructed. The main focus of the study is on the development of robust multilevel preconditioning methods for strongly anisotropic problems. The first part is devoted to the construction and analysis of Algebraic MultiLevel Iteration (AMLI) methods in the case of coefficient jumps which are aligned with the interfaces of the initial mesh $\mathcal{T}_0$. The presented condition number estimates are uniform with respect to both mesh and/or coefficient anisotropy, the jumps, as well as the size of the discrete problem. The case of higher order FEs is discussed in the second part of the talk. For instance, for quadratic FEs the standard hierarchical basis techniques do not result in splittings in which the angle between the coarse space and its hierarchical complement is uniformly bounded with respect to the anisotropy ratio. Here some recent alternative results are presented based on additive Schur complement approximations, including the case of anisotropy which is not aligned with the mesh. The two-level method is recursively applied in the construction of robust multilevel preconditioners with optimal or nearly optimal order of computational complexity.

Robust Domain Decomposition Multigrid Methods using Additive Schur Complement Approximation

Session Schedule Author Index Session Index M13

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Abstract

Sparse Schur complement approximations play a key role in various iterative methods for solving systems of linear algebraic equations arising from finite element discretization of partial differential equations. In this talk we consider an algorithm for additive Schur complement approximation that is based on computing and assembling exact Schur complements of local (stiffness) matrices associated with a covering of the entire domain by overlapping subdomains. The resulting coarse-grid matrix is sparse and is shown to be spectrally equivalent to the (global) Schur complement with a bound on the relative condition number independent of the variations in the coefficients of the model elliptic equation. This approach allows for constructing a variational multigrid method that provides energy minimizing interpolation on an auxiliary space. The related two-grid method is analyzed using the fictitious space lemma. Several possibilities of exploiting this new type of coarse-grid operator are illustrated. Numerical experiments demonstrate uniform multigrid convergence for problems with highly oscillatory coefficients.

Mini Symposium M14

100% Parallelizable Algorithms for Symmetric, Indefinite and Non-Symmetric Problems

Schedule Author Index Session Index

Organizers: Ismael Herrera and Luis Miguel de la Cruz

Abstract

Domain decomposition methods (DDM) are the most efficient means for applying parallel-computing to the solution of partial differential equations. Thus, a main goal of DDM research has been to develop highly parallelizable algorithms. As a result, since the international community began intensively studying DDM, attention has shifted from overlapping to non-overlapping methods, mainly because algorithms derived from non-overlapping methods can achieve a higher level of parallelization. Furthermore, the impressive progress of parallel hardware that has taken place in recent years demands the availability of 100%-in-parallel-software. On the other hand, at present it is recognized that competitive algorithms need to incorporate constraints, such as continuity on primal-nodes. This, however, poses a new challenge for developing 100% parallelizable algorithms, which has been difficult to overcome. This mini-symposium is devoted to present and discuss four algorithms with constraints of wide applicability, all of them 100% parallelizable. Such algorithms can be applied to symmetric, non-symmetric and indefinite problems. The new algorithms have been derived in the realm of a framework recently introduced: the DVS-framework [1-3]. Two of them are the DVS versions of BDDC and FETI-DP, respectively, while we could not identify in the literature algorithms with clear similarities to the other two. This mini-symposium is made of four lectures, two of them devoted to explain the new algorithms in general and the other two to applications.

REFERENCES

[1] Herrera, I. et al. Geofisica Internacional, 50, pp 445-463, 2011.
[2] Herrera, I. et al. NUMER. METH. PART D. E. 27, pp. 1262-1289, 2011.
[3] Herrera, I. et al. NUMER. METH. PART D. E. 26, pp. 874-905, 2010.

M14 Schedule

100% Parallelizable Algorithms for Symmetric, Indefinite and Non-Symmetric Problems

Schedule Author Index Session Index

Date: Monday, June 25
Time: 10:30-12:15
Location: Petri
Chairman: Ismael Herrera and Luis Miguel de la Cruz

10:35-11:00 : Ismael Herrera
Four Massively Parallel Algorithms for Symmetric, Indefinite and Non-Symmetric Matrices: Overview
Abstract

11:00-11:25 : Luis Miguel de la Cruz
Four Massively Parallel Algorithms for Symmetric, Indefinite and Non-Symmetric Matrices: Implementation Issues
Abstract

11:25-11:50 : Alberto Rosas
Four Massively Parallel Algorithms for Symmetric, Indefinite and Non-Symmetric Matrices: Applications to a Single Equation
Abstract

11:50-12:15 : Iván Contreras
Four Massively Parallel Algorithms for Static Elasticity
Abstract

Four Massively Parallel Algorithms for Symmetric, Indefinite and Non-Symmetric Matrices: Overview

Session Schedule Author Index Session Index M14

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Abstract

In this lecture a set of four general purpose algorithms that are very suitable for building the software that is required for efficiently programming the most powerful parallel computers available at present are introduced and explained. The most effective procedures for approaching the ideal of achieving totally parallelized algorithms are derived from non-overlapping domain decomposition methods. In principle, the goal is to develop algorithms capable of constructing the global solution by solving local problems, in each partition subdomain, exclusively. At present, however, competitive algorithms need to incorporate constraints, such as continuity on primal-nodes. This poses an additional challenge, which has been difficult to overcome. This mini-symposium is devoted to present and discuss four general preconditioned algorithms with constraints in which such a goal is achieved; namely, the global solution is obtained by solving local problems exclusively. Each one of such four algorithms can be applied to symmetric, non-symmetric and indefinite problems. The new algorithms have been derived in the realm of the *DVS-framework*, recently introduced [1-3]. Two of them are the DVS versions of BDDC and FETI-DP, respectively. As for the other two, we have not identified in the literature algorithms with clear similarities to them. Keywords: Massively-parallel algorithms; parallel-computers; non-overlapping DDM; DDM with constraints; BDDC; FETI-DP

[1] Herrera, I., Carrillo-Ledesma A. & Rosas-Medina A. A Brief Overview of Non-overlapping Domain Decomposition Methods, Geofisica Internacional, Vol. 50(4), pp 445-463, 2011.

[2] Herrera, I. & Yates R. A. The Multipliers-Free Dual Primal Domain Decomposition Methods for Nonsymmetric Matrices, NUMER. METH. PART D. E. 27(5) pp. 1262-1289, 2011. DOI 10.1002/Num. 20581. (Published on line April 28, 2010)

[3] Herrera, I. & Yates R. A. The Multipliers-free Domain Decomposition Methods, NUMER. METH. PART D. E. 26(4) pp. 874-905, 2010, DOI 10.1002/num. 20462 (Published on line April 23, 2009).

Four Massively Parallel Algorithms for Symmetric, Indefinite and Non-Symmetric Matrices: Implementation Issues

Session Schedule Author Index Session Index M14

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Abstract

Nowadays parallel computing is ubiquitous and almost all new computational resources contain more than one processing unit. This outstanding progress gives us the opportunity to develop parallel codes that take advantage of the current parallel architectures. On the other hand, domain decomposition methods (DDM) allow us to model macroscopic systems applying efective parallel algorithms, and in particular, the non-overlapping methods can achieve a higher level of parallelization. The first talk of this mini-symposium was devoted to present a set of four general purpose algorithms that are very suitable for efficiently programming the powerful parallel computers available at present. These new algorithms have been derived in the realm of the DVS-framework [1-3]. The numerical and computational issues, as well as some examples of application are presented in this talk. From the point of view of software engineering, the DVS-framework offers a general plataform which give us a natural separation of the concepts and operations, that results in general, efficient and elegant codes. In this implementation we use the Finite Volume Method (FVM) to obtain the numerical model, although we could have used almost any other discretization procedure. We apply object oriented and generic programming paradigms in order to generate several generic units that in turn can be used to construct the codes for the algorithms of DVS. These algorithms are iterative in nature, and are based on some well known Krylov methods, to say CGM, GMRES or some others. Finally, we present some parallelization metrics that measure the speedup and efficiency of our implementations.

[1] Herrera, I. et al. Geofisica Internacional, 50, pp 445-463, 2011.

[2] Herrera, I. et al. NUMER. METH. PART D. E. 27, pp. 1262-1289, 2011.

[3] Herrera, I. et al. NUMER. METH. PART D. E. 26, pp. 874-905, 2010.

Four Massively Parallel Algorithms for Symmetric, Indefinite and Non-Symmetric Matrices: Applications to a Single Equation

Session Schedule Author Index Session Index M14

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Abstract

In the introductory lecture of this minisymposium a set of four general purpose algorithms that are very suitable for building the software that is required for efficiently programming the most powerful parallel computers available at present were introduced and explained. Such algorithms are equally applicable to a single partial-differential equation and to systems made of many such equations; thus, in this mini-symposium that is devoted to them, applications are dealt with in some detail in two lectures: the present one, in which single-equation applications are discussed and another one in which system-of-equations applications are treated. All the algorithms considered were derived from non-overlapping domain decomposition methods with constraints using the *derived-vector space (DVS) framework*, recently introduced [1-3], and are applicable to symmetric, indefinite and non-symmetric matrices. The feature, shared by they all, that permits making them massively-parallel is that the global solution is obtained by solving local problems, in each partition subdomain, exclusively. Keywords: Massively-parallel algorithms; parallel-computers; non-overlapping DDM; DDM with constraints; BDDC; FETI-DP

[1] Herrera, I., Carrillo-Ledesma A. & Rosas-Medina A. A Brief Overview of Non-overlapping Domain Decomposition Methods, Geofisica Internacional, Vol. 50(4), pp 445-463, 2011.

[2] Herrera, I. & Yates R. A. The Multipliers-Free Dual Primal Domain Decomposition Methods for Nonsymmetric Matrices, NUMER. METH. PART D. E. 27(5) pp. 1262-1289, 2011. DOI 10.1002/Num. 20581. (Published on line April 28, 2010)

[3] Herrera, I. & Yates R. A. The Multipliers-free Domain Decomposition Methods, NUMER. METH. PART D. E. 26(4) pp. 874-905, 2010, DOI 10.1002/num. 20462 (Published on line April 23, 2009).

Four Massively Parallel Algorithms for Static Elasticity

Session Schedule Author Index Session Index M14

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Abstract

In order to profit from the parallel hardware available nowadays, massively-parallelized hardware is required. There are bounds for the level of parallelization that can be actually achieved beyond which it is not possible to go. For non-overlapping domain decompositions the goal is to develop algorithms capable of constructing the global solution by solving local problems, in each partition subdomain, exclusively. It has been generally recognized that for this purpose the introduction of constraints, as it is required today by competitive algorithms, constitutes an additional difficulty not easy to overcome. Fortunately, a set of four general purpose algorithms with constraints possessing such a feature and applicable to a broad class of matrices -symmetric, indefinite and non- symmetric- has recently been developed, as it is explained in the introductory lecture of this minisymposium. Thus, in the present talk we announce and explain four massively-parallel algorithms that have been derived from them for specifically treating the system of equations that govern Static Elasticity. All this work has been carried out in the *derived-vector space framework (DVS-framework)*, recently introduced by I. Herrera and his co-workers [1-3]. Keywords: Massively-parallel algorithms; parallel-computers; non-overlapping DDM; DDM with constraints; elasticity; BDDC; FETI-DP

[1] Herrera, I. et al. Geofisica Internacional, 50, pp 445-463, 2011.

[2] Herrera, I. et al. NUMER. METH. PART D. E. 27, pp. 1262-1289, 2011.

[3] Herrera, I. et al. NUMER. METH. PART D. E. 26, pp. 874-905, 2010.

Mini Symposium M15

Space-Time Parallel Methods

Schedule Author Index Session Index

Organizers: Martin J. Gander, Felix Kwok and Yvon Maday

Abstract

Often problems from real world applications are time dependent, and classical time stepping methods lend themselves only to parallelization in space. With the advent of new generation parallel computers with hundred-thousands of cores, the space direction is easily saturated, and one needs to include the time direction for parallelization. This minisymposium consists of recent research results on space-time parallel methods, with major new contributions like Dirichlet-Neumann and Neumann-Neumann methods for evolution problems, the application of the parareal algorithm to hyperbolic systems of conservation laws, Hamiltonian problems, spectral deferred correction variants of the parareal algorithm, and also new results for Schwarz waveform relaxation methods, moving mesh methods and control.

Part 1

Part 2

Part 3

M15P1 Schedule

Space-Time Parallel Methods

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 10:30-12:15
Location: Markov
Chairman: Martin J. Gander, Felix Kwok and Yvon Maday

10:35-11:00 : Yvon Maday
Parareal in Time Algorithm for Hyperbolic Systems
Abstract

11:00-11:25 : Michael Minion
Efficient Implementation of a Multi-Level Parallel in Time Algorithm
Abstract

11:25-11:50 : Rim Guetat
Coupling Parareal Algorithm with Domain Decomposition Methods
Abstract

11:50:12:15 : Felix Kwok
Neumann-Neumann Waveform Relaxation Methods for the Time-Dependent Heat Equation
Abstract

M15 Abstract

Part 2

Part 3

Parareal in Time Algorithm for Hyperbolic Systems

Session Schedule Author Index Session Index M15

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Abstract

The parareal in time algorithm allows to perform parallel simulations of time dependent problems. This algorithm has been implemented on many types of time dependent problems with some success. Recent contributions have allowed to extend the domain of application of the parareal in time algorithm so as to handle long time simulations of Hamiltonian systems. This improvement has managed to avoid the fatal large lack of accuracy of the plain parareal in time algorithm consequence of the fact that the plain parareal in time algorithm does not conserve invariants. A somehow similar difficulty occurs for problems where the solution lacks regularity, either initially or in the evolution, like for the solution to hyperbolic system of conservation laws. In this paper we identify the problem of lack of stability of the parareal in time algorithm and propose a simple way to cure it. The new method is used to solve a linear wave equation and a non linear Burger's equation, the results illustrate the stability of this variant of the parareal in time algorithm.

Efficient Implementation of a Multi-Level Parallel in Time Algorithm

Session Schedule Author Index Session Index M15

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Abstract

We explain how the communication between processors in a multi-level parallel-in-time algorithm for PDEs can be scheduled to reduce blocking communication. The particular time-parallel method examined is the parallel full approximation scheme in space and time (PFASST), which utilizes a heirarchy of spatial and temporal discretization levels. By decomposing the update to initial conditions passed between processors into multiple spatial resolutions, the communication at the finest level can be scheduled to overlap with computation at coarser levels. We demonstrate the cost savings with a three dimensional PDE example.

Coupling Parareal Algorithm with Domain Decomposition Methods

Session Schedule Author Index Session Index M15

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Abstract

Numerical temporal evolution schemes are sequential in nature and thus have been viewed as providing limited parallel performance gains. Parallelization with respect to the time variable appears as a seductive alternative. For solving temporal evolution problems using domain decomposition methods, the classical approach consists of using a parallel solver to solve the equations in space of an implicit time scheme at each time step. To avoid the major drawback of this approach, i.e., solving a stationary problem at each time step, an original method, called the parareal algorithm has been introduced a decade ago. This algorithm splits the global temporal evolution problem into a series of independent evolution problems on smaller time intervals. Both a coarse and fine grids have been introduced for the solution in time and the parareal algorithm can easily be summarized as a predictor-corrector method that allows to get parallelization through the time. For complex problems involving large size geometries, the parareal algorithm suffers from the size of the spatial problems to solve. In this paper, we propose to combine the parareal algorithm and the domain decomposition methods. We demonstrate that the combination of these two approaches (decomposition in space and decomposition in time) leads to an effective and robust algorithm, reducing significantly the computational time. The coupling parareal algorithm and the domain decomposition method have been implemented in our proper library within the C++ language. Numerical experiments illustrate the performance of this library on massive parallel computers. Applications on realistic industrial problems illustrate the extraordinary convergence properties (computational time, speed-up, scalability) of this new method.

Neumann-Neumann Waveform Relaxation Methods for the Time-Dependent Heat Equation

Session Schedule Author Index Session Index M15

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Abstract

We propose a waveform relaxation version of the Neumann–Neumann method for parabolic problems. Just like for the steady case, one step of the method consists of solving the subdomain problems using Dirichlet traces, followed by a correction step involving Neumann interface conditions. However, each subdomain problem is now in both space and time, and the interface data to be exchanged are also functions of time. Using a Laplace transform argument, we show for the heat equation that when we consider finite time intervals, the Neumann-Neumann method converges superlinearly both in one spatial dimension and for 2D decompositions into strips. The convergence rate depends on T/H^2, where T is the length of the time window and H is the size of the subdomain.

M15P2 Schedule

Space-Time Parallel Methods

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 16:00-17:45
Location: Markov
Chairman: Martin J. Gander, Felix Kwok and Yvon Maday

16:05-16:30 : Stefan Güttel
On the Convergence of Parallel Deferred Correction Methods
Abstract

16:30-16:55 : Martin J. Gander
Analysis of the Parareal Algorithm and a Symmetrized Variant for Hamiltonian Problems
Abstract

16:55-17:20 : Jacques Laskar
Time-Parallel Integrations for Long Term Solar System Studies
Abstract

17:20-17:45 : Julien Salomon
Time-Parallelization and Optimal Control for NMR
Abstract

M15 Abstract

Part 1

Part 3

On the Convergence of Parallel Deferred Correction Methods

Session Schedule Author Index Session Index M15

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Abstract

We discuss some preliminary convergence results for parallel deferred correction methods, with an emphasis on the Parareal-SDC variant proposed by Minion & Williams (2008) and Minion (2011), and a novel variant based on barycentric rational interpolation.

Analysis of the Parareal Algorithm and a Symmetrized Variant for Hamiltonian Problems

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Abstract

Hamiltonian problems have many geometric properties, like energy preservation and symplectic flows. When integrating such problems numerically, it is desirable to use numerical integrators that preserve some of these properties, in order to get good approximations over long time intervals. There are currently many well established geometric integrators, but when they are combined with the parareal algorithm, all good geometric properties are lost. We present a backward error analysis for the parareal algorithm and a recent symmetrized variant, which shows that indeed the length of the time interval on which the parareal algorithm is efficient has a precise bound, which unfortunately also holds for the symmetrized variant. We then optimize the parameters in the parareal algorithm for performance when applied to Hamiltonian problems, and present numerical results illustrating our analysis.

Time-Parallel Integrations for Long Term Solar System Studies

Session Schedule Author Index Session Index M15

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Abstract

In 1997, Saha Stadel and Tremaine have introduced a time-parallel algorithm for almost integrable Hamiltonian systems applied to the Solar System dynamics. Such an algorithm is based in an iterative predictor-corrector scheme and the perturbation theory for Hamiltonian systems. We remark that they have used a second order symplectic method as underlying integrator. In a slightly different approach, in 2001 Lions, Maday and Turinici have introduced another time-parallel scheme for PDEs knew as the "Parareal" scheme, which is independent of the numerical scheme. With the advent of the GPU computing systems, we have recently developed an extension of the Saha-Stadel-Tremaine algorithm which uses high order symplectic integrators taking advantage of the performance of the shared memory GPU technology. In this talk we explain the details of the algorithm and discuss some results about the speed-up we can reach. We propose an hybrid algorithm which combines the pure parareal scheme and our extended algorithm to simulate long term Solar System dynamics and we will discuss preliminary results for inner, and outer Solar System simulations.

Time-Parallelization and Optimal Control for NMR

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Abstract

From the mathematical point of view, Nuclear Magnetic Resonance (NMR) has the advantage that the results obtained in the simulations can be directly tested experimentally. Numerical methods play consequently a significant role in the design of efficient magnetic fields. After a short description of the model used in NMR, the Block equations, we will present an algorithm based on a relevant time parallelization that enables us to compute rapidly optimal magnetic fields.

M15P3 Schedule

Space-Time Parallel Methods

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 10:30-12:15
Location: Markov
Chairman: Martin J. Gander, Felix Kwok and Yvon Maday

10:35-11:00 : Bankim Chandra Mandal
Dirichlet-Neumann Waveform Relaxation for the Time Dependent Heat Equation
Abstract

11:00-11:25 : Mohamed Kamel Riahi
Parareal in Time Intermediate Targets Methods for Optimal Control Problem
Abstract

11:25-11:50 : Ron Haynes
A RIDC-DD Space-Time Algorithm for Time Dependent Partial Differential Equations
Abstract

11:50:12:15 : Olga Mula Hernandez
Parareal for Neutronic Core Calculations
Abstract

M15 Abstract

Part 1

Part 2

Dirichlet-Neumann Waveform Relaxation for the Time Dependent Heat Equation

Session Schedule Author Index Session Index M15

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Abstract

We present a waveform relaxation version of the Dirichlet-Neumann method for the time dependent heat equation. Like the Dirichlet-Neumann method for steady problems, the method is based on a non-overlapping spatial domain decomposition, and the iteration involves subdomain solves with Dirichlet boundary conditions followed by subdomain solves with Neumann boundary conditions. However, each subdomain problem is now in space and time, and the interface conditions are also time-dependent. An analysis using Laplace transforms shows linear convergence for unbounded spatial domains, except for a very specific choice of the relaxation parameter, for which the method converges in a finite number of steps. A more refined analysis on bounded domains reveals then that for this optimal choice of the relaxation parameter, we get superlinear convergence when we consider finite time windows, similar to the case of Schwarz waveform relaxation algorithms. The convergence rate depends on the length of the subdomains as well as the size of the time window. For any other choice of the relaxation parameter, convergence is only linear. We illustrate our theoretical results with numerical experiments.

