Seoul National University
Active Aeroelasticity and Rotorcraft Lab.

Development of Nonlinear Structural Analysis using Co-rotational Finite Elements with improved Domain Decomposition Method

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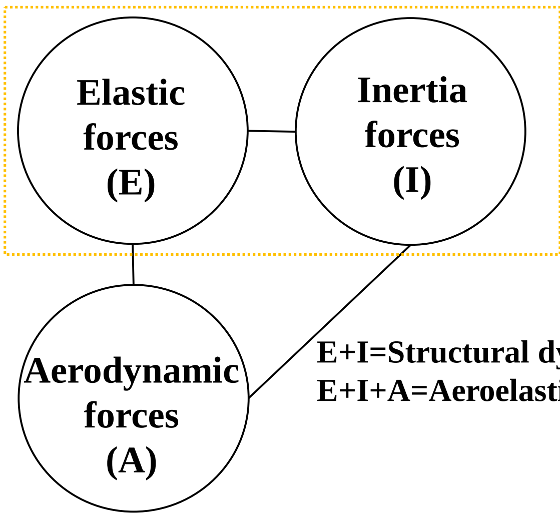
- Formulations
- Numerical results
- Conclusions and Future works

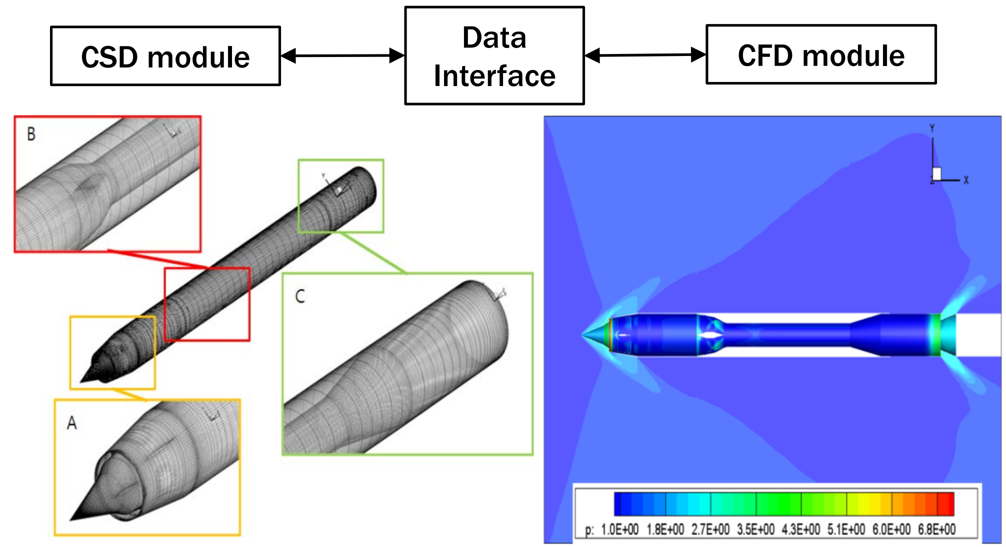
Motivation

❖ Large-size analysis in fluid-structure interaction problem



E+I=Structural dynamics
E+I+A=Aeroelasticity

▲ Multidisciplinary analysis (Gupta, 2000)



▲ Example of large-size FSI analysis

- **Advancement of the computer hardware/software technologies**
  - Large-size analysis in the field of aerospace engineering
- **Multidisciplinary analysis involves interactions among a number of disciplines.**
  - Structural analysis, Aerodynamic analysis, Fluid-structure interaction analysis

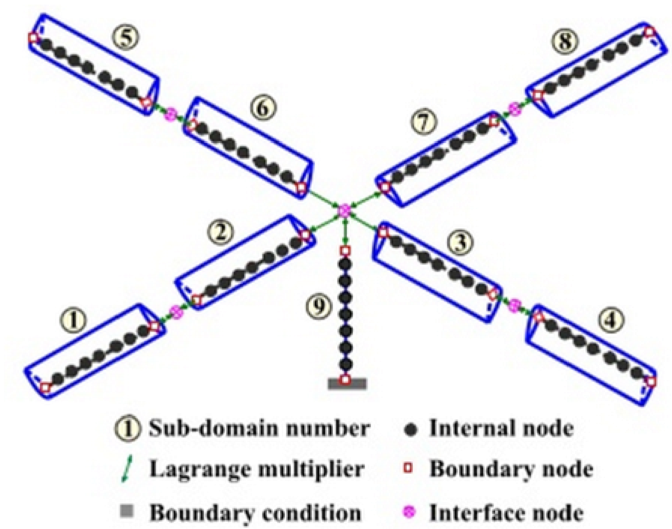
An effective solution methodology in the large-size structures has grown significantly in the field of the mechanical and aerospace engineering.