Parareal in Time Intermediate Targets Methods for Optimal Control Problem

Session Schedule Author Index Session Index M15

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Abstract

We present a method that enables us to solve in parallel the Euler-Lagrange system associated with the optimal control of a parabolic equation. Our approach is based on the definition and an iterative update of a sequence of intermediate targets that gives rise to independent sub-problems that can be solved in parallel. In order to accelerate the time-resolution, this method can be coupled with the parareal in time algorithm. Moreover, multi-dimensional optimization can be used with full parallelization to achieve the best scalability. Numerical experiments show the efficiency of our methods.

A RIDC-DD Space-Time Algorithm for Time Dependent Partial Differential Equations

Session Schedule Author Index Session Index M15

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Abstract

Recently, the Revisionist Integral Deferred Correction (RIDC) approach has been shown to be a relatively easy way to add small scale parallelism (in time) to the solution of time dependent PDEs. In this talk I will show how large scale spatial parallelism can be added to RIDC using relatively simple domain decomposition strategies. This results in a truly parallel space-time method for PDEs suitable for hybrid OpenMP/MPI implementation. Initial scaling studies will demonstrate the viability of the approach.

Parareal for Neutronic Core Calculations

Session Schedule Author Index Session Index M15

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Abstract

The parareal in time algorithm is a time domain decomposition method for the approximation of evolution problems. Its easy implementation in a parallel fashion allows for significant speed-ups in the computing time and opens the door to long time computations that involve accurate propagators. In this talk, we first propose to overview the different strategies for the parallelization of the algorithm. Then we will illustrate the efficiencies of these different strategies in a concrete PDE: the kinetic neutron diffusion equation in a nuclear reactor core. Implementations have been carried out with the MINOS solver, which is a tool developped at CEA in the framework of the APOLLO3® project. As a conclusion, we will discuss the possibility of using neutron diffusion as a coarse propagator for neutron transport.

Mini Symposium M16

Domain Decomposition with Mortars

Schedule Author Index Session Index

Organizers: Yvon Maday and Caroline Japhet

Abstract

Mortar methods are domain decomposition techniques based on a weak coupling between subdomains with nonconforming meshes, allowing different discretization schemes or even different physical models on each sides of the non-conforming interface. Originally introduced for the coupling of finite element with spectral elements methods, these techniques are used in a large class of finite element discretizations and for applications in computational electromagnetics, mechanics and fluid dynamics. There are still interesting theoretical and numerical difficulties in analysing a priori estimates for mortar coupling and in extending these methods to elastodynamics equations, to heterogeneous problems, to incompressible flows in axisymmetric channels, to optimized Schwarz methods and to FETI methods. The aim of this minisymposium is to report on recent advances in this field and on implementation issues with FreeFem++.

Part 1

Part 2

M16P1 Schedule

Domain Decomposition with Mortars

Schedule Author Index Session Index

Date: Monday, June 25
Time: 10:30-12:15
Location: Amphi
Chairman: Yvon Maday and Caroline Japhet

10:35-11:00 : Frédéric Hecht
Mortar Method to Solve Problem with Non-matching Grids in Freefem++
Abstract

11:00-11:25 : Alfio Quarteroni
Discontinuous Approximation of Elastodynamics Equations
Abstract

11:25-11:50 : Caroline Japhet
Mortar Methods with Optimized Transmission Conditions
Abstract

11:50:12:15 : Zakaria Belhachmi
Spectral Element Discretization of Incompressible Flows in Axisymmetric Channels
Abstract

M16 Abstract

Part 2

Mortar Method to Solve Problem with Non-matching Grids in Freefem++

Session Schedule Author Index Session Index M16

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Abstract

For a sake of simplicity, I will show how to solve classical physical problems with multiple domains with non-matching grids. The examples are

1. A Poisson equation solved with a domain decomposition based on three fields mortar method.
2. Couplings between two thermal models with a Contact thermal (i.e $F = [u]$ where F is the thermal flux, and $[u]$ is the jump of the temperature).
3. A Signoriny interface problem (example of the `IpOpt` interface).

In these three examples, we show some mortar technics, and how they solve this problem numerically on a parallel computer with MPI interface and in two or three dimensions.

Discontinuous Approximation of Elastodynamics Equations

Session Schedule Author Index Session Index M16

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Abstract

The possibility of inferring the physical parameter distribution of the Earth's substratum, from information provided by elastic wave propagations, has increased the interest towards computational seismology. Recent developments have been focused on spectral element methods. The reason relies on their flexibility in handling complex geometries, retaining the spatial exponential convergence for locally smooth solutions and a natural high level of parallelism. In this talk, we consider a Discontinuous Galerkin spectral element method (DGSEM) as well as discontinuous Mortar methods (DMORTAR) to simulate seismic wave propagations in three dimensional heterogeneous media. The main advantage with respect to conforming discretizations as those based on Spectral Element Method is that DG and Mortar discretizations can accommodate discontinuities, not only in the parameters, but also in the wavefield, while preserving the energy. The domain of interest Ω is assumed to be union of polygonal substructures Ω_i. We allow this substructure decomposition to be geometrically non-conforming. Inside each substructure Ω_i, a conforming high order finite element space associated to a partition $\mathcal{T}_{h_i}(\Omega_i)$ is introduced. We allow the use of different polynomial approximation degrees within different substructures. Applications to simulate benchmark problems as well as realistic seismic wave propagation processes are presented.
This work is in collaboration with P. Antonietti, I. Mazzieri and F. Rapetti.

Mortar Methods with Optimized Transmission Conditions

Session Schedule Author Index Session Index M16

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Abstract

For many applications in mechanics or fluid dynamics, one need to use different discretizations in different regions of the computational domain to match with the physical scales. Mortar methods are domain decomposition techniques based on a weak coupling between subdomains and enable the use of nonconforming grids. On the other hand, the optimized Schwarz methods, based on Robin or Ventcel transmission conditions greatly enhance the information exchange between subdomains and lead to robust and fast algorithms. A new cement has been developed over the last years which allows to glue nonconforming grids with Robin transmission conditions for Schwarz type methods.

We present this new cement for piecewise polynomials of low and high order in 2d and extended in 3d for P_1 elements. The nonconforming domain decomposition method is proved to be well posed, and the error analysis is performed. Then we present numerical results that illustrate the method.

Spectral Element Discretization of Incompressible Flows in Axisymmetric Channels

Session Schedule Author Index Session Index M16

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Abstract

We consider the Stokes and Navier-Stokes equations in a three-dimensional axisymmetric domain with non zero boundary conditions at the inward and outward faces. We propose a mortar spectral element discretization of this problem that relies on the use of truncated Fourier series in the angular direction and spaces of polynomials on each element of a partition of the meridian domain. A similar discretization in the framework of finite elements was considered in [1]. The key problem in the application of the method here is the choice of the quadrature formula: Indeed, the use of cylindrical coordinates leads to the appearance of a weight depending on the radial variable. Moreover, since we intend to use curved elements to handle the geometry of the initial domain, the discretization relies on the use of mappings which send a reference rectangle onto the curved element, so that we are led to consider problems with variable coefficients [2]. An idea to overcome this difficulty, due to Y. Maday et E. Rnquist [3], consists in using over-integration, in the following sense: The number of nodes in the quadrature formula is larger than the number of degrees of freedom for each unknown. We perform the numerical analysis of the discrete problem. Some numerical experiments for the Stokes system enable us to check the efficiency of the discretization.

[1] Z. Belhachmi, C. Bernardi, S. Deparis and F. Hecht, A truncated Fourier/finite element discretization of the Stokes equations in an axisymmetric domain, *Math. Models Methods Appl. Sci.*, **16** (2006). 233–263.

[2] Z. Belhachmi, A. Karageorghis, Spectral element discretization of the Stokes equations in deformed axisymmetric geometries, Adv. Appl. Math. Mech. **3**. **4** (2011), 448-469.

[3] Y. Maday and E. M. Ronquist, Optimal error analysis of spectral methods with emphasis on non-constants and deformed geometries, *Comput. Methods Appl. Mech. Engrg.*, **80** (1990.), 91-115.

M16P2 Schedule

Domain Decomposition with Mortars

Schedule Author Index Session Index

Date: Monday, June 25
Time: 16:00-18:10
Location: Amphi
Chairman: Yvon Maday and Caroline Japhet

16:05-16:30 : Christian Waluga
Quasi-Optimal a priori Estimates for the Lagrange Multiplier in Mortar Type Couplings
Abstract

16:30-16:55 : Yvon Maday
Some Recent Applications of Non Conforming Approximations
Abstract

16:55-17:20 : François-Xavier Roux
FETI-2LM for Localizing the Mortars
Abstract

17:20-17:45 : Oldřich Vlach
On Effective Implementation of the Non-penetration Condition for Non-matching Grids Preserving Scalability of FETI Based Algorithms
Abstract

17:45-18:10 : Todd Arbogast
Multiscale Mortar Mixed Methods for Heterogeneous Elliptic Problems
Abstract

M16 Abstract

Part 1

Quasi-optimal A Priori Estimates for the Lagrange Multiplier in Mortar Type Couplings

Session Schedule Author Index Session Index M16

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Abstract

We present quasi-optimal a priori convergence results for the approximation of surface based Lagrange multipliers such as those employed in mortar type finite element couplings. In classical estimates based on the standard saddle point theory, the error estimates for both the primal and dual variables are obtained simultaneously, which results in suboptimal a priori estimates for the dual variable. While improved estimates are fairly easily achievable if optimal order L^∞ bounds are available, the present analysis is based on significantly weaker regularity requirements. By using new estimates for the primal variable in strips of width $\mathcal{O}(h)$ near the mortar interfaces, we illustrate that an additional factor $\sqrt{h}|\ln h|$ in the a priori bound for the dual variable can be recovered, where the logarithmic factor can even be dropped for higher order elements ($p > 1$). We outline the analysis of a second order elliptic model problem and discuss possible extensions of the theory. Finally, we give different numerical examples to support the theoretical results.

Some Recent Applications of Non Conforming Approximations

Session Schedule Author Index Session Index M16

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Abstract

The original mortar element method has been introduced in order to be able to couple different type of variational discretizations used on different non overlapping subdomains. The matching on the interface between two subdomains is done through a Lagrange multiplier and the mortar method specifies an easy way to define these multipliers in order that the approximation is optimal in terms of degrees of freedom.

We present here some new applications of the mortar technology in the different framework of reduced basis element approximation for parameter dependent PDE's. The solution to such problems over each subdomains is approximated by a linear combination of well chosen solutions of a similar problem on the considered subdomain, the different approximations need to be glued at the interface with mortar type approaches. The approach can be used in conjunction with data assimilation where in some subdomains, the approximation is constructed in order, not to satisfy the PDE but match some measured data.

FETI-2LM for Localizing the Mortars

Session Schedule Author Index Session Index M16

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Abstract

Non conforming interfaces are generally introduced for modelling purpose, in specific areas. In such a context, there are only a few subdomains that are generally not balanced. The design of efficient parallel solution methods using domain decomposition requires a second level splitting that may create crosspoints on the non conforming interfaces where the mortar conditions couple more than two subdomains in a complex way. In order to avoid this drawback, all interface nodes associated with one mortar on each side of the initial non conforming interface must be attached to only one subdomain. Attaching extra nodes to a subdomain leads to ill posed local problems. With the FETI-2LM method this problem can be handled thanks to the Robin interface conditions. In this paper, we present the technique for relocalizing mortar conditions with the FETI-2LM method in the case of multi-level splitting. Results of numerical experiments will illustrate the presentation.

On Effective Implementation of the Non-penetration Condition for Non-matching Grids Preserving Scalability of FETI Based Algorithms

Session Schedule Author Index Session Index M16

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Abstract

Mathematical models of contact include the inequalities which make the contact problems strongly nonlinear. In spite of this, a number of interesting results have been obtained by modifications of the methods that were known to be scalable for linear problems, in particular of the FETI domain decomposition method introduced by Farhat and Roux for parallel solution of linear problems. The point of this paper is to extend our results obtained for elastic contact problems to the contact problems with non-matching grids which necessarily emerge, e.g., in the solution of transient contact problems or in contact shape optimization. We want to get good approximation and the constraint matrix B with nearly orthogonal rows. We consider both standard engineering approaches such as node to segment, or mortar elements. We give simple bounds on the singular values of the resulting matrix B and results of numerical experiments, including both the academic examples and some problems of practical interest. We conclude that the normalized orthogonal mortars proposed by Wohlmuth can be used to approximate the non-penetration conditions in a way that complies with the requirements of the FETI methods.

Multiscale Mortar Mixed Methods for Heterogeneous Elliptic Problems

Session Schedule Author Index Session Index M16

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Abstract

We consider the problem of computing flow fields in porous media with extreme natural heterogeneities. The system is modeled by a second order elliptic problem with a heterogeneous coefficient, which we write in mixed form. We develop numerical approximations suitable for parallel computation through the use of nonoverlapping domain decomposition mortar methods with a restricted set of degrees of freedom on the interfaces. We devise an effective but purely local multiscale method that incorporates information from homogenization theory. In the case of a locally periodic heterogeneous coefficient of period ϵ, we prove that the new method achieves both optimal order error estimates in the discretization parameters and convergence when ϵ is small. We also use this mortar approach to devise preconditioners that incorporate exact coarse-scale information to iteratively solve the full fine-scale problem. Moreover, we present numerical examples to assess the performance of the techniques.

Mini Symposium M17

Domain Decomposition Methods based on Robin Conditions for Large and / or Nonlinear Problems

Schedule Author Index Session Index

Organizers: Heiko Berninger, Sébastien Loisel, Oliver Sander

Abstract

Domain decomposition methods can be used to solve a wide variety of problems. In parallelized nonoverlapping settings, at each iteration, processors exchange data along an interface that separates the subdomains. In order to give improved convergence properties, one can choose optimized transmission conditions across the interface. These optimized transmission conditions should be chosen in such a way that the subdomain problems can be solved readily while maintaining fast convergence. In recent years, Robin boundary conditions have been used and optimized in a variety of linear and nonlinear problems, and some large-scale implementations are being developed. In this minisymposium, we will discuss recent developments in domain decomposition methods with Robin transmission conditions as well as their optimizations.

Part 1

Part 2

M17P1 Schedule

Domain Decomposition Methods based on Robin Conditions for Large and / or Nonlinear Problems

Schedule Author Index Session Index

Date: Thursday, June 28
Time: 10:30-12:15
Location: Amphi
Chairman: Heiko Berninger, Sébastien Loisel, Oliver Sander

10:35-11:00 : Sébastien Loisel
Large-Scale Implementation of Optimized Decomposition Methods
Abstract

11:00-11:25 : Florence Hubert
Optimized Schwarz Algorithms for Anisotropic Elliptic Operators in the Framework of DDFV Schemes
Abstract

11:25-11:50 : Oliver Sander
The 2-Lagrange-Multiplier Method for the Richards Equation
Abstract

11:50:12:15 : Minh Binh Tran
Optimized Schwarz Methods for the Primitive Equations
Abstract

M17 Abstract

Part 2

Large-Scale Implementation of Optimized Decomposition Methods

Session Schedule Author Index Session Index M17

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Abstract

Domain decomposition methods are used to solve large elliptic problems in parallel. The 2-Lagrange multiplier method is a domain decomposition method which uses optimized Robin boundary conditions on the artificial interface and which is related to the optimized Schwarz method. In this talk, we will describe a massively parallel implementation of the 2-Lagrange multiplier method which runs on the Hector supercomputer.

Optimized Schwarz Algorithms for Anisotropic Elliptic Operators in the Framework of DDFV Schemes

Session Schedule Author Index Session Index M17

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Abstract

Classical and optimized Schwarz algorithms have been developped for anisotropic elliptic problems using Discrete Duality Finite Volume techniques (DDFV) over the last five years. Like for Discontinuous Galerkin method (DG), it is not a priori clear how to appropriately discretize transmission conditions in DDFV, and numerical experiments have shown that very different scalings both for the optimized parameters and the contraction rates of the algorithms can be obtained, depending on the discretization. We explain in this presentation how the DDFV discretization can influence the performance of Schwarz algorithms, and also propose a new DDFV discretization technique of interface conditions which leads to the expected convergence rate of the Schwarz algorithms obtained from an analysis at the continuous level.

The 2-Lagrange-Multiplier Method for the Richards Equation

Session Schedule Author Index Session Index M17

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Abstract

The 2-Lagrange-multiplier method was originally introduced to solve linear elliptic equations on domains with a nonoverlapping partition with cross-points. It works by rewriting the problem as a set of local Robin boundary problems, and iterating on the Robin boundary data. In this talk we generalize the method to the Richards equation for unsaturated porous media flow. As in the linear case, local Robin problems have to be solved. By applying the Kirchhoff transformation to each subdomain problem we obtain a set of strictly convex minimization problems, for which a fast and robust multigrid solver is available. This works even if the permeability and saturation functions are different on each subdomain. Therefore, the 2-Lagrange-multiplier method extends a previous approach of Berninger, Kornhuber, and Sander for layered soils to general decompositions with cross-points.

Optimized Schwarz Methods for the Primitive Equations

Session Schedule Author Index Session Index M17

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Abstract

The primitive equations are equations, describing large scale dynamics of oceans and atmosphere. These equation are derived from the Navier-Stokes equations, with rotation, coupled to thermodynamics and salinity diffusion-transport equations, which account for the buoyancy forces and stratification effects under the Boussinesq approximation. In this talk, we will give numerical and theoretical results for Schwarz domain decomposition methods with Robin transmission conditions for this equation.

M17P2 Schedule

Domain Decomposition Methods based on Robin Conditions for Large and / or Nonlinear Problems

Schedule   Author Index   Session Index

Date: Friday, June 29
Time: 10:30-12:15
Location: Amphi
Chairman: Heiko Berninger, Sébastien Loisel, Oliver Sander

10:35-11:00 : Soheil Hajian
Discontinuous Galerkin, Block Jacobi and Schwarz Methods
Abstract

11:00-11:25 : Ronald Haynes
An Optimized Schwarz Method for the Generation of Equidistributed Grids
Abstract

11:25-11:50 : Joel Phillips
Schwarz Methods for Plane Wave Discontinuous Galerkin Methods
Abstract

11:50:12:15 : Yingxiang Xu
The Influence of Interface Curvature on Transmission Conditions in Domain Decomposition Methods
Abstract

M17 Abstract

Part 1

Discontinuous Galerkin, Block Jacobi and Schwarz Methods

Session Schedule Author Index Session Index M17

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Abstract

For classical discretizations of elliptic partial differential equations, like conforming finite element methods (FEM) or finite difference methods (FDM), block Jacobi iterations are equivalent to classical Schwarz iterations with Dirichlet transmission conditions. This is however not necessarily the case for discontinuous Galerkin finite element methods (DGFEM). We will show in this talk for the model problem $(\eta - \Delta)u = f$ and various DGFEM discretizations that a block Jacobi method applied to the discretized problem can be interpreted as a Schwarz method with different transmission conditions from the classical Dirichlet ones. We illustrate our results with numerical experiments.

An Optimized Schwarz Method for the Generation of Equidistributed Grids

Session Schedule Author Index Session Index M17

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Abstract

Adaptively choosing an underlying grid for computation has proven to be a useful, if not essential, tool for the solution of boundary value problems and partial differential equations. One way of generating adaptive meshes is through the so-called equidistribution principle (EP). In one spatial dimension the required mesh can be obtained through the solution of a nonlinear BVP. In this talk I will review the idea of EP and consider the solution of the resulting BVP via an optimized Schwarz iteration. Recent work on the generation of 2D locally equidistributed grids will be presented.

Schwarz Methods for Plane Wave Discontinuous Galerkin Methods

Session Schedule Author Index Session Index M17

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Abstract

The Ultra-Weak Variational Formulation (UWVF) and Plane Wave Discontinuous Galerkin (PWDG) methods use bases composed of element-wise particular solutions. For Helmholtz' problem, the weak form is

$$\int_{\partial K} \hat{u}_h \cdot \overline{\nabla v \cdot n} dS - ik \int_{\partial K} \hat{\sigma}_h \cdot n \overline{v} dS = \int_K f \overline{v} dV,$$

where the fluxes, $\hat{u}_h$ and $\hat{\sigma}_h$ are given in terms of jumps and averages. For example, on interior faces, we take

$$\hat{u}_h = \{\!\{u_h\}\!\} + \tau \cdot [\![u_h]\!] - \frac{\beta}{ik} [\![\nabla_h u_h]\!].$$

These methods give accurate approximations with small numbers of degrees of freedom but also suffer from poor conditioning. In this talk, we will demonstrate how this can be improved using a Schwarz-type domain decomposition.

We will also show how a (purely algebraic) block-Jacobi relaxation of the DG formulation is equivalent to a non-overlapping Schwarz method with Robin transmission conditions of the form,

$$\frac{\partial u_1^{(n+1)}}{\partial n_1} + s u_1^{(n+1)} = \frac{\partial u_2^{(n)}}{\partial n_1} + s u_2^{(n)},$$

with $s = ik$, and how, with a little extra work, we can let s take more general values.

The Influence of Interface Curvature on Transmission Conditions in Domain Decomposition Methods

Session Schedule Author Index Session Index M17

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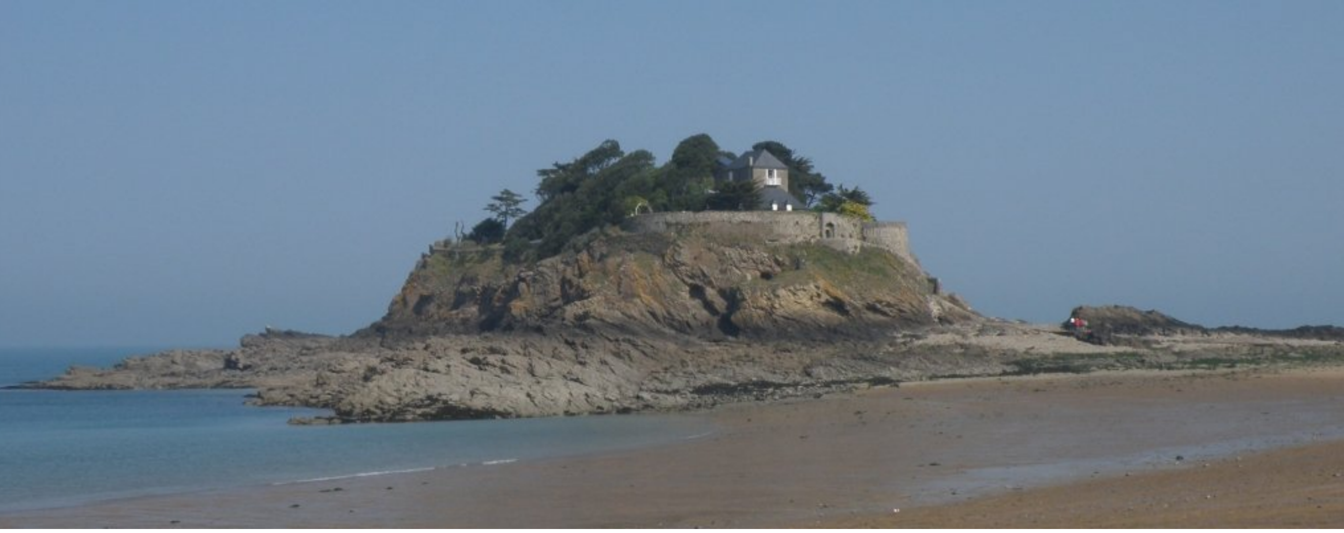
Abstract

The interface curvature, generally given by the concrete problem to be solved and domain decomposition used, may affect greatly the convergence properties for domain decomposition methods. We show in this talk for the Schwarz algorithm applied to symmetric positive definite problems several transmission conditions, derived from both micro-local analysis and optimization of the convergence factors on a model problem. We observe that many of our transmission conditions involve the interface curvature parameter, and this leads to better performance of the associated method generally. Overlap permits to accelerate the method, and the optimization-based methods achieve the best performance. Our theoretical analysis and our numerical experiments show that for getting the best possible performance, the interface curvature should definitely be taken into account.



Mini Symposium M18

Solvers for Discontinuous Galerkin Methods

Schedule Author Index Session Index

Organizers: Blanca Ayuso de Dios, Susanne C. Brenner

Abstract

Discontinuous Galerkin (DG) finite element methods were introduced in the late 1970s, and they have undergone a rapid development in recent years. For this family of numerical techniques, the finite element spaces are not subject to inter-element continuity conditions and local element spaces can be defined independently from each other. They possess many advantageous properties (local conservation; flexibility in handling irregular meshes with hanging nodes and in designing hp-refinement strategies; built-in parallelism) which have rendered them suitable for the approximation of a wide variety of problems; including elliptic, first-order hyperbolic and unsteady problems. They also provide a far-reaching framework to develop new appropriate discretization schemes for several applications where classical approaches might fail.

Despite the versatility of DG techniques, their practical use has been often limited by the much larger number of degrees of-freedom it requires compared with other classical discretization methods. For this reason, the development of efficient solvers and preconditioning strategies for the solution of the algebraic systems resulting from DG discretizations is becoming crucial. Over the last ten years, different solvers and preconditioning strategies based on domain decomposition (DD), multigrid and multilevel methods have been developed and analyzed for DG discretizations of several (still simple) problems. Their complete understanding is however still lacking.

The aim of this mini-symposium is to bring together experts in the field to discuss and identify the most relevant aspects of the current development of solution techniques for DG methods. Sample topics include the design, the theoretical analysis and issues related to the implementation and applications of the various solution techniques.

Part 1

Part 2

M18P1 Schedule

Solvers for Discontinuous Galerkin Methods

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 16:00-17:45
Location: Markov
Chairman: Blanca Ayuso de Dios, Susanne C. Brenner

16:05-16:30 : Alexandre Pieri
BDDC Preconditioners: from hp-Continuous to Discontinuous Galerkin Schemes with Different Local Polynomial Degrees
Abstract

16:30-16:55 : Kolja Brix
Robust Preconditioners for DG-Discretizations with Arbitrary Polynomial Degrees on Locally Refined Meshes
Abstract

16:55-17:20 : Christoph Lehrenfeld
DD Preconditioning for High Order Hybrid DG Methods on Tetrahedral Meshes
Abstract

17:20-17:45 : Eun-Hee Park
A BDDC Method for a Symmetric Interior Penalty Galerkin Method
Abstract

M18 Abstract

Part 2

BDDC Preconditioners: from hp-Continuous to Discontinuous Galerkin Schemes with Different Local Polynomial Degrees

Session Schedule Author Index Session Index M18

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Abstract

Spectral element methods can achieve exponential convergence rate for elliptic problems and can be associated with domain decomposition algorithms to further achieve scalable parallel solvers. The regularity of the solution may vary locally and adaptive meshes with locally varying spectral discretization can improve accuracy. Here, we first extend the classic spectral element method with uniform polynomial degrees to a spectral formulation allowing local changes in polynomial degrees (relative to a spectral element). Then, we propose an efficient preconditioner to solve the associated algebraic system, extending the so-called Balancing Domain Decomposition by Constraints (BDDC) method developed for finite and spectral elements with uniform polynomial degrees to the case with jumps in polynomial degrees between elements. We also study a Discontinuous Galerkin formulation based on the Auxiliary Space Method and we extend the BDDC preconditioner to this formulation. The results of several numerical tests in two dimensions for both continuous and discontinuous Galerkin formulations show the scalability, quasi-optimality and efficiency of the proposed method, as well as its robustness with respect to jumps in the elliptic coefficients and local spectral degrees.

Robust Preconditioners for DG-Discretizations with Arbitrary Polynomial Degrees on Locally Refined Meshes

Session Schedule Author Index Session Index M18

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Abstract

Discontinuous Galerkin (DG) methods offer enormous flexibility regarding local grid refinement and variation of polynomial degrees rendering such concepts powerful discretization tools. At the same time they have proven to be well-suited for a variety of different problem classes. While initially the main focus has been on transport problems like hyperbolic conservation laws, interest has meanwhile shifted towards diffusion problems. We therefore consider DG discretizations for elliptic boundary value problems and in particular focus on the efficient solution of the linear systems of equations that arise from the Symmetric Interior Penalty DG method. We propose preconditioners that are based on the concept of the auxiliary space method in combination with techniques from spectral element methods such as Legendre-Gauss-Lobatto grids. Under mild grading conditions on the grid refinement and the variation of the polynomial degrees, we can show the resulting condition numbers to be bounded even for locally refined grids with hanging nodes and arbitrary polynomial degrees. Special measures have to be taken in the case of varying polynomial degrees around a hanging node. We present some numerical experiments that demonstrate the efficiency of the preconditioners.

DD Preconditioning for High Order Hybrid DG Methods on Tetrahedral Meshes

Session Schedule Author Index Session Index M18

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Abstract

Discontinuous Galerkin methods are popular discretization methods in applications from fluid dynamics and many others. The concept of hybridization offers a structure to reduce the discrete system to unknowns on the element interfaces [1] which makes arising linear systems more suitable for an efficient solution. After the presentation of the method, we show for an elliptic model equation discretized on tetrahedral meshes how standard domain decomposition techniques like non-overlapping Schwarz type methods or balancing domain decomposition with constraints (BDDC) [2,3] can be easily applied to the Hybrid Discontinuous Galerkin formulation. Therefore we consider one element as a sub-domain and divide the degrees of freedom into mean values on faces and the remainder as primal and dual unknowns. We prove poly-logarithmic (in the polynomial order) condition number estimates for the preconditioned matrix. In order to show that, we have to develop technical tools, specially that an optimal extension from faces to elements with Dirichlet constraints are nearly as good as an extension without constraints.

[1] B. Cockburn, J. Gopalakrishnan, and R. Lazarov. Unified hybridization of discontinuous Galerkin, mixed, and continuous Galerkin methods for second order elliptic problems. *SIAM J. Numer. Anal.*, 47:1319-1365, 2009.

[2] C. R. Dohrmann. A preconditioner for substructuring based on constrained energy minimization. *SIAM J. Sci. Comput.*, 25(1):246-258, 2003

[3] M. Dryja and O. B. Widlund. Towards a unified theory of domain decomposition algorithms for elliptic problems. In T. F. Chan, R. Glowinski, J. Périaux, and O. B. Widlund, editors, *Third International Symposium on Domain Decomposition Methods for Partial Differential Equations*, pages 3–21, Philadelphia, 1990. SIAM.

A BDDC Preconditioner for a Symmetric Interior Penalty Galerkin Method

Session Schedule Author Index Session Index M18

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Abstract

In this talk we will discuss a nonoverlapping domain decomposition preconditioner for a symmetric interior penalty Galerkin method for the heterogeneous elliptic problem. The preconditioner is based on balancing domain decomposition by constraints (BDDC). Theoretical results on the condition number estimate of the preconditioned system will be presented along with numerical results.

M18P2 Schedule

Solvers for Discontinuous Galerkin Methods

Schedule Author Index Session Index

Date: Thursday, June 28
Time: 10:30-12:15
Location: Markov
Chairman: Blanca Ayuso de Dios, Susanne C. Brenner

10:35-11:00 : Paola F. Antonietti
Schwarz Methods for a Preconditioned WOPSIP Discretization of Elliptic Problems
Abstract

11:00-11:25 : Andrew Barker
Additive Schwarz Preconditioners for the Discontinuous Petrov-Galerkin Method
Abstract

11:25-11:50 : Guido Kanschat
Multigrid Methods for a Divergence–Conforming DG Discretization of Incompressible Flow
Abstract

11:50:12:15 : Ludmil T. Zikatanov
A Preconditioner for $H(\mathrm{div})$-Conforming DG Discretizations of Stokes Equation
Abstract

M18 Abstract

Part 1

Schwarz Methods for a Preconditioned WOPSIP Discretization of Elliptic Problems

Session Schedule Author Index Session Index M18

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Abstract

We propose and analyze several two-level non-overlapping Schwarz methods for a preconditioned weakly over-penalized symmetric interior penalty (WOPSIP) discretization of a second order boundary value problem. We show that the condition number of the resulting preconditioned linear systems of equations is of order H/h, being H and h the granularity of the coarse and fine partitions, respectively. Numerical experiments that illustrate the performance of the proposed two-level Schwarz methods are also presented.

Additive Schwarz Preconditioners for the Discontinuous Petrov-Galerkin Method

Session Schedule Author Index Session Index M18

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Abstract

The discontinuous Petrov-Galerkin method allows for use of nearly optimal test functions at a reasonable computational cost, because the test functions can be solved for locally. The resulting methods can be very effective and show good stability properties, but solution of the resulting ill-conditioned linear systems is a challenge. We explore the effectiveness of additive Schwarz preconditioning for linear systems arising from the DPG discretization, considering both their theoretical properties and their practical efficiency.

Multigrid Methods for a Divergence-Conforming DG Discretization of Incompressible Flow

Session Schedule Author Index Session Index M18

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Abstract

A multigrid method based on an overlapping domain decomposition smoother is presented. The smoother operates in the divergence free subspace and thus does not require to be embedded into a block preconditioner for the saddle point problem. Its efficiency is documented with numerical examples.

A Preconditioner for $H(\mathrm{div})$-Conforming DG Discretizations of Stokes Equation

Session Schedule Author Index Session Index M18

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Abstract

We present a preconditioner for $H(\mathrm{div})$ conforming, DG-discretizations of the Stokes equation. We focus on preconditioning the linear system resulting from the lowest order Brezzi-Douglas-Marini (BDM) elements discretization for the velocity. We solve the problem on the divergence free subspace using an auxiliary space preconditioner. The action of the preconditioner amounts to solving two scalar Laplace equations and a vector Laplace equation. We present numerical tests that show that such preconditioner is optimal and we also discuss some of the theoretical estimates on the condition number of the preconditioned system.

Mini Symposium M19

Domain Decomposition in Computational Cardiology

Schedule Author Index Session Index

Organizers: Rolf Krause and Luca Pavarino

Abstract

The numerical simulation of the mechanical and electrical activity within the human heart is a challenging task. Different effects as the propagation of the elecrical activation front, the chemical reactions within the ion channels, or the influence of the fiber orientation on the mechanical contraction of the heart have to be taken into acount. High spatial and temporal resolution is necessary in order to resolve, e.g., electrophysiological phenomena as the activation front, or mechanicaly important quantities as the micro-structure of the tissue. As a consequence, the employed simulation methods have to be adapted carefully. This includes possibly adaptive discretization methods as well as efficient parallel solution techniques for the arising large scale problems. This minisymposium is intended to provide a platform to present and to discuss newest developments in computational cardiology with a strong focus on domain decomposition and multigrid techniques.

Part 1

Part 2

M19P1 Schedule

Domain Decomposition in Computational Cardiology

Schedule Author Index Session Index

Date: Thursday, June 28
Time: 10:30-12:15
Location: Petri
Chairman: Rolf Krause and Luca Pavarino

10:35-11:00 : Luca Gerardo-Giorda
Optimized Schwarz Coupling and Model Adaptivity for Numerical Elctrocardiology
Abstract

11:00-11:25 : Dorian Krause
Scalable Solvers for Electrocardiology on Massively Parallel Architectures
Abstract

11:25-11:50 : Stefano Zampini
Exact and Inexact BDDC Methods for the Cardiac Bidomain Model
Abstract

11:50-12:15 : Charles Pierre
A Preconditioner with Almost Linear Complexity for the Bidomain Model
Abstract

M19 Abstract

Part 2

Optimized Schwarz Coupling and Model Adaptivity for Numerical Elctrocardiology

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Abstract

The Bidomain model is nowadays one of the most accurate mathematical descriptions of the action potential propagation in the heart. However, its numerical approximation is in general fairly expensive as a consequence of the mathematical features of this system. For this reason, a simplification of this model, called Monodomain problem is often adopted in order to reduce computational costs. Reliability of this model is questionable in the presence of applied currents and in the regions where the upstroke or the late recovery of the action potential is occurring, but in the absence of applied currents it provides a reasonable approximation for the action potential propagation at the heart scale. An heterogeneous approach aiming at reducing computational costs and maintaining accuracy can be devised by solving the Bidomain problem only over "critical" regions of the domain (the term"critical" being driven by physiopathological arguments), and solving the Monodomain problem in areas where the potential propagation dynamics does not require the most sophisticated model. This approach falls in the general framework of "model adaptivity".

In this talk, stemming from an intermediate model called *Hybridomain*, we will describe a model adaptive strategy: the computational domain is subdivided into regions, coupled through an Optimized Schwarz Method, in which either the Bidomain or the Monodomain problem is solved. The model choice is driven by a model error estimator following the spatio-temporal evolution of the action potential propagation.

Scalable Solvers for Electrocardiology On Massively Parallel Architectures

Session Schedule Author Index Session Index M19

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Abstract

The accurate simulation of the electrical propagation in cardiac tissue is a challenging problem as a large span of length- and timescales needs to be resolved. This is even more the case when dealing with full heart simulations on complicated geometries obtained from medical imaging data. High performance computing and – in particular – massively parallel processing are important enabling techniques for computational electrocardiology. In this talk we describe algorithms and implementation techniques for exploiting contemporary massively parallel architectures for solving mono- and bidomain reaction diffusion equations. We present the hybrid parallelization of a heart model and analyze the advantages of multithreading for this application for explicit and implicit-explicit time integration on up to 8,448 cores of a Cray XT5 system. Strong and weak scalability of several preconditioners (ranging from Block-Jacobi ILU(0) to algebraic multigrid) for the linear system arising in the parabolic-elliptic formulation of the bidomain equation are investigated. We conclude the presentation by discussing the results of a recent study of the accuracy of signals computed at low resolution using sources from high resolution propagation simulations. This application showcases the capabilities of our approach on massively parallel architectures and highlights the gains achievable by carefully optimizing the mesh resolution used at the different stages of our simulation workflow.

Exact and Inexact BDDC Methods for the Cardiac Bidomain Model

Session Schedule Author Index Session Index M19

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Abstract

Balancing Domain Decomposition by Constraints (BDDC) preconditioners are constructed and analyzed for an implicit-explicit discretization of the parabolic-parabolic formulation of the cardiac Bidomain model, discretized with low-order conforming isoparametric Q1 elements in three dimensions. Unlike more conventional non-overlapping methods, the preconditioner's construction is carried out on the whole set of dofs allowing for the use of approximate solvers on the subdomains. After having introduced the formulation of the Bidomain variational problem and preconditioner details, theoretical estimates will be provided for the upper bound of the average operator in the Bidomain Schur norm; parallel experimental results will confirm quasi-optimality and scalability of the exact BDDC method for the cardiac Bidomain model in three-dimensions. Theoretical and experimental results will also confirm robustness of the BDDC method in case of jumps in cardiac conductivity coefficients aligned with the interface of the non-overlaping partition. Finally, since memory consumption is the most important bottleneck of all non-overlapping methods in three dimensions, an inexact BDDC formulation is considered, substituting exact factorizations of local problems by the action of algebraic multigrid preconditioners: different types of smoothers will be also taken into account. Robustness and scalability of the approximate preconditioner will be theoretically proved and experimentally validated; experimental results for large simulations up to 60 million of unknowns will confirm the efficiency of the inexact approach.

A Preconditioner with Almost Linear Complexity for the Bidomain Model

Session Schedule Author Index Session Index M19

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Abstract

The bidomain model is widely used in electro-cardiology to simulate spreading of excitation in the myocardium and electrocardiograms. It consists of a system of two parabolic reaction diffusion equations coupled with an ODE system. Its discretisation displays an ill-conditioned system matrix to be inverted at each time step: simulations based on the bidomain model therefore are associated with high computational costs. In this paper we propose a preconditioning for the bidomain model in an extended framework including a coupling with the surrounding tissues (the torso). The preconditioning is based on a formulation of the discrete problem that is shown to be symmetric positive semi-definite. A block LU decomposition of the system together with a heuristic approximation (referred to as the monodomain approximation) are the key ingredients for the preconditioning definition. Numerical results are provided for two test cases: a 2D test case on a realistic slice of the thorax based on a segmented heart medical image geometry, a 3D test case involving a small cubic slab of tissue with orthotropic anisotropy. The analysis of the resulting computational cost (both in terms of CPU time and of iteration number) shows an almost linear complexity with the problem size, i.e. of type $n \log^{\alpha}(n)$ (for some constant α) which is optimal complexity for such problems.