Motivation

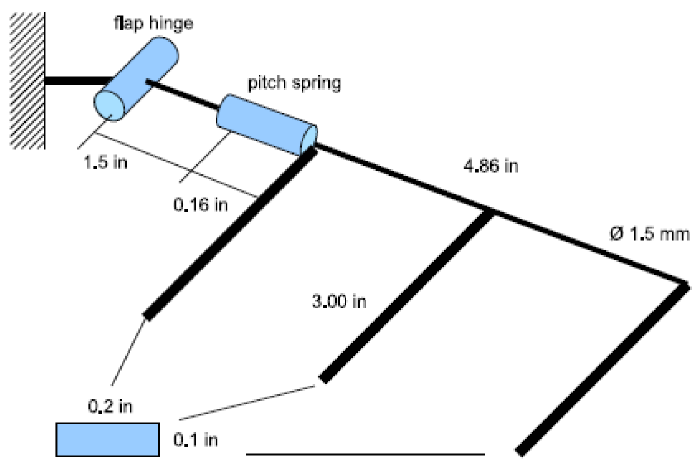
❖ Efficient strategies for nonlinear structural analysis

- **Complex structures consisting of many mechanical components**
  - Multi-body dynamics including motion of various joints
- **Flexible structures, i.e., rotor blades, flapping wing, show geometrically nonlinear behavior.**

An effective solution methodology to flexible multibody systems involving nonlinear kinematic constraints



▲ **Dynamics of the helicopter rotor (Heo, 2014)**



▲ **Multi-body configuration of the flapping wing (Masarati P., 2013)**

Motivation

❖ Solution techniques for structural analysis (1)

Numerical algorithms for large-size problems

Direct solution algorithm

- **Most of the commercial finite element analyses use a direct solver.**
- **Limited success in terms of scalability and simplicity in implementation.**
- **Generally, direct solvers require much larger storage than iterative solver does.**

Iterative solution algorithm

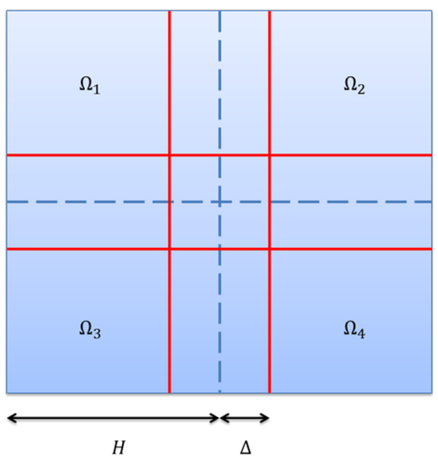
- **Iterative solvers have been preferred for a large-size parallel finite element analysis.**
- **Efficiency depends on the system matrix condition number and good preconditioners.**
- **Difficult to construct pre-conditioners**

Motivation

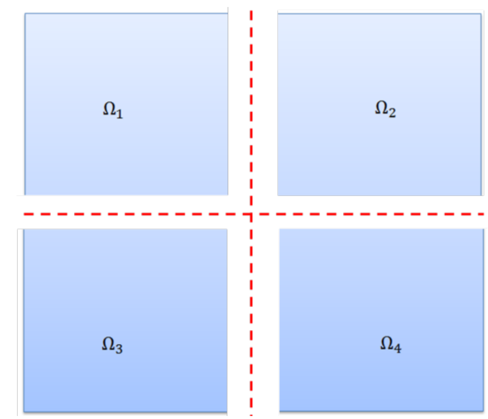
❖ **Solution techniques for structural analysis (2)**

Domain decomposition method (DDM)