M19P2 Schedule

Domain Decomposition in Computational Cardiology

Schedule Author Index Session Index

Date: Friday, June 29
Time: 10:30-12:15
Location: Petri
Chairman: Rolf Krause and Luca Pavarino

10:35-11:00 : Martin Weiser
Delayed Residual Compensation for Bidomain Equations
Abstract

11:00-11:25 : Gernot Plank
GPU Accelerated Strongly Scalable Simulations of Cardiac electro-mechanics
Abstract

11:25-11:50 : Ricardo Ruiz Baier
An Eulerian Finite Element Method for the Simulation of Cardiomyocyte Active Contraction
Abstract

11:50:12:15 : Maxime Sermesant
Interactive Electromechanical Model of the Heart for Patient-Specific Simulation
Abstract

M19 Abstract

Part 1

Delayed Residual Compensation for Bidomain Equations

Session Schedule Author Index Session Index M19

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Abstract

The biodomain model of cardioelectric excitation consists of a reaction-diffusion equation, an elliptic algebraic constraint, and a set of pointwise ODEs. Fast reaction enforces small time steps, such that for common mesh sizes the reaction-diffusion equation is easily solved implicitly due to a dominating mass matrix. In contrast, the elliptic constraint does not benefit from small time steps and requires a comparably expensive solution. We propose a simple delayed residual compensation that improves the solution of the elliptic constraint at virtually no computational cost and thus alleviates the need for long iteration times.

GPU Accelerated Strongly Scalable Simulations of Cardiac Electro-Mechanics

Session Schedule Author Index Session Index M19

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Abstract

Anatomically realistic and biophysically detailed multiscale computer models of the heart are playing an increasingly important role in advancing our understanding of integrated cardiac function in health and disease. Such detailed simulations, however, are computationally vastly demanding, which is a limiting factor for a wider adoption of in-silico modeling. While current trends in high performance computing (HPC) hardware promise to alleviate this problem, exploiting the potential of such architectures remains challenging since strongly scalable algorithms are necessitated. Alternatively, acceleration technologies such as graphics processing units (GPUs) are being considered. While the potential of GPUs has been demonstrated in various applications, benefits in the context of bidomain simulations where large sparse linear systems have to be solved in parallel with advanced numerical techniques, are less clear. In previous studies, using up to 16k cores we demonstrated strong scalability for solving the monodomain [1] as well as bidomain [2] equations. More recently, we implemented a proper domain decomposition algebraic multigrid bidomain solver which can be compiled for execution on both CPUs and GPUs in distributed memory environments [3]. In this study first results are presented on extending our solver framework to apply these methods to electro-mechanically coupled multi-physics simulations. Scalability results of weakly coupled electromechanical simulations will be presented and challenges that need to address with regard to GPU implementations will be discussed.

[1] S. Niederer, L. Mitchell, N. Smith, and G. Plank, “Simulating human cardiac electrophysiology on clinical time-scales.” *Front Physiol*, vol. 2, p. 14, 2011.

[2] L. Mitchell, M. Bishop, E. Hötzl, A. Neic, M. Liebmann, G. Haase, and G. Plank, “Modeling cardiac electrophysiology at the organ level in the peta flops computing age,” *AIP Conference Proceedings*, vol. 1281, no. 1, pp. 407–410, 2010. [Online]. Available: `http://link.aip.org/link/?APC/1281/407/1`

[3] Liebmann M. Hoetzl E. Mitchell L. Vigmond E.J. Haase G. Plank G. Neic, A. Accelerating cardiac bidomain simulations using graphics processing units. *IEEE Trans Biomed Eng*, 2012 (under review).

An Eulerian Finite Element Method for the Simulation of Cardiomyocyte Active Contraction

Session Schedule Author Index Session Index M19

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Abstract

In this work we are interested in the modeling of the deformations of a single cardiomyocyte surrounded by a Newtonian fluid, and their mechano-chemical interactions with the kinematics of calcium concentrations and active contraction. We choose to formulate our coupled problem in an Eulerian setting. Following [1], we track the boundary of the elastic body using a level set method with the particularity that the level set function also delivers information about the stretching of the interface. Here we incorporate several variants on the method and on the model itself. Namely, we employ a modified level set approach based on the imposition of additional constraints via Lagrange multipliers (see [2]). Moreover, we consider an active strain description of the activation mechanism which is based on the assumption of a multiplicative decomposition of the deformation gradient [3,4], and we use slightly different models for the calcium-driven mechanical activation.

[1] G.-H. Cottet, E. Maitre, T. Milcent, Eulerian formulation and level set models for incompressible fluid-structure interaction, *ESAIM Math. Model. Numer. Anal.* 2008; **42**:471–492.

[2] A. Laadhari, C. Misbah, P. Saramito, On the equilibrium equation for a generalized biological membrane energy by using a shape optimization approach, *Physica D* 2010; **239**:1567–1572.

[3] F. Nobile, A. Quarteroni, R. Ruiz-Baier, An active strain electromechanical model for cardiac tissue, *Int. J. Numer. Meth. Biomed. Engrg.*, to appear.

[4] S. Rossi, R. Ruiz-Baier, L.F. Pavarino, A. Quarteroni, An orthotropic active strain model for the numerical simulation of cardiac biomechanics, *Int. J. Numer. Meth. Biomed. Engrg.*, to appear.

Interactive Electromechanical Model of the Heart for Patient-Specific Simulation

Session Schedule Author Index Session Index M19

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Abstract

The software platform SOFA[1] is an open-source project for interactive multi-physics simulation of medical interventions. The objective is to design a modular framework so that biophysical simulations of the human body can be run in real-time and integrated in a simulator for clinical training. Specific choices have to be made at the modelling and implementation level in order to tackle that real-time constraint. Moreover methods to adapt generic models to patient data are needed to obtain realistic simulations on personalised models. I will present some of the computational methods used in SOFA and results on cardiac electrophysiology and mechanics, as well as for cardiac catheter-based interventions. Approaches used to obtain patient-specific models will also be detailed.

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[1] Simulation Open Framework Architecture: http://www.sofa-framework.org

Mini Symposium M20

Domain Decomposition and Multiscale Methods

Schedule Author Index Session Index

Organizers: Petter E. Bjørstad

Abstract

Over the past several years, there is increased interest in Multiscale Methods and the use of such methods to model multiscale phenomena. At the same time, it has also been discovered that some of these methods share important properties with Domain Decomposition. Having different motivation and purpose, these methods often focus on the interaction between a coarse and a fine level. This mini-symposium will invite participants from both communities and focus on methods where shared properties perhaps may be analyzed in a common framework.

M20 Schedule

Domain Decomposition and Multiscale Methods

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 16:00-17:45
Location: Petri
Chairman: Petter E. Bjørstad

16:05-16:30 : Talal Rahman
Alternative Coarse Spaces for Additive Schwarz Methods for Multiscale Elliptic Problems
Abstract

16:30-16:55 : Juan Galvis
Domain Decomposition and Multiscale Methods for High-contrast Elliptic Equations
Abstract

16:55-17:20 : Robert Scheichl
Weak Approximation Properties of Elliptic Projections with Functional Constraints
Abstract

17:20-17:45 : Rui Du
Two-Level Additive Schwarz Methods with Adaptive Sampling Coarse Spaces for Multiscale Problems in High Contrast Media
Abstract

Alternative Coarse Spaces for Additive Schwarz Methods for Multiscale Elliptic Problems

Session Schedule Author Index Session Index M20

Talal Rahman
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Abstract

In this talk, we will discuss alternative ways to construct the coarse problem for the multiscale problem, without +the need of introducing a coarse triangulation. Such coarse problems will have coarse spaces which are based on the partition +of unity and the energy minimizing principle, and at the same time offer certain flexibility in constructing robust methods +for generally highly varying coefficients.

Domain Decomposition and Multiscale Methods for High-contrast Elliptic Equations

Session Schedule Author Index Session Index M20

Juan Galvis
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Abstract

We explore the relations between the construction of coarse spaces in "Numerical Homogenization/MsFEM (Multiscale Finite Element Methods)" and the construction of coarse solvers in multilevel domain decomposition methods. With coarse space that use local spectral information we construct robust dd methods and robust multiscale (upscaling) approximations. We present representative numerical examples.

Weak Approximation Properties of Elliptic Projections with Functional Constraints

Session Schedule Author Index Session Index M20

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Abstract

This paper is on the construction of energy-minimizing coarse spaces that obey certain functional constraints and can thus be used, for example, to build robust coarse spaces for elliptic problems with large variations in the coefficients. In practice they are built by patching together solutions to appropriate local saddle point or eigenvalue problems. We develop an abstract framework for such constructions, akin to an abstract Bramble–Hilbert-type lemma, and then apply it in the design of coarse spaces for discretizations of PDEs with highly varying coefficients. The stability and approximation bounds of the constructed interpolant are in the weighted L2 norm and are independent of the variations in the coefficients. Such spaces can be used, for example, in two-level overlapping Schwarz algorithms for elliptic PDEs with large coefficient jumps generally not resolved by a standard coarse grid or for numerical upscaling purposes. Some numerical illustration is provided.

Two-Level Additive Schwarz Methods with Adaptive Sampling Coarse Spaces for Multiscale Problems in High Contrast Media

Session Schedule Author Index Session Index M20

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Abstract

In this talk we introduce novel coarse spaces, within the framework of two level additive Schwarz methods for elliptic problems with highly varying coefficients. The basis functions of coarse spaces are constructed by discrete a-harmonic extension of the standard piecewise linear basis functions over sampling cells that cover the high conductivity regions. We prove that the proposed preconditioners converge independent of the contrast in the coefficients. Numerical results are also presented to confirm this.

Contributed Talks C1

Contact and Mechanics Problems

Schedule Author Index Session Index

Date: Monday, June 25
Time: 10:30-12:15
Location: I51
Chairman: Christian Rey

10:35-11:00 : Jaroslav Haslinger
A Domain Decomposition Algorithm for Contact Problems with Coulomb's Friction
Abstract

11:00-11:25 : Brahim Nouiri
Multiplicative Schwarz Method for Nonlinear Quasi-Variational Inequalities and their Application in Contact Mechanics
Abstract

11:25-11:50 : Ihor I. Prokopyshyn
Parallel Domain Decomposition Methods for Multibody Contact Problems of Nonlinear Elasticity
Abstract

11:50:12:15 : Alexandros Markopoulos
Total FETI Method in Mechanics Problems
Abstract

A Domain Decomposition Algorithm for Contact Problems with Coulomb's Friction

Session Schedule Author Index Session Index C1

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Abstract

Contact problems take an important place in the computational mechanics. Many numerical procedures have been proposed in engineering literature. The discretization of such problems leads to very large and ill-conditioned systems. Domain decomposition methods represent a possible remedy how to overcome this difficulty. This contribution deals with an iterative method for numerical solving contact problems with Coulomb friction for two elastic bodies Ω_1 and Ω_2. Each iterative step of our algorithm consists of four auxiliary problems separately formulated on the individual bodies. First, the non-penetration and the friction conditions are treated, after that two auxiliary Neumann problems are used to ensure continuity of contact stresses along a common part of the boundary. There are more variants how to realize this idea. For instance, one can solve the Dirichlet problem on Ω_1 while the non-penetration and friction conditions are considered on Ω_2. Alternatively, the friction and non-penetration conditions may be decomposed between bodies Ω_1 and Ω_2, respectively. Finally, the Gauss-Seidel splitting can be used for problems on Ω_1 and Ω_2 to get smaller subproblems in terms of the normal and tangential contact stresses.

Numerical experiments indicate scalability of the variants of our algorithms for some choices of the relaxation parameter.

Multiplicative Schwarz Method for Nonlinear Quasi-Variational Inequalities and their Application in Contact Mechanics

Session Schedule Author Index Session Index C1

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Abstract

We present and analyze subspace correction method for the solution of nonlinear quasi-variational inequalities and apply these theoretical results to non smooth contact problems in nonlinear elasticity with slip-rate dependent friction. We introduce this method in a Hilbert space, prove that it is globally convergent and give error estimates. In the context of finite element discretization, where my method turns out to be one-and two-level Schwarz methods, we specify their convergence rate and its dependence on the discretization parameters and conclude that our methods converge optimally. Transferring this results to frictional contact problems, we thus can overcome the mesh dependence of some fixed-point schemas which are commonly employed for contact problems with slip-rate dependent Tresca and Coulomb friction.

Parallel Domain Decomposition Methods for Multibody Contact Problems of Nonlinear Elasticity

Session Schedule Author Index Session Index C1

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Abstract

We propose on continuous level several classes of parallel domain decomposition methods for solution of multibody contact problems of nonlinear elasticity.

We consider the contact problems of two types. The problems of the first type are the problems of frictionless unilateral contact between several nonlinear elastic bodies. The problems of the second type are the problems of contact between several nonlinear elastic bodies through intermediate nonlinear Winkler layers. We give variational formulation of the first type problems in the form of nonlinear variational inequality on the closed convex set, and of the second type problems in the form of nonlinear variational equation in Hilbert space. For the problems of the first type the penalty method is used to reduce the nonlinear variational inequality on the convex set to the nonlinear variational equation in the whole space. The existence of a unique solution of all variational problems is shown, and the strong convergence of the solution of penalty variational equation to the solution of initial variational inequality is proved.

For solution of nonlinear variational equations, which correspond to contact problems of both types, we propose such stationary and nonstationary iterative methods, which lead to the domain decomposition. In each iterative step of these methods we have to solve in a parallel way some linear variational equations in separate subdomains, which correspond to linear elasticity problems with Robin boundary conditions on possible contact areas and with some additional volume forces. We prove theorems about the strong convergence of these domain decomposition methods.

Total FETI Method in Mechanics Problems

Session Schedule Author Index Session Index C1

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Abstract

We briefly review the TFETI based domain decomposition methodology adapted to the solution of 2D and 3D multibody contact problems with Tresca and Coulomb friction. Recall that TFETI enforces the imposed Dirichlet boundary conditions (given displacements) by Lagrange multipliers, so that all the subdomains are floating and their kernels are a priori known. We present our in a sense optimal algorithms for the solution of resulting quadratic programming problems together with their powerful ingredients such as massively parallel and scalable implementation and stable generalized inverse computation. The unique feature of these algorithms is their capability to solve the class of quadratic programming problems with spectrum in a given positive interval in O(1) iterations. The theory yields the error bounds that are independent of conditioning of constraints and the results are valid even for linearly dependent equality constraints. The performance is demonstrated by the solution of difficult real world problems as analysis of the yielding clamp connection of the arc steel support used in mining industry, the roller bearing of wind generator etc.

Contributed Talks C2

Contact and Mechanics Problems

Schedule Author Index Session Index

Date: Monday, June 25
Time: 16:00-18:10
Location: I51
Chairman: Leonardo Baffico

16:05-16:30 : Daniel Choi
A Posteriori Error Estimates and Domain Decomposition Algorithm for Contact Problems
Abstract

16:30-16:55 : Vincent Visseq
Scalability Study of the NonSmooth Contact Domain Decomposition Method (NSCDD)
Abstract

16:55-17:20 : Geoffrey Desmeure
A Mixed Domain Decomposition Method for Structural Assemblies with Interface Tractions Represented in $H^{1/2}$
Abstract

17:20-17:45 : Julien Riton
A Robin Domain Decomposition Algorithm for Contact Problem with given Friction
Abstract

17:45-18:10 : Philippe Karamian
A Numerical Implementation of Homogenization Technique to Evaluate the Effective Mechanical Properties of Reinforced Polymer Composites in the Frame of Domain Decomposition
Abstract

A Posteriori Error Estimates and Domain Decomposition Algorithm for Contact Problems

Session Schedule Author Index Session Index C2

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Abstract

We consider an iterative domain decomposition method associated to a finite element method to approximate a unilateral contact problem between two elastic bodies. Each iterative step consists of a Dirichlet problem for the one body, a contact problem for the other one and two Neumann problems to coordinate contact stresses. We present a global error estimator that takes into acount as well of the error introduced by finite element analysis as the error commited by the iterative resolution of the domain decomposition algorithm. The control of these errors sources is a key point in order to introduce adaptive techniques based on errors indicators that estimate the contribution of each source of error.

Some numerical results are presented, showing the pratical efficiency of the estimator.

Scalability Study of the NonSmooth Contact Domain Decomposition Method (NSCDD)

Session Schedule Author Index Session Index C2

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Abstract

From few years a FETI-like Domain decomposition called NonSmooth Contact Domain Decomposition (NSCDD) is developed to handle simulations of large discrete systems, such as a railway ballast submitted to cyclic loading or the behavior of the Nîmes arena and Arles aqueduct subjected to seismic loading. A Sequential Multidomain Implementation of the NSCDD method is performed to analyze its optimal efficiency. Moreover, parallel implementation in 2D and 3D, on the LMGC90 platform, using MPI library is herein studied as regards of two communications scheme. Those studies highlight influences of specificities of discrete systems as: corner grains, large rigid bodies and reorganizations of the contact network.

A Mixed Domain Decomposition Method for Structural Assemblies with Interface Tractions Represented in $H^{1/2}$

Session Schedule Author Index Session Index C2

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Abstract

Mechanical industries' increasing need of liability in numerical simulations leads to evermore fine and complex models. They must take into account complicated physical behaviours among which non-linarities and multiple physical scales are recurrent. With the aim of modelling large complex structural assemblies, we propose a non-overlapping mixed domain decomposition method based on a LaTIn-type iterative solver. It relies on splitting the studied domain into substructures and interfaces, both being able to bear mechanical behaviours so that interface behaviours like perfect cohesion, contact or delamination can be modelled. The associated Uzawa solver enables to treat at small scales nonlinear phenomena. The method, thus, can be easily parallelized and scalabilty is then ensured by a coarse problem. One main difficulty of the method is that its efficiency strongly depends on parameters, like search directions, varying with the geometry and materials of the structure. Its effectiveness also relies on the choice of appropriate mechanical fields representation and discretization , among which interface tractions. The method presented uses the Riesz representation theorem to represent interface tractions in $H^{1/2}$ so that it is possible to discretize them accordingly to the displacements. This allows to achieve an independance of convergence from mesh. The presentation will also evidence that high precision can be reached in few iterations. It will assess the method for perfect and contact interfaces that are the basic behaviours found in structural assemblies.

A Robin Domain Decomposition Algorithm for Contact Problem with given Friction

Session Schedule Author Index Session Index C2

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Abstract

Development of numerical methods for the solution of contact problems is a challenging task whose difficulty lies in the non-linear conditions for non-penetration and friction. In this contribution, we propose and study a Robin domain decomposition algorithm to approximate contact problems with Tresca friction beween two bodies. Indeed this algorithm combines, in the contact zone, the Dirichlet and Neumann boundaries conditions (Robin boundary condition). The advantage consists in solving in parallel the same variational inequality in each body. By numerical experiments, we illustrate that the algorithm is mesh independent for a suitable choice of parameters.

A Numerical Implementation of Homogenization Technique to Evaluate the Effective Mechanical Properties of Reinforced Polymer Composites in the Frame of Domain Decomposition

Session Schedule Author Index Session Index C2

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Abstract

In the frame of the domain decomposition we present an approach to evaluate the effective mechanical properties of short fibres reinforced polymer composites in 2 or 3 dimension. In the framework of double-scale homogenization method one must build a representative volume element (RVE) which must be large enough to represent the mechanical aspect of the medium. We have implemented a fully automated fast and reliable model to conceive RVEs which are big enough especially in the three-dimensional case. Despite the computer performances, these RVEs cannot be directly used to evaluate the Young and shear moduli in the frame work of the homogenization process. Due to the large number of degree of freedom the calculation of the elasticity tensor and thus the compliance tensor is out of reach. The aim of this talk is to present some numerical results obtained with the help of direct Schur complement method and parallel programming. Actually, in the 2D-case the RVE is split in 4 partitions for instance and each portion is individually treated by given cpu. The main difficulties stay first in the mesh generation and second in the treatment of periodic boundary conditions for which we present a natural way to treat them even though the method has its benefit and inconvenience but affordable.

Contributed Talks C3

Optimized Schwarz Methods

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 10:30-12:15
Location: I50
Chairman: Kévin Santugini

10:35-11:00 : Florence Hubert
Optimized Schwarz Algorithms for Finite Volume Schemes
Abstract

11:00-11:25 : Lahcen Laayouni
On the Algebraic Optimized Schwarz methods (AOSM): Performances and Applications
Abstract

11:25-11:50 : Erell Jamelot
Domain Decomposition for the Neutron SP_N Equations
Abstract

11:50:12:15 : Frédéric Magoulès
A Stochastic-based Optimized Schwarz Method for the Gravimetry Equations on GPU Clusters
Abstract

Optimized Schwarz Algorithms for Finite Volume Schemes

Session Schedule Author Index Session Index C3

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Abstract

We present a finite volume discretization of Ventcell type optimized Schwarz algorithms for advection-diffusion equations. The approach includes a wide range of convection approximation. We prove convergence of the discrete algorithm with energy estimates. Numerical illustrations of the properties of the scheme and of the Schwarz algorithm will be given.

On the Algebraic Optimized Schwarz methods (AOSM): Performances and Applications

Session Schedule Author Index Session Index C3

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Abstract

Recently, the Algebraic Optimized Schwarz Methods (AOSM) have been introduced to solve differential equations. The idea of AOSM has been inspired from the well-known optimized Schwarz methods (OMS). The AOSM methods are based on the modification of the block matrices associated to the transmission conditions between sub-domains. The transmission blocks are replaced by modified blocks to improve the convergence of the corresponding methods. In the optimal case, the convergence can be achieved in two iterations. We are interested in how the algebraic optimized Schwarz methods, used as preconditioner solvers, perform in solving partial differential equations. We are also interested in their asymptotic behavior with respect to change in problems parameters. We will present different numerical simulations corresponding to different type of problems in two- and three-dimensions

Domain Decomposition for the Neutron SP_N Equations

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Abstract

Studying numerically the steady state of a nuclear core reactor is expensive, in terms of memory storage and computational time. In order to address both requirements, one can use a domain decomposition method, implemented on a parallel computer. We present here such a method for the neutron SP_N equations, which are an approximation of the transport neutron equation. This method is based on the Schwarz iterative algorithm with Robin interface conditions to handle communications (see *P.-L. Lions (1988) in: Glowinski, R., et al. Eds.*). From a computational point of view, this method is rather easy to implement. We will analyse the domain decomposition from the continuous equations to their discretization, and we will detail on how to optimize its convergence (see *F. Nataf, F. Nier, (1997) in Numer. Math. 75*). Finally, we will give some numerical results in a realistic $3D$ configuration. Computations are carried out with the MINOS solver (see *A.-M. Baudron, J.-J. Lautard (2007) in Nuclear Science and Engineering 155*), which is a multigroup SP_N solver of the APOLLO3®[2] neutronics code. Numerical experiments show that the method is robust and efficient, and that our choice of the Robin parameters is satisfactory.

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[2] APOLLO3 is a trademark registered in France

A Stochastic-based Optimized Schwarz Method for the Gravimetry Equations on GPU Clusters

Session Schedule Author Index Session Index C3

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Abstract

Low order, sequential or non-massively parallel finite elements are generaly used for three-dimensional gravity modelling as well as popular block semi-analytical solutions. Here, in order to obtain better gravity anomaly solutions in heterogeneous media, we solve the gravimetry problem using massively parallel high order finite elements method on hybrid multi-CPU/GPU clusters. Parallel algorithms which are well suited to such architectures have to be designed, such as optimized Schwarz methods. In this paper, theoretical and numerical results are presented to define a new stochastic-based optimization procedure for the Schwarz method for the gravimetry equations. Then, we use graphical cards processors units to accelerate the solution. In hybrid architectures (CPU-GPU), each subdomain could easily be allocated to one single processor, each processor dealing with the iteration of the optimized Schwarz method, whereas at each iteration the solution of the equations on each subdomain is performed on the GPU card. Unfortunately, to obtain high speed-up, several implementation optimizations should be carrefully performed, such as data transfert between CPU and GPU, matrix data storage, etc. In this paper, we investigate, describe and present the optimizations we have developed for finite elements problems, leading to better efficiency than existing librairies (CUSP, CUSPARSE, etc) for the solution on each subdomain. Numerical experiments performed on a reallistic case of Chicxulub crater, demonstrates the robustness, efficiency and high speed-up of the proposed method and of its implementation on massive multi-CPU/GPU architecture.

Contributed Talks C4

Domain Decomposition for Helmholtz Equation

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 14:45-15:35
Location: Markov
Chairman: Ana Alonso Rodriguez

14:45-15:10 : Chris Stolk
Domain Decomposition for Helmholtz Equations with PML Boundary Conditions
Abstract

15:10-15:35 : Dalibor Lukáš
BEM–based Domain Decomposition Methods
Abstract

Domain Decomposition for Helmholtz Equations with PML Boundary Conditions

Session Schedule Author Index Session Index C4

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Abstract

We study a domain decomposition method for finite difference discretizations of the variable coefficient Helmholtz equation. On the subdomains we use a general purpose multifrontal sparse LU decomposition. In 2-D, the complexity of the multifrontal method is $O(N^{3/2})$ operations and $O(N \log N)$ storage space, where N is the number of unknowns. This is favorable, but can still lead to memory limitations for very large problems on a serial machine. In 3-D the multifrontal method requires $O(N^2)$ operations and $O(N^{4/3})$ storage space, which can be very costly for large problems. By domain decomposition these issues can be addressed. We derive a new type of boundary conditions at the boundaries between the subdomains based on a careful analysis of the constant coefficient problem. The resulting method is used as a preconditioner in the GMRES iterative solution method. We present numerical experiments for a number of different choices for the variable coefficient in the Helmholtz equation. In these experiments the preconditioned iterative method converged in only a few iterations for both small and large problem sizes.

BEM–based Domain Decomposition Methods

Session Schedule Author Index Session Index C4

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Abstract

We present two scalable domain decomposition methods that rely on boundary integral formulations.

First, we consider Galerkin boundary element method (BEM) accelerated by means of hierarchical matrices (H-matrices) and adaptive cross approximation (ACA). This leads to almost linear complexity $O(n \log n)$ of a serial code, where n denotes the number of boundary nodes or elements. Once the setup of an H-matrix is done, parallel assembling is straightforward via a load-balanced distribution of admissible (far-field) and nonadmissible (near-field) parts of the matrix to N concurrent processes. This traditional approach scales the computational complexity as $O((n \log n)/N)$. However, the boundary mesh is shared by all processes. We propose a novel method, which leads to memory scalability $O((n \log n)/\sqrt{N})$, which is optimal due to the dense matter of BEM matrices. The method relies on our recent results in cyclic decompositions of directed graphs. Each process is assigned to a subgraph and to related boundary submesh. We apply the method to 3d Helmholtz problem. The parallel scalability is documented on a distributed memory computer up to 91 cores.

In the second part of the talk, we propose a boundary element preconditioner to the primal domain decomposition method introduced by Bramble, Pasciak, and Schatz in 1986 under the name DD1. Their Schur complement system on the skeleton acts as Dirichlet-to-Neumann operator discretized by finite elements. We decompose the skeleton into faces, build the related preconditioner as a block diagonal (face-wise) BEM approximation of Neumann-to-Dirichlet operator and accelerate the diagonal blocks by H-matrices and ACA. The method is robust with respect to coefficient jumps.

Contributed Talks C5

Heterogeneous Problems and Coupling Methods

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 14:45-15:35
Location: Turing
Chairman: Eric Blayo

14:45-15:10 : Jonathan Youett
A Time Discretization for a Heterogeneous Knee Model involving Contact Problems
Abstract

15:10-15:35 : Manel Tayachi
Design of a Schwarz Coupling Method for a Dimensionally Heterogeneous Problem
Abstract

A Time Discretization for a Heterogeneous Knee Model involving Contact Problems

Session Schedule Author Index Session Index C5

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Abstract

We present a dynamical heterogeneous knee model consisting of geometrically exact elastic continua and Cosserat rods. The non-penetration condition between femur and tibia leads to a dynamical large-deformation contact problem. Due to the difference of the mechanical systems at hand we apply different time integration schemes for the rods and the continua. The continua are discretized using a contact-stabilized midpoint rule and we use the Energy–Momentum method for the Cosserat rods. The resulting coupled spatial problems are then solved using a Dirichlet–Neumann algorithm. A monotone multigrid SQP method is used as the solver for the local nonlinear contact subproblems. We investigate the energy behaviour of the time discretization and illustrate our results by numerical experiments.

Design of a Schwarz Coupling Method for a Dimensionally Heterogeneous Problem

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Abstract

When dealing with simulation of complex physical phenomena, one may have to couple several models which levels of complexity and computational cost are adapted to the local behavior of the system. In order to avoid heavy numerical simulations, one can use the most complex model only at locations where the physics makes it necessary, and the simplest ones - usually obtained after simplifications - everywhere else. Such simplifications in the models may involve a change in the geometry and the dimension of the physical domain. In that case, one deals with dimensionally heterogeneous coupling.

Our final objective is to derive an efficient coupling strategy between 1-D/2-D shallow water equations and 2-D/3-D Navier-Stokes system. As a first step in this direction and in order to identify the main questions that we will have to face, we will present in this talk a preliminary study in which we couple a 2-D Laplace equation with non symmetric boundary conditions with a corresponding 1-D Laplace equation. We will first show how to obtain the 1-D model from the 2-D one by integration along one direction, by analogy with the link between shallow water equations and the Navier-Stokes system. Then, we will focus on the design of an efficient Schwarz-like iterative coupling method. We will discuss the choice of boundary conditions at coupling interfaces. We will prove the convergence of such algorithms and give some theoretical results related to the choice of the location of the coupling interface, and the control of the error between a global 2-D reference solution and the 2-D coupled one. These theoretical results will be illustrated numerically. Finally we will present some first numerical results of a test case coupling a 3-D Navier-Stokes system with a 1-D shallow water model.

This work is performed in the context of a collaboration with EDF R&D.

Contributed Talks C6

Heterogeneous Problems and Coupling Methods

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 10:30-11:50
Location: I51
Chairman: Rolf Krause

10:35-11:00 : Marco Discacciati
Domain-Decomposition Preconditioners for the Darcy-Stokes Problem
Abstract

11:00-11:25 : Marina Vidrascu
Matched Asymptotic Expansion and Domain Decomposition for an Elastic Structure
Abstract

11:25-11:50 : Christian Engwer
Heterogeneous Coupling for Implicitly Described Domains
Abstract

Domain-Decomposition Preconditioners for the Darcy-Stokes Problem

Session Schedule Author Index Session Index C6

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Abstract

In this talk we present some preconditioning strategies based on domain decomposition theory for the finite element approximation of the coupled Darcy-Stokes problem. In particular, we consider possible interface equations associated to the Darcy-Stokes problem that can be obtained using either the classical Steklov-Poincaré approach or a new one that we call "augmented".

We compare the different linear systems arising from those methodologies and we characterize suitable preconditioners of additive or multiplicative type that require, at each iteration, to solve independently the fluid and/or the porous-media subproblems.

We provide estimates for the condition number of the preconditioned systems and we discuss the effectiveness of the different approaches taking also into account their computational cost.

Finally, we illustrate the behavior of the proposed methods on several numerical tests of physical relevance.

Matched Asymptotic Expansion and Domain Decomposition for an Elastic Structure

Session Schedule Author Index Session Index C6

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Abstract

The study of the behavior of a structure made of a material with a large number of heterogeneities using a standard finite element method is very expensive because the characteristic size of a heterogeneity is much smaller than the whole structure. An alternative approach is to use a multi-scale method such as those defined in the framework of matched asymptotic expansions. We assume that the displacement and stress fields admit two asymptotic expansions, one far from the heterogeneities (the outer one) another one close (the inner one). Basically the matched asymptotic expansions allow to replace the initial problem by a set of new ones where the layer of heterogeneities is replaced by a surface (in 3d) or a line in (2d) on which particular jumping conditions for displacement and stresses are defined. We show that the order 0 outer problem is independent of the heterogeneities. For the first order outer problem the transmission coefficients are given by several elementary inner problems posed on a representative cell. The number of such problems depends on the nature of the heterogeneities. They all have the same structure, and can be solved by domain decomposition of Neumann-Neumann or Robin-Robin type. More precisely on on each subdomain, we will look for a solution $\boldsymbol{u}^i$ which takes the following form $\boldsymbol{u}^i = \boldsymbol{w}^i + \beta_i \boldsymbol{z}^i$ with β_i two real numbers conveniently chosen. In this decomposition $\boldsymbol{z}^i$ is the solution of a standard problem and takes into account the gap in displacements while $\boldsymbol{w}^i$ is continuous on the interface but there is a gap in the stresses. When writing the interface problem for $\boldsymbol{w}^i$ using the Steklov-Poincaré operator we notice that it differs from a standard elasticity problem only in the right hand side. Numerical results will validate this approach by comparing the solution obtained using this method to the standard solution of the problem.

Heterogeneous Coupling for Implicitly Described Domains

Session Schedule Author Index Session Index C6

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Abstract

Modern imaging techniques yield high quality information of complex shaped microscopic structures. The unfitted discontinuous Galerkin method (UDG) offers an approach to solve PDEs on implicitly described sub-domains, e.g. obtained using micro-CT imaging, without the need to construct a geometry-resolving mesh. The domain description uses a level-set based formulation; still sub-domain boundaries are incorporated explicitly. While UDG allows an easy application to multi-domain problems, these techniques are not sufficient for many biological application, which involve coupling of volume and surface processes.

We discuss an extension of the UDG method to incorporate processes on manifold in a heterogeneous domain-decomposition framework. UDG constructs basis functions from a simple background mesh and restrict the support according to the actual sub-domain boundaries, i.e. the implicitly prescribed domain. Using the explicit reconstruction of the implicit sub-domain boundary it is possible to couple level-set based surface problems on the interface with sub-domain problems. First results show a model problem, coupling surface diffusion on a membrane with the surrounding volume.

Contributed Talks C7

Domain Decomposition with Preconditionners

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 16:00-17:45
Location: I51
Chairman: Damien Tromeur-Dervout

16:05-16:30 : Daniel Szyld
Additive Schwarz with variable weights is better
Abstract

16:30-16:55 : Feng-Nan Hwang
Parallel Multilevel Polynomial Jacobi-Davidson Eigensolver for Dissipative Acoustic Problems
Abstract

16:55-17:20 : Santiago Badia
On the Scalability of Balanced Domain Decomposition Preconditioners for Large Scale Computing: Galerkin-based and Efficient Coarse Corrections
Abstract

17:20-17:45 : Laurent Berenguer
Low-Rank Update of the Restricted Additive Schwarz Preconditioner for Nonlinear Systems
Abstract

Additive Schwarz with variable weights is better

Session Schedule Author Index Session Index C7

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Abstract

We develop a proposal for the simultaneous use of multiple preconditioners for GMRES (or other minimal residual methods). At each step, we apply all available preconditioners, and choose a minimal solution from the thus constructed larger subspaces. We charachterize these subspaces, and considere "truncated" versions, where specific smaller subspaces are chosen. We apply these new ideas to the case that each preconditioner corresponds to a local solve in an overlapping domain decomposition setting, considering additive and restricted additive Schwarz versions. Numerical results show the advantage of the proposed approach.

Parallel Multilevel Polynomial Jacobi-Davidson Eigensolver for Dissipative Acoustic Problems

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Abstract

Many scientific and engineering applications require accurate, fast, robust, and scalable numerical solution of large sparse algebraic polynomial eigenvalue problems (PEVPs) arising from some appropriate disretization of partial differential equations. The polynomial Jacobi-Davidson (PJD) algorithm has been numerically shown as a promising approach for the PEVPs to effectively find their interior spectrum and has gained its popularity. The PJD algorithm is a subspace method, which extracts the candidate approximate eigenpair from a search space and the space undated by embedding the solution of the correction equation at the JD iteration. In this talk, we propose the multilevel PJD algorithm for PEVPs with emphasis on the application of the dissipative acoustic cubic eigenvalue problem. The proposed multilevel PJD algorithm is based on the Schwarz framework. The initial basis for the search space is constructed on the current level by using the solution of the same eigenvalue problem, but defined on the previous coarser grid. On the other hand, a parallel efficient multilevel Schwarz preconditioner is designed for the correction equation to enhance the scalability of the PJD algorithm, which plays a crucial property in parallel computing for large-scale problem solved by using a large number of processors. Some numerical examples obtained on a parallel cluster of computers are given to demonstrate the robustness and scalability of our PJD algorithm.

On the Scalability of Balanced Domain Decomposition Preconditioners for Large Scale Computing: Galerkin-based and Efficient Coarse Corrections

Session Schedule Author Index Session Index C7

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Abstract

The coarse space correction in domain decomposition sub-structuring algorithms is basic for attaining algorithmic and weak scalability. However, this coarse correction has also a negative impact, since its assembly/solution requires global dense collectives and increases the parallelization overheads due to idling or wasted computation. Among these type of preconditioners, we have the BNN and BDDC algorithms. The BDDC preconditioner is considered superior to the BNN one since: 1) it allows to solve the coarse problem only approximately and additive preconditioners; 2) the local solvers do not require to deal with a singular matrix and sparse direct solvers can be used; 3) the coarse problem is sparser than the one of BNN and its sparsity pattern is similar to the one of the original matrix.

We rehabilitate BNN with respect to point 1) and 2) using a similar (or even simpler) approach to the one of BDDC for the imposition of the local coarse constraints, allowing only approximately balanced residuals as well as the use of efficient and robust sparse direct (local) solvers. Further, the BNN algorithm has some benefits when compared to BDDC preconditioning. The size of the BNN coarse problem in dimension 3 can be smaller than the BDDC one (using both corners and edges) and point 3) is not important when a sparse direct solver is used for the coarse correction problem. Further, the coarse correction in BNN is of Galerkin type, allowing to solve the coarse problem only once per iteration, even for its multiplicative version, with an impact on its scalability.

We analyze the effect of the coarse solver on the weak/strong scalabity of these preconditioners, as well as the effect of a multiplicative Galerkin-type correction (as

for BNN).

Low-Rank Update of the Restricted Additive Schwarz Preconditioner for Nonlinear Systems

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Abstract

The Backward differentiation formula (BDF) discretization of nonlinear PDEs requires efficient Newton methods to solve the nonlinear system $F(x) = 0$ at each time step. In particular Inexact Newton methods with restricted additive Schwarz (RAS) preconditioner [2] to approximate the inverse Jacobian has already been developed. Such technique involves the solution of local linear systems at each application of the preconditioner which can be avoided if the preconditioning matrix is computed and updated from one iteration to another. The generalized Broyden's update of the RAS preconditioner is investigated here. Quasi-Newton methods update the approximation the Jacobian $J_k \approx J(x_k)$ at the inner iteration k minimizing the change $||J_k - J_{k-1}||_F$ under the secant condition $J_k\,(x_k - x_{k-1}) = F(x_k) - F(x_{k-1})$. This update can also be applied to J_k^{-1} via the Sherman-Morison formula. Indeed, there is no more linear systems to solve at each quasi-Newton iterations, but the major drawback is the computation of $J(x_0)^{-1}$. A natural extension is to perform Newton steps, updating the preconditioner as an approximation of J^{-1} [1], taking advantage of the Newton methods convergence properties, and of the cheap low-rank updating procedure. We first propose to investigate this methodology in the domain decomposition context: updating the restricted additive Schwarz preconditioner from one Newton iteration to another. Then, a quasi-Newton algorithm is proposed, updating the preconditioned system at once. Numerical experiments will be provided on different formulations of the driven cavity problem.

[1] L. Bergamaschi, S. Bru., and A. Martinez. Low-rank update of preconditioners for the inexact Newton method with spd jacobian. *Mathematical and Computer Modelling*, 54:1863–1873, 2011.

[2] X.-C. Cai and M. Sarkis. A restricted additive Schwarz preconditioner for general sparse linear systems. *SIAM J. Sci. Comput.*, 21(2):792–797 (electronic), 1999.