Overlapping method



Non-overlapping method



- **Schwarz alternating method (Dryja, 1987)**
- **The original domain is split into overlapping sub-domains**

- **FETI method (Farhat, 1991)**
- **Lagrange multipliers enforce continuity along the interface**

Previous investigation

❖ Previous FETI approaches

➢ **Farhat (1991), (1994), (1998), (2001)**

- Method of finite element tearing and interconnecting and its parallel solution algorithm(1991)

→ Fewer inter-processor communications

- **Transient FETI methodology for large-size parallel implicit computations** in structural mechanics(1994)

→ Substructure version of Newmark integrator

- **Two-level FETI** method part I: an optimal iterative solver for bi-harmonic systems(1998)

→ Extension into the fourth order problems

- **FETI-DP**: **Dual-primal unified FETI method** part I: faster alternative to two-level FETI method(2001)

→ Unified all previously developed FETI algorithms into a single dual-primal FETI method

➢ **Hackbusch (1994), Li (2010), Gueye (2011), Tak (2013)**

- DDM with direct methods have been attempted.

Present research objectives

❖ Required enhancement in FETI method

➢ Farhat (1991), (1994), (1998), (2001)

- Preconditioner is required. → Additional mathematical algorithm, i.e., PCPG algorithm

→ difficulty to extending the algorithm to nonlinear problem or applying for multibody system.

➢ Proposed FETI approach

- the augmented Lagrangian formulation and direct solver → natural preconditioning and securement of numerical efficiency

→ effective extension to nonlinear problem or multibody system.

ALF with Lagrange multiplier

Direct solver

→

Numerical efficiency

Concise algorithm

→

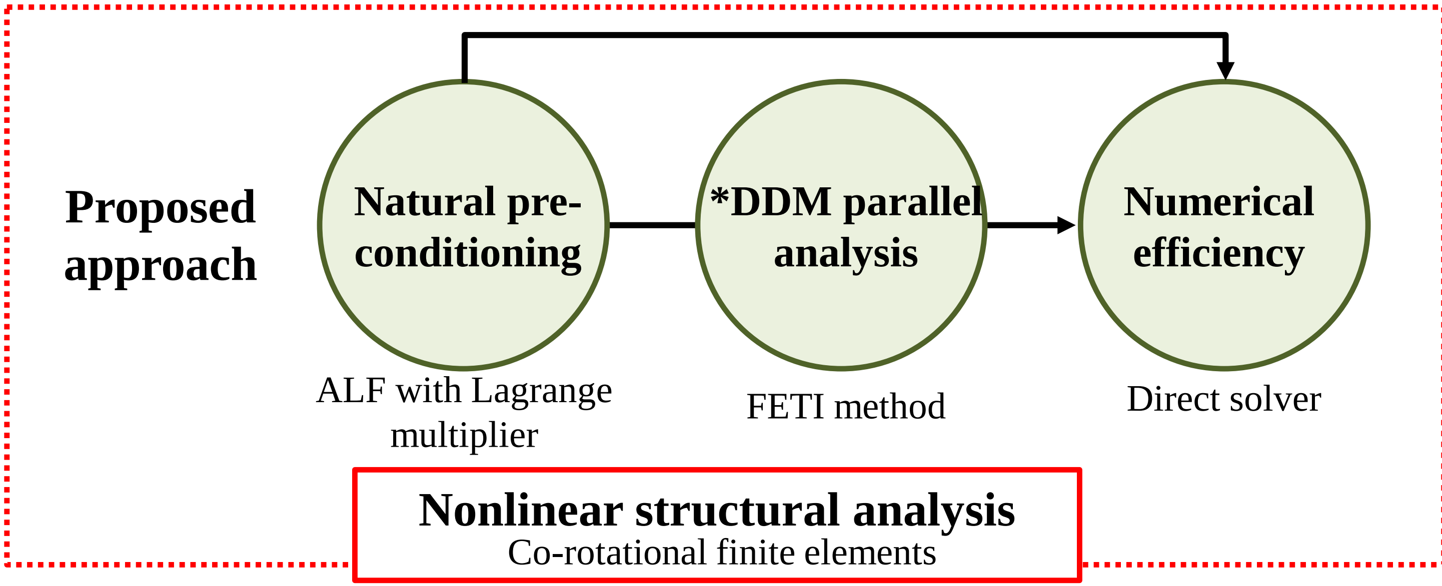
✓ **Multibody system**

✓ **Nonlinear structural problem**

Present research objectives

*DDM: Domain decomposition method

❖ Present research objectives

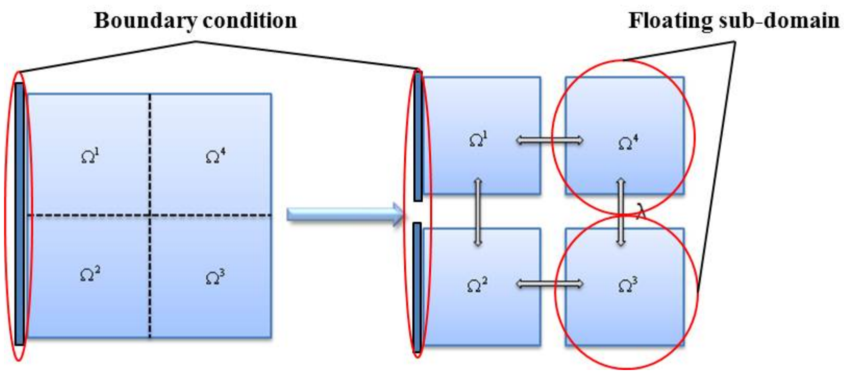


- **Derive an augmented Lagrangian formulation as a penalty term of the present proposed FETI method and develop the equation of motion.**
- **Develop a computation algorithm** based on a finite element domain decomposition technique for the analysis of large-size structural problems and its **parallelization for a parallel computer hardware.**
- Develop **a computation algorithm of nonlinear structural analysis based on co-rotational finite element** in the presently proposed FETI method.

- Introduction
- **Formulations**
- Numerical results
- Conclusions and Future works

Original FETI methods

❖ Algorithm of the original FETI method (1)



$$\underline{\underline{K}}\,\underline{u} = \underline{f}$$

$$\underline{\underline{K}}^{(s)}\underline{u}^{(s)} = \underline{f}^{(s)} + \sum_{s=1}^{a^{(s)}} \underline{\underline{B}}_j^{(s)T}\underline{\lambda}$$

$$\underline{\underline{B}}_j^{(s)}\underline{u}^{(s)} = \underline{\underline{B}}_s^{(j)}\underline{u}^{(j)}$$

$$s = 1, 2, \cdots, N_s$$

- FETI method is an approach in which **the computational domain is divided into non-overlapping sub-domains**.
- In the FETI method, **Lagrange's multipliers are introduced** to enforce compatibility **at the interface nodes as the interface connecting forces**.
- In the static analysis, **each floating sub-domain**, which is under non-boundary condition, **induces a local singularity**.

Original FETI methods

❖ Algorithm of the original FETI method (2)

Constrained sub-domains

$$\underline{u}^{(s)} = \underline{\underline{K}}^{(s)-1}\left(\underline{f}^{(s)} + \underline{\underline{B}}^{(s)T}\underline{\lambda}\right)$$

$$\left[\sum_{s=1}^{N_s}\underline{\underline{B}}^{(s)}\underline{\underline{K}}^{(s)-1}\underline{\underline{B}}^{(s)T}\right]\underline{\lambda} = \left[\sum_{s=1}^{N_s}\underline{\underline{B}}^{(s)}\underline{\underline{K}}^{(s)-1}\underline{f}^{(s)}\right]$$

$$s = 1, 2, \cdots, N_s$$

Floating sub-domains

Pseudo-inverse

$$\underline{u}^{(s)} = \underline{\underline{K}}^{(s)^{\dagger}}\left(\underline{f}^{(s)} + \underline{\underline{B}}^{(s)T}\underline{\lambda}\right) + \underline{\underline{R}}^{(s)}\underline{\alpha}^{(s)}$$

$$\underline{\underline{R}}^{(s)T}\left(\underline{f}^{(s)} - \underline{\underline{B}}^{(s)T}\underline{\lambda}\right) = 0$$

Rigid body modes

$$s = 1, 2, \cdots, N_s$$

- **The solution of the problem is obtained in two steps.**
  - ✓ First, the solution of the interface problem yields the Lagrange multipliers.
  - ✓ Second, the displacement field in each sub-domains is evaluated.