Contributed Talks C8

Application to Flow Problems

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 10:30-12:15
Location: I51
Chairman: Taoufik Sassi

10:35-11:00 : Aivars Zemitis
On Domain Decomposition Based Software Tool for Flow Simulation in Containment Pools of Nuclear Reactors
Abstract

11:00-11:25 : Leonardo Baffico
A Fluid Structure Interaction Problem with Friction–Type Slip Boundary Condition
Abstract

11:25-11:50 : François Pacull
Krylov Acceleration of Schur Complement Type Iterations for Linearized CFD Systems: a Numerical Examination
Abstract

11:50:12:15 : Daniel Loghin
Interface Preconditioners for Flow Problems
Abstract

On Domain Decomposition Based Software Tool for Flow Simulation in Containment Pools of Nuclear Reactors

Session Schedule Author Index Session Index C8

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Abstract

A large number of scenario has to be simulated in connection with nuclear reactors safety. Here we deal with simulation of unsteady non-isothermal fluid flow for a class of containment pools of nuclear reactors, coupled with unsteady heat transfer in containment's walls and in various obstacle. A specialized software tool is developed, CoPool, which allows for a fast simulation, and at the same time provide the required accuracy in computing the thermal stratification in the fluid. Advantage is taken from the fact that in the general case a containment geometry can be created using primitive geometrical objects (spheres, cylinders, parallelepipeds) and Boolean operations on them. Overlapping Domain Decomposition method, namely, additive Schwarz method, is used to solve for the heat transfer in the walls. Local orthogonal coordinate systems are attached to each primitive, and a local orthogonal grid is generated in each primitive. Finite volume method is used to discretize the heat conduction equation in each primitive, resulting in necessity to solve SPD problems for each primitive. The usage of local grids for each object result in non-matching composite grid. Different approaches are used to approximate the temperature on the boundary of the primitives: e.g., interpolation, moving least squares, etc. Further on, non-overlapping Domain Decomposition is used to decouple the heat transfer in the fluid part and in the walls. Special attention is paid to energy conservation in this case. Results from tests and simulations for particular containments are presented and discussed.

A Fluid Structure Interaction Problem with Friction-Type Slip Boundary Condition

Session Schedule Author Index Session Index C8

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Abstract

We study a simple model of the stationary interaction between a rigid tube and a viscous fluid. In the original model, introduced in the late '80's, the fluid motion is governed by the Stokes equation and the displacements of the tube's section are determined using a simple algebraic equation. In the present work, instead of the classic no-slip condition for the fluid velocity on the fluid-solid interface, we suppose that the fluid can slip on this interface if the shear stress reach a threshold value. In the first part of this talk we will present the fluid-solid interaction problem and we will show an existence result based on a fixed point argument. In the second part, we will expose the numerical approximation (based on Lagrange multipliers) implemented using FreeFEM++, and we will point out some drawbacks of this mixed approach that could be corrected using a domain decomposition approach. Finally, we will address possible extensions of this model (e.g., to consider a deformable structure, unsteady case), both from theoretical (existence/uniqueness result) and numerical (domain decomposition/stabilization technics) point of view.

Krylov Acceleration of Schur Complement Type Iterations for Linearized CFD Systems: a Numerical Examination

Session Schedule Author Index Session Index C8

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Abstract

In the field of CFD optimization, efficient, robust and accurate algebraic solvers for large, sparse and real linear systems arising from the full linearization of the compressible Navier-Stokes equations, including the turbulence model, are critical, whether in the discrete direct or adjoint approaches. To this regard, parallel hybrid direct/iterative solvers with local direct solvers (e.g. multifrontal sparse LU), algebraic domain decomposition preconditioners (e.g. RAS) and iterative Krylov subspace solvers (e.g. GMRES) are often used. However, these last two types of methods are penalized by the fact that the linear systems considered are severely lacking most of the enjoyable properties such as symmetry, diagonal dominance, definite positiveness and well-conditioning, especially in the convection-dominated cases. The associated convergence rate is often being strongly diminished by the tight anisotropic coupling between neighboring sub-domains as well as the large number of artificial interfaces that need to be crossed by the information during the process, when no coarse approximation of the global solution is employed. Because the linkage between sub-domains is crucial, we investigate in the present paper the Krylov acceleration of iterations on Schur complement type systems, that is, of sequences of unknowns located at each sub-domain external interface nodes. Although this approach might not be competitive, the interface system exhibit some distinct spectral properties from the global one, which we will indirectly deduce from the GMRES convergence behavior. It is also smaller, despite the fact that the interface/interior nodes ratio is rather large in 3D with high-order discretizations. Note that the Schur complement type matrix does not have to be built explicitly: its product with a given vector can be deduced from a single Schwarz iterate with homogeneous local problems, provided that the initial guess is carefully chosen and that some constant vector terms are added. This means that the same numerical tools as for the traditional hybrid linear solvers can be used for the investigation. Finally, the indirect effect of the global partitioning on the interface system will be studied.

Interface Preconditioners for Flow Problems

Session Schedule Author Index Session Index C8

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Abstract

We describe a class of constrained interface preconditioners for the Stokes and the Oseen equations. Our approach involves constructing preconditioners based on discrete interpolation norms via a suitable Krylov approximation method. We show that the coercivity and continuity of the bilinear forms induced by the interface operator provide mesh-independent preconditioners for both the Stokes and Oseen problems. Numerical results using standard test problems are included to illustrate the procedure and verify the optimality of the proposed solver technique.

Contributed Talks C9

Multidomains and Time Domain Decomposition

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 16:00-17:45
Location: I50
Chairman: Xiao-Chuan Cai

16:05-16:30 : Chao Yang
Parallel Implicit Method for Phase-Field Problems
Abstract

16:30-16:55 : Felix Kwok
Analysis of a Predictor-Corrector Method with Many Subdomains
Abstract

16:55-17:20 : Martin Cermak
Total-FETI Domain Decomposition Method for Solving Elasto-Plastic Problem with Contact
Abstract

17:20-17:45 : Damien Tromeur-Dervout
Non Linear Boundary Conditions for Time Domain Decomposition Method
Abstract

Parallel Implicit Method for Phase-Field Problems

Session Schedule Author Index Session Index C9

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Abstract

Phase-field modeling has found numerous applications in materials science. Due to the multiscale nature, partial differential equations arising in many phase-field models are typically high-order nonlinear parabolic PDEs containing both diffusive and anti-diffusive terms and are often stiff and highly ill-conditioned. In this work, stabilized implicit schemes with an adaptive time-stepping strategy for some typical phase-field problems are investigated. We apply a Newton-Krylov-Schwarz algorithm to solve the nonlinear system of equations arising at each time step. Low-order homogeneous boundary conditions for the overlapping subdomains are imposed in the Schwarz preconditioner to achieve promising convergence results. Numerical tests on a supercomputer with thousands of processor cores are provided to show the scalability of the parallel solver.

Analysis of a Predictor-Corrector Method with Many Subdomains

Session Schedule Author Index Session Index C9

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Abstract

We analyze a predictor-corrector analogue of the Crank-Nicolson method for solving linear reaction-diffusion equations in parallel on many subdomains, which has been proposed by Rempe and Chopp in 2006. Unlike the standard time-step-and-precondition strategy or the waveform relaxation approach, it is not necessary to iterate to convergence; one advances in time simply by predicting the interface values using forward Euler, solving for the remaining variables parallel, and finally correcting the interface values with a backward Euler step. We show that the method is unconditionally stable and formally second order in time for a fixed spatial grid. However, if we refine the spatial and temporal grid simultaneously, the method is only second order when $\Delta t = O(h^\alpha)$ for $\alpha \geq 3/2$, unlike the classical Crank-Nicolson method, which requires $\alpha \geq 1$ only. We also show that the error is inversely proportional to the diameter of the subdomains.

Total-FETI Domain Decomposition Method for Solving Elasto-Plastic Problem with Contact

Session Schedule Author Index Session Index C9

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Abstract

In this paper, we consider a rate-independent elasto-plastic problem with isotropic hardening and with the Signorini contact boundary conditions. We assume the von Mises plastic criterion and the associated plastic flow rule. The time discretization of the problem is based on the implicit Euler method. The corresponding one-time-step problem is formulated in incremental form with respect to unknown displacement. We approximate the problem by the finite element method. For such a problem, we propose an "external" algorithm based on a linearization of the elasto-plastic stress-strain relation by the corresponding tangential operator. It means, that in each iteration, we solve a variational inequality of the first kind, that is similar to an elastic problem with contact. The considered algorithm has a local quadratic convergence and coincides with the semismooth Newton method if the contact boundary conditions are not considered. The arising variational inequalities can be formulated as problems of quadratic programming with equality and inequality constraints. Such problems are solved by the "internal" algorithm, which is based on the SMALSE method in combination with the Total-FETI domain decomposition method [Dostal, Z., Kozubek, T., Markopoulos, A., Brzobohaty, T., Vondrak, V., Horyl, P.: Theoretically supported scalable TFETI algorithm for the solution of multibody 3D contact problems with friction. Computer Methods in Applied Mechanics and Engineering 205-208 (1) , pp. 110-120, 2012]. The main idea of the algorithm is based on enforcing the contact boundary conditions, the Dirichlet boundary conditions and the intersubdomain continuity conditions by Lagrange multipliers. The algorithm enables a parallel implementation and has parallel scalability. The elasto-plastic problem with contact was implemented into the MatSol library in parallel Matlab environment [Kozubek, T., Markopoulos, A., Brzobohatý, T., Kučera, R., Vondrák, V. and Dostál, Z.: MatSol - MATLAB efficient solvers for problems in engineering, http://matsol.vsb.cz/]. We illustrate the performance of our algorithm on benchmarks for 2D and 3D.

Non Linear Boundary Conditions for Time Domain Decomposition Method

Session Schedule Author Index Session Index C9

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Abstract

We developed parallel time domain decomposition methods to solve systems of ODEs based on the Aitken-Schwarz (C. R. Math. Acad. Sci. Paris, 349(15-16):911-914, 2011) or primal Schur complement domain decomposition methods (Advances in Parallel Computing 19:75-82,2010). The method claims the transformation of the initial value problem in time defined on $[0, T]$ into a boundary values problem in time. To overcome the lack of knowledge of the boundary value at $t = T$, the time interval is symmetrized with respect to T in order to integrate backward the ODEs with a symmetric scheme and to come back to $t = 0$ where the value of the time derivative is known. Then classical domain decomposition methods can be applied with no overlapping time slices. A Dirichlet (respectively Neumann) boundary condition at the begin (respectively end) of the time slice are imposed for the local problem. In this talk, we will present some recent developments of the method where we show that Dirichlet-Neumann boundary conditions for non linear ODEs are not optimal. Then we will propose non linear boundary conditions that accelerate the convergence and simplify the methodology also.

Contributed Talks C10

Application to Flow Problems

Schedule Author Index Session Index

Date: Wednesday, June 27
Time: 16:00-17:45
Location: I51
Chairman: Frédéric Magoulès

16:05-16:30 : Jyri Leskinen
Distributed Shape Optimization Using the Coupling of DDM of Nonlinear Flows and GDM of Shapes on a Hybrid CPU/GPU Platform
Abstract

16:30-16:55 : Mohamed Khaled Gdoura
Domain Decomposition for Stokes Problem with Tresca Friction: Augmented Lagrangian Approach
Abstract

16:55-17:20 : Thu Huyen Dao
A Schur Complement Method for Two-Phase Flow Models
Abstract

17:20-17:45 : Guillaume Houzeaux
Schur or not Schur: not so sure
Abstract

Distributed Shape Optimization Using the Coupling of DDM of Nonlinear Flows and GDM of Shapes on a Hybrid CPU/GPU Platform

Session Schedule Author Index Session Index C10

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Abstract

In this work we present the latest results of a new distributed one-shot method using DDM/GDM techniques with Nash algorithms. In inverse shape design problems, algorithmic convergence can be improved significantly by splitting the design vector into subvectors and computing efficiently the Nash equilibrium solution of the associated multi-objective problem using competitive (Nash) games due to the reduction of the search space. With DDM techniques reconstruction of nonlinear flows can also be formulated as a Nash game where a traditional decomposition method is augmented with an optimizer which minimizes the deviation of the solutions at the overlapped interfaces of subdomains. Considerable speed-ups in wall-clock time can be achieved by combining these two games into one global "one shot" Nash game where the flow is reconstructed simultaneously with the geometry.

The improved efficiency and design quality are presented and discussed from the computational results of the new method on aeronautical configurations like multi element airfoils subjected to large geometric changes using a hybrid CPU/GPU platform consisting of several CPU nodes and state-of-the-art GPUs and its extension to more complex geometries and nonlinear mathematical flow modeling emphasized.

Domain Decomposition for Stokes Problem with Tresca Friction: Augmented Lagrangian Approach

Session Schedule Author Index Session Index C10

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Abstract

Development of numerical methods for the solution of Stokes system with slip boundary conditions (Tresca friction conditions) is a challenging task whose difficulty lies in the non-linear conditions. Such boundary conditions have to be taken into account in many situations arising in practice.

We propose and study a domain decomposition method for the Stokes problem with slip boundary conditions which treats the constraint of velocity at the interfaces by Lagrangian multiplier method. Based on augmented Lagrangian techniques, it first rewrites the original global minimization problem as a saddle-point problem and then solve it by an Uzawa block relaxation method involving three supplementary conditions . The numerical realization of such problems will be discussed and results of a model example will be shown.

A Schur Complement Method for Two-Phase Flow Models

Session Schedule Author Index Session Index C10

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Abstract

In this talk, we will report our recent efforts to apply a Schur complement method for nonlinear hyperbolic problems. We use the finite volume method and an implicit version of the Roe approximate Riemann solver. With the interface variable introduced in [1] in the context of single phase flows, we are able to simulate two-fluid models ([2]) with various schemes such as upwind, centered or Rusanov. Moreover, we introduce a scaling strategy to improve the condition number of both the interface system and the local systems. Numerical results for the isentropic two-fluid model and the compresible Navier-Stokes equations in various 2D and 3D configurations and various schemes show that our method is robust and efficient. The scaling strategy considerably reduces the number of GMRES iterations in both interface system and local system resolutions. Comparisons of performances with classical distributed computing with up to 512 processors are also reported.

[1] T.H. Dao, M. Ndjinga, F. Magoulès: A Schur Complement Method for the Compressible Navier-Stokes Equations *Accepted in Proceeding of 20th International Conference on Decomposition Methods.*

[2] M. Ndjinga, A. Kumbaro, F. De Vuyst, P. Laurent-Gengoux: Numerical simulation of hyperbolic two-phase flow models using a Roe-type solver *Nucl. Eng. Design*, 238(2008), 2075-2083.

Schur or not Schur: not so sure

Session Schedule Author Index Session Index C10

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Abstract

We present in this work a comparison between various algebraic solvers for solving fluid flow problems in bio-mechanics, namely the airflow in the human large airways and the brain hemodynamics, which geometries come from medical imaging. We focus on the pressure equation coming from a stabilized finite element method for the Navier-Stokes equations, which leads to a symmetric positive definite matrix. The use of complex geometries, like the ones envisaged in this work share, a common feature. They involve very elongated domains (airways, hundereds of brain arteries), which render the solution of such problems tricky. In fact, low frequency modes are barely damped by classical solvers and render the use of multigrid or deflation based solver uncircumventable for such cases. In this talk, we compare classical parallelized iterative solvers, Schur-based parallel solver and algebraic optimized Schwarz-based solver for the efficient solution of this problem. The solvers considered are tested on a supercomputer, using up to 1024 CPU'S, for large meshes involving more than 100 million of elements. The comparison is carried out using the following criteria: CPU time, speedup, efficiency and convergence rate.

Contributed Talks C11

Time Parallel - Parareal Methods

Schedule Author Index Session Index

Date: Thursday, June 28
Time: 10:30-11:50
Location: I50
Chairman: Michael Minion

10:35-11:00 : Noha Makhoul-Karam
Ratio-Based Parallel Time Integration
Abstract

11:00-11:25 : Daniel Ruprecht
Hybrid Space-Time Parallel Solution of Burger's Equation
Abstract

11:25-11:50 : Rolf Krause
A Massively Space-Time Parallel N-Body Solver
Abstract

Ratio-Based Parallel Time Integration

Session Schedule Author Index Session Index C11

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Abstract

Because time-integration of time-dependent problems is inherently sequential, time parallelism aims mainly at reducing the computational time of some real-time evolutionary problems and may be done through predictor-corrector schemes.

In a previous paper, we have presented a new approach for solving some of such problems in a time-parallel way that uses an end-of-slice condition, together with a rescaling methodology, and automatically decomposes the time-domain. This approach is possible when the end-of-slice values of the solution exhibit a ratio-property and it results in a Ratio-based Parallel Time Integration (RaPTI) algorithm that has been successfully tested on some particular bounded problems. It differs from other time-domain decomposition techniques by the fact that the time-slices are not predefined, they are rather deduced gradually (together with the solution) as a measure of the progress, within a local time, of the prediction-correction procedure.

We now apply the rescaling method onto initial value problems having an explosive solution, in infinite time. We show how a relevant choice of the end-of-slice condition and the time-rescaling factor might lead to rescaled systems having an "asymptotic similarity" (i.e. uniform convergence to a limit problem). When exploited by RaPTI algorithm, such asymptotic similarity provides much better predictions and enhances the relevance of RaPTI that consists mainly of (i) the little sequential computations it involves (predictions and corrections are done in parallel), (ii) the relatively low communication cost it induces and (iii) the similarity of the computation on all slices yielding similar computational times on all processors. Hence, significant speed-ups are achieved. This is illustrated on two problems: a non-linear diffusion-reaction problem having an explosive solution, and a membrane problem having an oscillatory and explosive solution.

Keywords: Initial value problems, rescaling, end-of-slice condition, uniform convergence, parallel time-integration.

Hybrid Space-Time Parallel Solution of Burger's Equation

Session Schedule Author Index Session Index C11

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Abstract

We present a hybrid shared-memory/distributed-memory implementation of Parareal [1] combined with spatial domain decomposition and investigate its performance for the two-dimensional Burger's equation using up to 128 cores. Time-parallel methods, like Parareal, provide an additional direction for parallelization beyond spatial parallelization by decomposing the computational domain [2]. Space-time parallel schemes combining Parareal or other time-parallel methods with spatial parallelization can potentially help to utilize the anticipated massive numbers of cores available in upcoming high-performance computing systems.

In our approach, Parareal is implemented using shared-memory while the spatial domain decomposition uses MPI. This eliminates the need to communicate full volume data in the iteration of Parareal. We demonstrate that the hybrid [3] space-time parallel implementation can provide speedup beyond the saturation of the spatial parallelization. Details on the organization of the MPI communication of ghost-cells for the multi-threaded code are presented and the overhead resulting from the need to exchange data for multiple points in time simultaneously is addressed.

[1] J. Lions and Y. Maday and G. Turinici, *A "parareal" in time discretization of PDE's*, C. R. Acad. Sci. – Ser. I – Math. **332**, 661–668, 2001.

[2] Y. Maday and G. Turinici, *The parareal in time iterative solver: A further direction to parallel implementation*, LNCSE **40**, 441–448, 2005.

[3] R. Rabenseifner et al., *Hybrid MPI/OpenMP Parallel Programming on Clusters of Multi-core SMP Nodes*, 17th Eurom. Int. Conf. on Parallel, Distributed and Network-based processing, 427–436, 2009.

A Massively Space-Time Parallel N-Body Solver

Session Schedule Author Index Session Index C11

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Abstract

We present a novel space-time parallel version of the hybrid Barnes-Hut tree code PEPC that uses the *Parallel Full Approximation Scheme in Space and Time* (PFASST) algorithm to perform parallelized time integration. For small problem sizes strong scaling of the spatial parallelization within PEPC naturally becomes saturated. To further reduce time-to-solution, an additional direction of parallelism is needed. For this purpose, PFASST uses an iterative ODE method based on deferred correction sweeps within a parareal-like iterative time-parallel strategy. These sweeps are combined with multiple space-time discretization levels to reduce the computational cost per iteration, hence relaxing the strict bound on parallel effciency of parareal. We illustrate the potential of space-time parallel algorithms to extend the intrinsic strong scaling limit of purely spatial parallelism, which is an indispensable requirement to efficiently use the fast-growing number of cores in upcoming HPC architectures.

Contributed Talks C12

Optimization Methods/Probabilistic Methods

Schedule Author Index Session Index

Date: Friday, June 29
Time: 10:30-12:15
Location: I51
Chairman: Hui Zhang

10:35-11:00 : Andreas Langer
Domain Decomposition Methods for a Class of Non-smooth Convex Variational Problems
Abstract

11:00-11:25 : Firmim Andzembe Okoubi
Domain Decomposition with Nesterov's Minimization Method
Abstract

11:25-11:50 : Francisco Bernal
A Meshfree Scheme for PDEs on Large Domains using Probabilistic Domain Decomposition
Abstract

11:50:12:15 : Samia Riaz
A Domain Decomposition Method for Elliptic Variational Inequalities
Abstract

Domain Decomposition Methods for a Class of Non-smooth Convex Variational Problems

Session Schedule Author Index Session Index C12

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Abstract

In the last decades, in the literature, there have been introduced many different approaches and algorithms for minimizing non-smooth convex variational problems. These standard techniques are iterative-sequentially formulated and therefore not able to solve large scale simulations in acceptable computational time. For such large problems we need to address methods that allow us to reduce the problem to a finite sequence of subproblems of a more manageable size, perhaps computed by one of the standard techniques. With this aim, we introduce domain decomposition methods for non-smooth convex variational problems. A prominent example of such problems is the minimization of the total variation. The main idea of domain decomposition is to split the space of the initial problem into several smaller subspaces. By restricting the function to be minimized to the subspaces, a sequence of local problems, which may be solved easier and faster than the original problem, is constituted. Then the solution of the initial problem is obtained via the solutions of the local subproblems by gluing them together. In the case of domain decomposition for non-smooth and non-additive problems the crucial difficulty is the correct treatment of the interfaces of the domain decomposition patches. Due to the non-smoothness and non-additivity, one encounters additional difficulties in showing convergence of more general subspace correction strategies to global minimizers. Nevertheless, we are able to propose an implementation of overlapping and non-overlapping domain decomposition algorithms for non-smooth, non-additive, and convex functionals with the guarantee of convergence to a minimizer of the original functional and the monotonic decay of the energy. We provide several numerical experiments, showing the successful application of the algorithms.

Domain Decomposition with Nesterov's Minimization Method

Session Schedule Author Index Session Index C12

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Abstract

A nonoverlapping domain decomposition with N subdomains can be formulated as the following constrained minimization problem

$$\min \sum_{i=1}^{N} J_i(u_i) \tag{5}$$

$$[u_{ij}] = 0, \tag{6}$$

where

- J_i are convex functions,
- $[u_{ij}] = (u_i - u_j)_{|\Gamma_{ij}}$, the solution gap across the interface $\Gamma_{ij} = \partial\bar{\Omega}_i \cap \partial\bar{\Omega}_j$.

Nesterov's gradient descent method [Nesterov Y., A method of solving a convex programming problem with convergence rate of $O(1/k^2)$, *Soviet. Math. Dokl.* 27 (2), 372-376 (1983)] is a convex optimization method with convergence rate of $O(1/k^2)$, where k is the current iteration. We apply this method to solve problem (5)-(6). Since the Nesterov method is a first order method, we do not solve linear systems during the iterative process. The saving of computational time can therefore be significant.

A Meshfree Scheme for PDEs on Large Domains using Probabilistic Domain Decomposition

Session Schedule Author Index Session Index C12

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Abstract

Probabilistic Domain Decomposition (PDD) is a novel approach to the numerical solution of partial differential equations (PDEs) on a parallel computer. Like many other state-of-the-art domain decomposition methods, PDD splits the domain along internal interfaces into non-overlapping subdomains, which are handed over to a PDE solver. The hallmark of PDD is a previous step in which the solution to the PDE is interpolated along the artificial interfaces, exploiting the stochastic representation of the PDE through a Monte Carlo approach. Therefore, each of the subdomains holds a well-posed problem that can be solved independently, and hence without inter-processor communication. We show several examples on irregular geometries which use collocation meshfree methods based on radial basis functions (RBFs) as the PDE subdomain solver.

A Domain Decomposition Method for Elliptic Variational Inequalities

Session Schedule Author Index Session Index C12

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Abstract

Variational inequalities have found many applications in applied science. A partial list includes: obstacles problems, fluid flow in porous media, management science, traffic network, and financial equilibrium problems. However, solving variational inequalities remain a challenging task as they are often subject to some set of complex constraints.

Domain decomposition methods provide great flexibility to handle these type of problems. In our presentation we consider the following general variational inequality

$$\begin{cases} \mathcal{L}u & \geq f \text{ in } \Omega, \\ u & \geq \psi \text{ in } \Omega, \\ u & = 0 \text{ on } \partial\Omega, \end{cases}$$

where $\mathcal{L}$ is an elliptic operator. We will present a non-overlapping domain decomposition formulation for variational inequalities. In our formulation, the original problem is reformulated into two subproblems such that the first problem is a variational inequality in some subdomains Ω_i^i and the other is a variational equality in the complementary subdomains Ω_i^e. This new formulation will reduce the computational cost as the variational inequality is solved on a smaller region. However one of the main challenges here is to obtain the global solution of the problem, which is to be coupled together through an interface problem. We validate our approach on two dimensional obstacle test problems using quadratic programming.

Contributed Talks C13

FETI Methods

Schedule Author Index Session Index

Date: Friday, June 29
Time: 14:00-15:45
Location: Markov
Chairman: François-Xavier Roux

14:05-14:30 : Leszek Marcinkowski
A Parallel Preconditioner for FETI-DP Method for a Crouzeix-Raviart Discretization of an Elliptic Problem
Abstract

14:30-14:55 : Ange Toulougoussou
Schur Complement Methods for the Solution of the Discrete Stokes System with Continuous Pressure
Abstract

14:55-15:20 : Hui Zhang
Optimized Interface Preconditioners for the FETI Methods
Abstract

15:20-15:45 : Christian Rey
Stopping Criterion for FETI Solver based on an Evaluation of the Discretization Error
Abstract

A Parallel Preconditioner for FETI-DP Method for a Crouzeix-Raviart Discretization of an Elliptic Problem

Session Schedule Author Index Session Index C13

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Abstract

In the talk we discuss a parallel preconditioner for a FETI-DP problem arising from a nonconforming Crouzeix-Raviart discretization of a model second order problem with discontinuous coefficients. Locally in the subdomains, we introduce local triangulations which form one global triangulation. Then the nonconforming Crouzeix-Raviart finite element space is introduced.The local interior variables are eliminated in subdomains and next a Feti-dp problem is introduced. Finally we construct and analyze a parallel preconditioner for the Feti-dp problem. We show that the preconditioner is quasi-optimal, i.e. the condition number of the preconditioned problem grows polylogarithmically with respect to the sizes of the local meshes and is independent of jumps of coefficients in the subdomains.

Schur Complement Methods for the Solution of the Discrete Stokes System with Continuous Pressure

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Abstract

We consider the solution of the linear system arising from the discretization of the Stokes system by finite element methods with continuous pressure spaces. For a system of large size, it is convenient to use iterative methods that take advantage of parallel high performance computing and save memory space. FETI and BDD belong to this class of methods and have proved efficiency for a large type of problems. They were successfully extended to the Stokes system discretized with discontinuous pressure spaces. When the approximate velocity and pressure are both continuous, the interface systems of FETI and BDD become mixed problems and have a bad condition number. We introduce a new method, the hybrid Schur method, that combines FETI and BDD in such a way that the interface unknowns are physically homogeneous. We give some theoretical analysis of the condition number of the preconditioned hybrid Schur method applied to the discrete Stokes system and some numerical comparisons with variants of FETI.

Optimized Interface Preconditioners for the FETI Methods

Session Schedule Author Index Session Index C13

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Abstract

It is well-known that using the Dirichlet preconditioner in the classical FETI method leads to very nice condition number estimates. However, the Dirichlet preconditioner can be quite expensive, and it was proposed in the literature to use as a cheaper alternative the lumped preconditioner instead. In this talk, we will show new interface preconditioners that are optimized for rapid convergence, while their costs are comparable to the cost of the lumped preconditioner.

Stopping Criterion for FETI Solver based on an Evaluation of the Discretization Error

Session Schedule Author Index Session Index C13

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Abstract

For the last decades, three trends have grown and reinforced each other: the fast growth of hardware computational capacities, the requirement of finer and larger finite element models for industrial simulations and the development of efficient computational strategies amongst which non-overlapping domain decomposition (DD) methods are very popular since they have proved to be scalable in many applications. One main fallout lies on the verification of the discretized models in order to warranty the quality of numerical simulations (global or goal-oriented error estimators). In this talk, we present some of our recent works that aim at building error estimators in a non-overlapping domain decomposition framework. We focus on the construction of fully parallel global error estimator based on interface admissibility conditions. Connection with both primal (BDD) and dual (FETI) iterative domain decomposition solver is outlined. It yields a guaranteed upper bound on the error whatever the state (converged or not) of the associated iterative solver. Eventually, we introduce first works that enable to separate algebraic error and discretization error. This leads to the definition of convergence criteria of DD iterative solvers based on discretization error estimator instead of purely algebraic criteria.

Contributed Talks C14

Time Dependent PDEs and Applications

Schedule Author Index Session Index

Date: Thursday, June 28
Time: 10:30-12:15
Location: I51
Chairman: Daniel Loghin

10:35-11:00 : Petros Aristidou
A Schur Complement Method for DAE Systems in Power System Simulation
Abstract

11:00-11:25 : Rodrigue Kammogne
Domain Decomposition Methods for Reaction-Diffusion Systems
Abstract

11:25-11:50 : Frederic Plumier
Combining Full Transients and Phasor Approximation Models in Power System Time Simulation
Abstract

11:50-12:15 : David Cherel
Domain Decomposition For Stokes Equations Using Waveform Relaxation Method
Abstract

A Schur Complement Method for DAE Systems in Power System Simulation

Session Schedule Author Index Session Index C14

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Abstract

The power system networks in North America and Europe are the largest man-made interconnected systems in the world. Many power system applications rely on time consuming dynamic simulations of large-scale power systems in order to optimize the operation and ensure the reliability of the electricity network. Dynamic simulations of power systems involve the solution of a series of initial value, stiff, hybrid DAE systems over a time window. To achieve this, the time window is discretized and a new DAE system is formed and solved at each time step, with initial values taken from the previous time step solution. At each new time step, the DAE system to be solved can be different because of the discrete variables involved in the formulation (e.g. a differential equation can become algebraic and vice versa). A non-overlapping domain decomposition is proposed to speed up the solution of the DAE system using the Schur Complement Method. The special structure of the physical system helps define the domain partitioning scheme and eliminates the need for a partitioning algorithm. It allows the formulation and solution of the reduced system using sparse, direct solvers to obtain the interface variables. Afterwards, the parallel evaluation of the internal subdomain variables is possible and efficient load balancing is achieved. Numerically, the method shows no convergence degradation when compared to the integrated method, which is traditionally used for solving power system DAEs. The aspects of decomposition, solution and optimization of the algorithm for the specific problem are discussed and results from the application of the DDM on realistic power system models are presented.

Domain Decomposition Methods for Reaction-Diffusion Systems

Session Schedule Author Index Session Index C14

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Abstract

Reaction diffusion systems are an important application in the area of modern mathematical modeling. They can be found in biology, engineering, medicine, pollution effect modeling, weather prediction, finance, to name but a few. However the numerical solution for reaction-diffusion problems remains a challenge as they often arise as a couple system of nonlinear PDE, which are solved on a complex domain. Domain Decomposition Methods provide a powerful and flexible tools for the numerical solution of reaction diffusions-systems. In our presentation we consider the following generic system:

$$\begin{cases} u_t - \Delta u = & f(u) & \text{in } \Omega \\ \mathcal{B}u = & g & \text{on } \partial\Omega \end{cases}$$

where $u = (u_1, u_2)$ and $f \in \mathbb{R}^2$.

We will present a rigorous formulation of non-overlapping domain decomposition methods for reaction-diffusion systems together with the arising Steklov−Poincaré operator. Our approach is based on the well known fact that the Steklov−Poincaré operators arising in a non-overlapping **DD**−algorithm for scalar elliptic problems are coercive and continuous with respect to Sobolev norms of index 1/2. Furthermore, this key property will motivate the construction of new interface preconditioners for the Steklov−Poincaré operator, which leads to solution techniques independent of the mesh parameters. The convergence analysis for the preconditioned GMRES together with various numerical examples are also included.

Combining Full Transients and Phasor Approximation Models in Power System Time Simulation

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Abstract

Power system time simulations are generally classified into two main categories, depending on the time scale of the dynamic phenomena under study. The slowest time-scale is the so-called Phasor Approximation (PA). It focuses on the time evolution of the magnitude and phase angle of the various phasors. PA is simulated using single-phase representation of the three-phase currents and voltages in the network. The other two phases are assumed to operate at the same frequency, to have the same amplitude but mutual phase difference of $\pm 120^o$. In steady-state condition, the above assumptions are almost perfectly met, but in transient condition it may not be the case at all. The faster time-scale, Full Transients (FT), relies on three-phase waveform representation of the system currents and voltages, without requiring further assumptions on amplitude, frequency or phase. This leads to a more precise simulation, although more time consuming. Generally the power system is simulated as a whole making use of one or the other modelling and simulation methods. However, in some cases one might take advantage of a domain decomposition into two subdomains, i.e. an FT subdomain and a PA subdomain. This is the case for instance when evaluating the effects of large disturbances in the system, e.g. short-circuits. Some disturbances cannot be simulated in PA programs and should then be modeled in FT, while some others can be modeled using both methods but would give a higher level of detail in FT. This domain decomposition method is applied to a simple but representative model of a small power system. Overlapping and non-overlapping approaches are thoroughly analysed.

Domain Decomposition For Stokes Equations Using Waveform Relaxation Method

Session Schedule Author Index Session Index C14

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Abstract

The overall objective of this work is to construct a coupling algorithm between the Navier-Stokes equations and the Primitive Equations, which are commonly used for describing the ocean circulation in the open sea and are derived from the Navier-Stokes system under Boussinesq and hydrostatic approximations. Such a coupling allows for the description of the ocean dynamics from the coast to the open ocean. Two intermediate steps are first dealt with: a domain decomposition method for the Navier-Stokes equations, and then a coupling algorithm between two Navier-Stokes equations with different aspect ratios. We shall present herein the first step of this work, namely a domain decomposition method for the Navier-Stokes equations. After a time discretization of the Navier-Stokes equations, we construct a Schwarz waveform relaxation algorithm. This algorithm uses Dirichlet-to-Neumann relaxed operator on the interface between subdomains. Using a perfect transparent operator would allow the Schwarz algorithm to converge in only two iterations. However, since such an operator is nonlocal in space, it must be approximated for actual applications. Different approximations of this operator are derived, which general forms correspond to different orders of a Taylor expansion with respect to a small parameter, and which degrees of freedom are chosen in order to optimize the convergence rate of the algorithm.
These approximations are numerically implemented in the well known test case of the lid driven cavity. It is shown first that a projection method, widely used for solving the Navier-Stokes equations, is inappropriate to treat complex interface conditions properly. Then numerical results are given using a code solving simultaneously for the velocity and the pressure fields.

Contributed Talks C15

Multigrid Methods

Schedule Author Index Session Index

Date: Friday, June 29
Time: 14:00-15:45
Location: Amphi
Chairman: James Brannick

14:05-14:30 : Kab Seok Kang
A Parallel Multigrid Solver on a Structured Triangulation of a Hexagonal Domain
Abstract

14:30-14:55 : Pawan Kumar
Parallel Aggregation Based Algebraic Multigrid
Abstract

14:55-15:20 : Lori Badea
Multigrid Method with Constraint Level Decomposition for Variational Inequalities with Contraction Operators
Abstract

A Parallel Multigrid Solver on a Structured Triangulation of a Hexagonal Domain

Session Schedule Author Index Session Index C15

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Abstract

Fast elliptic solvers are a key ingredient of massively parallel Particle-in-Cell (PIC) simulation codes for fusion plasmas. This applies for both, the gyrokinetic and fully kinetic models. The most efficient solver for large elliptic problems is the multigrid method, especially the geometric multigrid method which requires detailed information of the geometry for its discretization. In our particular case, we consider a parallel geometric multigrid solver for a structured triangulation of the hexagonal domain for an elliptic partial differential equation. Special care has been taken in optimizing also the parallel performance by making use of the available geometric information. The scaling properties of the multigrid solver on massively parallel computers as e.g. HPC-FF and IFERC-CSC are investigated. In addition, the performance results are compared with the results of solvers from public available libraries and our own implementation of the domain decomposition method.

Parallel Aggregation Based Algebraic Multigrid

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Abstract

Domain decomposition methods (DDM) are among the most popular methods for solving large sparse linear systems of the form $Ax = b$ on distributed memory architectures. The method is robust when the solution at the interface are approximated well, or when a coarse grid solve is used, both leading to exchange of information across the domains which is often essential for convergence.

In this talk, we present a parallel aggregation based algebraic multigrid method. Here the graph corresponding to the matrix is partitioned using a $k-way$ partitioning scheme. The smooother is the Gauss-Siedel method that ignores the non-local connections. The coarsening is done by aggregating the nodes locally based on strength of connection and ignoring the nodes that satisfy a diagonal dominance criterium.

The method has true black-box nature in the sense that the underlying problem may arise from a non-PDE problem and only input required from the user is the coefficient matrix A and the right hand side vector b.

The scalability and robustness of the method is studied on a wide range of problems including convection diffusion, and problems from the Florida matrix market collection. The method is also compared with the existing state of the art methods on a cluster.

Multigrid Method with Constraint Level Decomposition for Variational Inequalities with Contraction Operators

Session Schedule Author Index Session Index C15

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Abstract

We introduce a multigrid method for the variational inequalities with contraction operators, where the closed convex set is decomposed as a sum of closed convex level subsets. The method is described as multigrid V-cycles, but the results hold for other iteration types, the W-cycle iterations, for instance. We first introduce the method as a subspace correction algorithm in a general reflexive Banach space. Under an assumption on the level decomposition of the closed convex set of the problem, we prove the convergence of the algorithm and estimate the global convergence rate as a functions of the number of levels. In finite element spaces, the algorithm becomes a multigrid method for one-obstacle problems. We prove that the assumption we made holds, and compare the obtained convergence rate with that of other multigrid methods in the literature. In particular, the proposed algorithm supplies a multigrid method for the inequalities which do not arise from the minimization of a functional.

Contributed Talks C16

Finite Element Method for Domain Decomposition

Schedule Author Index Session Index

Date: Friday, June 29
Time: 14:00-15:45
Location: Petri
Chairman: Marina Vidrascu

14:05-14:30 : Patrick Le Tallec
Multidomain Calculations with Embedded Interfaces
Abstract

14:30-14:55 : Thomas Dickopf
Evaluating Local Approximations of the L^2-Orthogonal Projection Between Non-nested Finite Element Spaces
Abstract

14:55-15:20 : Debasish Pradhan
A Robin-Type Non-Overlapping Domain Decomposition Procedure for Second Order Parabolic Problems
Abstract

15:20-15:45 : Frédéric Magoulès
A Bacteria Evolution-based Partitioning Scheme for the Parallel Solution of Problems in Computational Mechanics
Abstract

Multidomain Calculations with Embedded Interfaces

Session Schedule Author Index Session Index C16

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Abstract

Many simulations in fluid structure interactions or in dynamic impact problems involve independent components interacting through complex interfaces. For such problems, standard strategies use a piecewise ALE approach with one finite element mesh per component and adequate matching strategies. These are are difficult to use in presence of large mesh distortions or when facing topology changes. An alternative is to use a Eulerian strategy describing the different substructures on a single grid using a single average velocity field, and developing ad hoc constitutive laws to handle the multiphase microstructure of each element. These models are usually quite crude on the interface physics. In this context, there is a renewed interest in models which will use a single global smooth mesh in the background, not matching the individual components, and independent finite element velocity fields for each subpart. The talk will discuss and validate such an approximation strategy. The first ingredient is a discontinuous interface reconstruction strategy based on volumic fraction gradients. The velocity fields are defined on the background mesh, meaning that a single node possibly carries two independent velocity unknowns. A different finite element formulation is then introduced for each subdomain by restricting the subdomain integration to the parts of the background finite elements which are inside the considered subdomain. The interface kinematic continuity constraint is imposed in average inside each finite element, which introduces an interface pressure unknown per element. In order to obtain a stable system, one must then enrich the finite element velocities next to the interface, which is done as in Dolbow by introducing bubble velocity shape functions on all elements with interface constraints. A specific treatment must be developed for inertia terms, and the convection terms are dealt with by projection of the convected quantities to the background mesh using exact mesh intersection.

Evaluating Local Approximations of the L^2-Orthogonal Projection Between Non-nested Finite Element Spaces

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Abstract

We present quantitative studies of transfer operators between finite element spaces associated with unrelated meshes. Several local approximations of the global L^2-orthogonal projection are reviewed and evaluated computationally. The numerical studies in 3D provide the first estimates of the quantitative differences between a range of transfer operators between non-nested finite element spaces. We consider the standard finite element interpolation, Clément's quasi-interpolation with different local polynomial degrees, the global L^2-orthogonal projection, a local L^2-quasi-projection via a discrete inner product, and a pseudo-L^2-projection defined by a Petrov–Galerkin variational equation with a discontinuous test space. Understanding their qualitative and quantitative behaviors in this computational way is interesting per se; it could also be relevant in the context of discretization and solution techniques which make use of different non-nested meshes. It turns out that the pseudo-L^2-projection approximates the actual L^2-orthogonal projection best. The obtained results seem to be largely independent of the underlying computational domain; this is demonstrated by four examples (ball, cylinder, half torus and Stanford Bunny).

A Robin-Type Non-Overlapping Domain Decomposition Procedure for Second Order Parabolic Problems

Session Schedule Author Index Session Index C16

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Abstract

This article deals with the analysis of an iterative non overlapping domain decomposition (DD) method for parabolic problems, using Robin-type boundary condition on the inter-subdomain boundaries, which can be solved in parallel with local communications. The proposed iterative method allows us to relax the continuity condition for Lagrange multipliers on the inter-subdomain boundaries. In order to derive the corresponding discrete problem, we apply a non-conforming Galerkin method using the lowest order Crouzeix-Raviart elements. The convergence of the iterative scheme is obtained by proving that the spectral radius of the matrix associated with the fixed point iterations is less than 1. For $\Delta t = O(h^2)$, we derive the upper bound of the rate of convergence which is of order 1 - $O(h^{1/2}H^{-1/2})$, where h is the finite element mesh parameter, H is the maximum diameter of the subdomains and Δt is the time step. The numerical experiments confirm the theoretical results established in this paper.

A Bacteria Evolution-based Partitioning Scheme for the Parallel Solution of Problems in Computational Mechanics

Session Schedule Author Index Session Index C16

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Abstract

For parallel execution a finite element mesh must be divided, in other words partitioned, so the parts can be distribute among the different processors or computers. The primary goal of mesh partitioning is to minimize communication time while maintaining load balance. In an explicit method, communication is associated with the nodes that lie on the boundaries between subdomains and are shared by more than one processor. Communication time depends on both the message sizes, which increase with the number of shared nodes, and the number of messages, which increases with the number of adjacent subdomains. In an implicit method, such as optimized Schwarz domain decomposition methods, additional quality criteria of the partitioning, strongly impact the convergence of the iterative algorithm: aspect ratio of the partitions, geometrical shape of the interface, etc. Unfortunately, some of these criteria are not implemented in standard libraries, such as METIS, SCOTCH, JOSTLE, Chaco, etc. In this paper, we propose an original partitioning procedure based on bacteria evolution, we have developed for domain decomposition methods. Bacteria propagate or reproduce most commonly by a kind of cell division called binary fission. In binary fission, a single cell divides and two identical cells are formed following given specific rules of the species and of the environments. Cohabitation of different bacteria species lie to an equilibrium of the global system. Inspired from this idea, and defining new bacteria evolution laws, we design a mesh partitioning methods which leads to a mesh partitioner with desirable properties, i.e. , producing subdomains of nearly equal size, and with as few nodes shared between processors as is reasonably possible, and leading to excellent convergence properties of the optimized Schwarz algorithm. This original algorithm is presented together with several applications on computational mechanics problems which outlines the robustness, efficiency and quality of the partition, compared with existing librairies.

Contributed Talks C17

Non-matching Grids/Nonconforming Discretization

Schedule Author Index Session Index

Date: Friday, June 29
Time: 14:00-15:45
Location: Turing
Chairman: Guillaume Houzeaux

14:05-14:30 : Kirill Pichon Gostaf
Finite Element Analysis of Multi-Component Assemblies: CAD - based Domain Decomposition
Abstract

14:30-14:55 : Ajit Patel
Mortar Finite Element Methods for Hyperbolic Problems
Abstract

14:55-15:20 : Eliseo Chacón Vera
Stabilization of a FETI-DP Mortar Method for the Stokes Problem
Abstract

15:20-15:45 : Beatriz Eguzkitza
A Chimera Method applied to Computational Solid Dynamics
Abstract

Finite Element Analysis of Multi-Component Assemblies: CAD - based Domain Decomposition

Session Schedule Author Index Session Index C17

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Abstract

Computer Aided Design (CAD) and Finite Element modeling are standards in a concept to manufacture industrial chain. Realistic simulations require not only huge computational resources, but may last several days or even years. We propose to apply the domain decomposition methodology (DDM) to carry out numerical simulations of multi-component CAD assemblies. The novelty of our research is the CAD - based domain decomposition. We consider design parts as independent subdomains, and we use assembly topology to define regions, where the interface boundary conditions should be applied. There are two key motivations for employing this strategy: an attempt to regularize the mathematical models and their parallel computations, and to facilitate finite element management of the essential CAD data. Obviously, our principal objective is to adapt domain decomposition methods and to develop consistent numerical schemes, which are inherently parallel and, therefore, are perfectly suited for high-performance computing (HPC). In order to enforce continuity of the global domain decomposition solution, the interface conditions have to be settled on the boundaries between all adjacent subdomains. These conditions are imposed iteratively and may vary in time. We examine the convergence rate of the global numerical solution, which is extremely sensitive and strongly depends on these interface conditions, especially in case of either non-matching or geometrically discontinuous discretizations. The Dirichlet-Neumann, the Neumann-Neumann and the FETI methods, as well as their mortar based extensions are studied in this work. We detail the proposed CAD - based domain decomposition strategy with numerical experiments, and we focus attention on the essence of a parallel implementation.

Mortar Finite Element Methods for Hyperbolic Problems

Session Schedule Author Index Session Index C17

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Abstract

In this article, the mortar finite element method is used for spatial discretization and a finite difference scheme is used for time discretization of a class of hyperbolic problems. Optimal error estimates in L^2- and H^1-norms for both semidiscrete and fully discrete schemes are discussed. The results of numerical experiments support the theoretical results obtained.

Stabilization of a FETI-DP Mortar Method for the Stokes Problem

Session Schedule Author Index Session Index C17

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Abstract

We couple a non standard FETI-DP Mortar with stabilization to the discretization of the Stokes equations. The flux term across interfaces is computed via the Riesz representation and this gives freedom to choose matching or not triangulations at interfaces. Moreover, the computational cost is reduced when stabilization techniques are also used. Theoretical analysis as well as some numerical tests will be presented. This research extends the ideas recently introduced in [1] and [2] for elliptic problems and in [3] for Stokes equations.

[1] C. Bernardi, T. Chacón Rebollo and E. Chacón Vera, *A FETI method with a mesh independent condition number for the iteration matrix* Computer Methods in Applied Mechanics and Engineering, Vol 197/13-16 pp 1410–1429., 2008.

[2] E. Chacón Vera, *A continuous framework for FETI-DP with a mesh independent condition number for the dual problem* Computer Methods in Applied Mechanics and Engineering, Vol 198, pp 2470–2483., 2009.

[3] E. Chacón Vera, D. Franco Coronil and A. Martínez Gavara *A non standard FETI-DP mortar method for Stokes Problem* Submitted.

Acknowledgments: Research partially funded by Spanish government MEC Research Project MTM2009-07719.

A Chimera Method applied to Computational Solid Dynamics

Session Schedule Author Index Session Index C17

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Abstract

We present in this work a Chimera method applied to Computational Solid Dynamics (CSD). The Chimera method was originally design as a technique for simplifying the mesh generation, by glueing overlapping and non-conforming meshes together. During many years, it has been almost excusively applied to CFD, although the concept is quite general. We propose here to extend the method to CSD, where very few examples are given in the literature. In CFD, the mehtod is useful to treat moving objects, each object having an independent attached mesh. In the context of CSD, the method can fairly simplify the meshing and enables to add, remove or move components to the geometry without having to remesh the global geometry. It can also be used for optimization purpose where the free degrees of freedom are the positions of the objects. The first step of the method is the classical hole cutting. The hole cutting consists in removing elements from the background mesh located inside the patch meshes. The second step consists in imposing some conditions on the subdomain interfaces in order to obtain continuous solution and flux across them. The proposed method impose Dirichlet conditions in an implicit way so the resulting algorithm does not introduce any additional iterative loop and is fully implicit. The method is also inherently parallel. We present some validation and application cases demonstrating the efficiency of the method on a supercomputer.

Contributed Talks C18

FETI Methods

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 11:50:12:15
Location: I51
Chairman: Rolf Krause

11:50:12:15 : K. C. Park
A Simple Explicit-Implicit FETI Transient Analysis Algorithm
Abstract

Date: Thursday, June 28
Time: 11:50-12:15
Location: I50
Chairman: Michael Minion

11:50-12:15 : Ulrich Langer
FETI-Solvers for Non-standard Finite Element Equations based on Boundary Integral Operators
Abstract

A Simple Explicit-Implicit FETI Transient Analysis Algorithm

Session Schedule Author Index Session Index C18

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Abstract

A simple explicit-implicit FETI (AFETI-EI) algorithm is presented for partitioned transient analysis of linear structural systems. The present algorithm employs two decompositions. First, the total system is partitioned via spatial or domain decomposition to obtain the governing equations of motions for each partitioned domain. Second, for each partitioned subsystem, the governing equations are *modally* decomposed into the rigid-body and deformational equations. The resulting rigid-body equations are integrated by an explicit integrator, for its stability is not affected by step-size restriction on account of zero frequency contents ($\omega = 0$). The modally decomposed partitioned deformation equations of motion are integrated by an unconditionally stable implicit integration algorithm. It is shown that the present AFETI-EI algorithm exhibits unconditional stability and that the resulting interface problem possesses the same solution matrix profile as the basic FETI static problems. The present simple dynamic algorithm, as expected, falls short of the performance of the FETI-DP but offers a similar performance of implicit two-level FETI-D algorithm with a much cheaper coarse solver; hence, its simplicity may offer relatively easy means for conducting parallel analysis of both static and dynamic problems by employing the same basic scalable FETI solver, especially for research-mode numerical experiments.

Acknowledgements: The first author has been partially funded by the local government of Andalucía (*Junta de Andalucía, Spain(P08-TEP-03804)*) and the Spanish Ministry of Science (*Ministerio de Educación y Ciencia(DPI2010-19331)*). The second author has been partially supported by WCU (World Class University) Program through the Korea Science and Engineering Foundation funded by the Ministry of Education, Science and Technology, Republic of Korea (Grant Number R31-2008-000-10045-0).

FETI-Solvers for Non-standard Finite Element Equations based on Boundary Integral Operators

Session Schedule Author Index Session Index C18

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Abstract

We present efficient Domain Decomposition solvers for a class of non-standard Finite Element Methods. These methods utilize PDE-harmonic trial functions in every element of a polyhedral mesh, and use boundary element techniques locally in order to assemble the finite element stiffness matrices. For these reasons, the terms BEM-based FEM or Trefftz-FEM are sometimes used. In the present talk, we show that Finite Element Tearing and Interconnecting (FETI) metho ds can be used to solve the resulting linear systems in a quasi-optimal and parallel manner. An important theoretical tool are spectral equivalences between FEM- and BEM-approximated Steklov-Poincaré operators.

Contributed Talks C19

Multiprocessors Applications

Schedule　　Author Index　　Session Index

Date: Tuesday, June 26
Time: 14:45-15:35
Location: Petri
Chairman: Eric Darrigrand

14:45-15:10 : Hatem Ltaief
Data-Driven Fast Multipole Method on Distributed Memory Systems with Hardware Accelerators
Abstract

15:10-15:35 : Menno Genseberger
Improved Parallel Performance on Supercomputers by Domain Decomposition in Jacobi-Davidson for Large Scale Eigenvalue Problems
Abstract

Data-Driven Fast Multipole Method on Distributed Memory Systems with Hardware Accelerators

Session Schedule Author Index Session Index C19

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Abstract

Fast N-body methods such as the fast multipole method (FMM) are essential for solving particle-based systems and boundary integral problems with a two-dimensional domain decomposition, because they can reduce the original complexity of $\mathcal{O}(N^2)$ to $\mathcal{O}(N)$. Besides the linear complexity, the FMM also has favorable characteristics such as; computationally intensive kernels, very few synchronization points, no iteration loops, and a hierarchical data structure that adopts to the memory hierarchy. However, when the application has an irregular distribution of particles/points the tree structure used in the FMM also becomes irregular, and simultaneous balancing of the work load load becomes a challenging problem.

Conventional techniques to handle the load-balancing problem in FMMs are based on bulk-synchronous static repartitioning of the next step by using the work load of the current step. The repartitioning can be done efficiently by updating the tree structure instead of rebuilding it, but the bulk-synchronous and static nature of the repartitioning poses certain limitations.

The present work proposes an alternative approach based on data flow programming model, where data-driven dynamic scheduling is employed to balance the work load within each subdomain. Previous work [1] by the authors showed promising results on shared memory systems. The idea is now to create per subdomain an instance of a dynamic runtime environment system (e.g., StarPU [2]) and to dynamically schedule the different computational tasks on the available x86 CPU cores as well as GPUs, taking advantage of all processing units provided by the system. The overall distributed FMM code can be therefore seen as a *hybrid* application in terms of scheduling policy, where a static scheduler drives the application at the MPI level and a dynamic scheduler balances the work load at the level of the local computational domains.

[1] H. Ltaief and R. Yokota. Data-Driven Execution of Fast Multipole Methods. http://arxiv.org/abs/1203.0889, *Submitted to EuroPar'12*, August 2012.

[2] C. Augonnet, S. Thibault, R. Namyst, and P.-A. Wacrenier. StarPU: a unified platform for task scheduling on heterogeneous multicore architectures. *Concurr. Comput. : Pract. Exper.*, 23:187–198, February 2011.

Improved Parallel Performance on Supercomputers by Domain Decomposition in Jacobi-Davidson for Large Scale Eigenvalue Problems

Session Schedule Author Index Session Index C19

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Abstract

Most computational work in Jacobi-Davidson, an iterative method for large scale eigenvalue problems, is due to a so-called correction equation. In earlier work a strategy for the computation of (approximate) solutions of the correction equation was proposed. The strategy is based on a domain decomposition preconditioning technique in order to reduce wall clock time and local memory requirements. However, there is more to gain. This talk discusses the aspect that the original strategy can be improved by taking into account that, for approximate solves of the correction equation by a preconditioned Krylov method, Jacobi-Davidson consists of two nested iterative solvers. For ease of presentation, consider the standard eigenvalue problem $\mathbf{A}\,\mathbf{x} = \lambda\,\mathbf{x}$ with an approximate eigenvalue θ, computed by Jacobi-Davidson so far, and preconditioner $\mathbf{M} \approx \mathbf{A} - \theta\,\mathbf{I}$. In the innerloop of Jacobi-Davidson a search subspace for the (approximate) solution of the correction equation is built up by powers of $\mathbf{M}^{-1}\,(\,\mathbf{A} - \theta\,\mathbf{I}\,)$ for fixed θ. In the outerloop a search subspace for the (approximate) solution of the eigenvalue problem is built up by powers of $\mathbf{M}^{-1}\,(\,\mathbf{A} - \theta\,\mathbf{I}\,)$ for variable θ. In the domain decomposition preconditioning technique was applied to the innerloop. But, as θ varies slightly in succeeding outer iterations, one may take advantage of the nesting by applying the same technique to the outerloop. For large scale eigenvalue problems this aspect turns out to be nontrivial. In the talk, the impact on the parallel performance will be shown by results of scaling experiments on supercomputers. This is of interest for large scale eigenvalue problems that need a massively parallel treatment.

Contributed Talks C20

Adaptive Meshing Paradigm

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 14:45-15:35
Location: I50
Chairman: Stéphane Lanteri

14:45-15:10 : Shuo Zhang
Norms of Trace Functions on Unstructured Grid
Abstract

15:10-15:35 : Cédric Lachat
PaMPA: Parallel Mesh Partitioning and Adaptation
Abstract

Norms of Trace Functions on Unstructured Grid

Session Schedule Author Index Session Index C20

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Abstract

Norm is an important concept that plays a key role in the design and analysis of numerical methods. Constructive or computational presentations of the appropriately chosen norms of discrete functions can bring much convenience in theory and practice. These presentations are not always available for the norms defined non-locally and especially the dual norms. In this talk, we introduce the constructive and computational presentations of several nonlocal norms of discrete functions defined on the unstructured grid on the boundary of a domain. The approach is based on the construction of an isomorphic extension operator on the trace space, and its conforming or nonconforming discretization. Discussions of exactly revertible Poincaré–Steklov operators are accompanied.

PaMPA: Parallel Mesh Partitioning and Adaptation

Session Schedule Author Index Session Index C20

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Abstract

This talk will present the structure and operations of PaMPA (*“Parallel Mesh Partitioning and Adaptation”*), a middleware library dedicated to the management of unstructured meshes distributed across the processors of a parallel machine. Its purpose is to relieve solver writers from the tedious and error prone task of writing again and again service routines for mesh handling, data communication and exchange, remeshing, and data redistribution. PaMPA represents meshes as graphs, whose data is distributed across the processors of the parallel machine. Graph vertices model the various entities of the mesh: its elements, faces, edges, nodes, etc. Edges connect interrelated entities: elements to all of their faces, faces to all of their nodes and edges, elements to their neighboring elements, etc. Numerical data of any type (either scalar, vector or structured) can be attached to either kind of mesh entity or sub-entity. A mesh overlap size can be defined by the user, so as to allow processors to access copies of mesh data located on neighboring processors. An overlap update routine allows users to propagate modifications of the data associated with locally owned vertices to their copies owned by neighboring processors. PaMPA iterators allow users to loop over entities and sub-entities of the mesh. By using iterators and accessing overlap data, users can easily express their numerical schemes without having to write data exchange routines by themselves. One of the key features of PaMPA is its ability to handle re-meshing in parallel. Parts of the mesh that need re-meshing are processed independently on each processor by a user-provides sequential remesher. This processed is repeated on yet un-remeshed areas until all of the mesh is processed. The re-meshed graph is then repartitioned so as to preserve load balance. In order to perform its task, PaMPA relies on several external libraries. Parallel graph partitioning is performed by PT-Scotch, while sequential remeshing of tetrahedra meshes is delegated to MMG3D.

Contributed Talks C21

FETI Methods

Schedule Author Index Session Index

Date: Tuesday, June 26
Time: 14:45-15:35
Location: I51
Chairman: Alexandros Markopoulos

14:45-15:10 : Marta Jarošová
Hybrid Total FETI
Abstract

15:10-15:35 : Michal Merta
Massively Parallel Implementation of Total-FETI DDM with Applications to Medical Image Registration
Abstract

Hybrid Total FETI

Session Schedule Author Index Session Index C21

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Abstract

We propose a hybrid FETI method based on our variant of the FETI type domain decomposition method called Total FETI. Our hybrid method was developed in an effort to overcome the bottleneck of classical FETI methods, namely the bound on the dimension of the coarse space due to memory requirements. We first decompose the domain into relatively large clusters that are completely separated, and then we decompose each cluster into smaller subdomains that are joined partly by Lagrange multipliers λ_0 in selected interface variables or in averages if the transformation of basis is applied. The continuity in the rest of interface variables and also the Dirichlet condition are enforced by Lagrange multipliers λ_1. This decomposition leads to the algorithm, where TFETI is used on two levels. The results of numerical experiments on benchmark from the linear elasticity will conclude the talk.

Massively Parallel Implementation of Total-FETI DDM with Applications to Medical Image Registration

Session Schedule Author Index Session Index C21

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Abstract

The FETI (Finite Element Tearing and Interconnecting) method turned out to be one of the most successful methods for the parallel solution of elliptic partial differential equations. The FETI-1 is based on the decomposition of the spatial domain into non-overlapping subdomains that are “glued” by Lagrange multipliers. Total-FETI (TFETI) by Dostal et al. simplifes the inversion of stiffness matrices of subdomains by using Lagrange multipliers not only for gluing the subdomains along the auxiliary interfaces, but also to enforce the Dirichlet boundary conditions. Thus bases of kernels of all subdomain stiffness matrices are known a priori and can be assembled directly from mesh data. In this work we compare two parallel implementations of TFETI method based on either PETSc or Trilinos software frameworks. Both these libraries are widely used for the developement of scientific codes. While PETSc is based almost entirely on pure C, Trilinos utilizes features of the modern C++ including templates and object oriented design. We focus on the parallel efficiency of both codes, mainly on the treatment of the solution of the coarse problem and the action of orthogonal projectors, which seem to be main bottlenecks of the TFETI parallel implementations. Although usual applications of TFETI method lie in the field of material sciences and structural mechanics, we demonstrate applicability of our codes to the problem of the image registration of computer tomography and magnetic resonance imaging data using elastic registration method. The numerical benchmarks were run on HECToR supercomputer at EPCC in the UK which is part of the PRACE HPC ecosystem.

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