Original FETI methods

❖ Algorithm of the original FETI method (3)

Interface problem

$$\begin{bmatrix} \underline{\underline{F}}_I & -\underline{\underline{G}}_I \\ -\underline{\underline{G}}_I^T & \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} \underline{\lambda} \\ \underline{\alpha} \end{bmatrix} = \begin{bmatrix} \underline{d} \\ -\underline{e} \end{bmatrix}$$

$$\underline{\underline{F}}_I = \sum_{s=1}^{N_s} \underline{\underline{B}}^{(s)} \underline{\underline{K}}^{(s)^\dagger} \underline{\underline{B}}^{(s)T} \qquad \underline{e} = \underline{\underline{R}}^{(s)T} \underline{f}^{(s)}$$

$$\underline{\underline{G}}_I = \left[\underline{\underline{B}}^{(1)} \underline{\underline{R}}^{(1)}, \cdots, \underline{\underline{B}}^{(N_s)} \underline{\underline{R}}^{(N_s)} \right]$$

$$\underline{\underline{K}}^{(s)^\dagger} = \underline{\underline{K}}^{(s)-1}$$

Preconditioner
+iterative solver

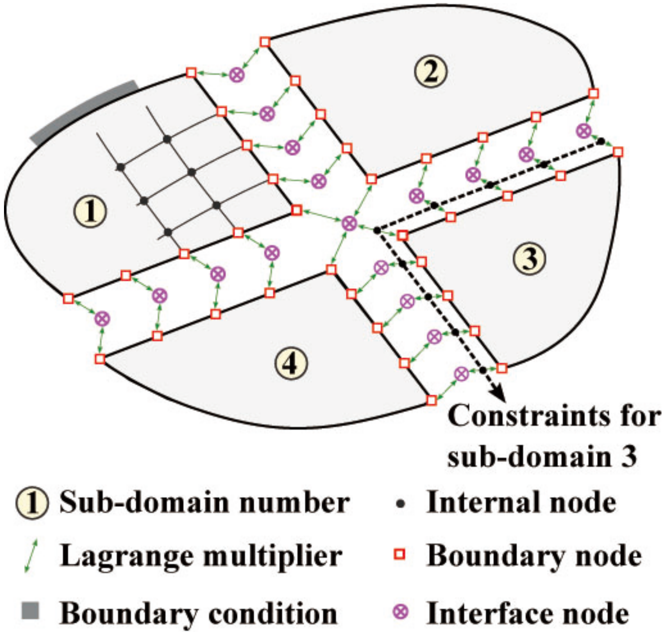
$$\underline{\alpha}^T = \left[\underline{\alpha}_1^T, \cdots, \underline{\alpha}_{N_s}^T \right] \qquad \underline{d} = \sum_{s=1}^{N_s} \underline{\underline{B}}^{(s)} \underline{\underline{K}}^{(s)^\dagger} \underline{f}^{(s)}$$

- **Unknown variables $\underline{\lambda}$ and $\underline{\alpha}$ are solved by using an iterative method.**
  - Preconditioned conjugate projected gradient (PCPG) is required.

Complex algorithm due to the iterative solver, such as PCPG is required.
→ difficulty in understanding and implementation

Proposed FETI approach

❖ Features of the proposed FETI approach



Localized Lagrange multipliers:

$$\underline{\lambda}^{(i)T} = \left\{ \underline{\lambda}^{[1]T}, \underline{\lambda}^{[2]T}, \cdots, \underline{\lambda}^{\left[N_b^{(i)}\right]T} \right\}$$

Nodal DOFs and Lagrange multipliers of sub-domain *i*:

$$\underline{\breve{u}}^{(i)T} = \left\{ \underline{u}^{(i)T}, \underline{\lambda}^{(i)T} \right\}$$

Array storing the DOFs of all sub-domains:

$$\underline{\breve{u}}^{(i)T} = \left\{ \underline{\breve{u}}^{(1)T}, \underline{\breve{u}}^{(2)T}, \cdots, \underline{\breve{u}}^{(N_s)T} \right\}$$

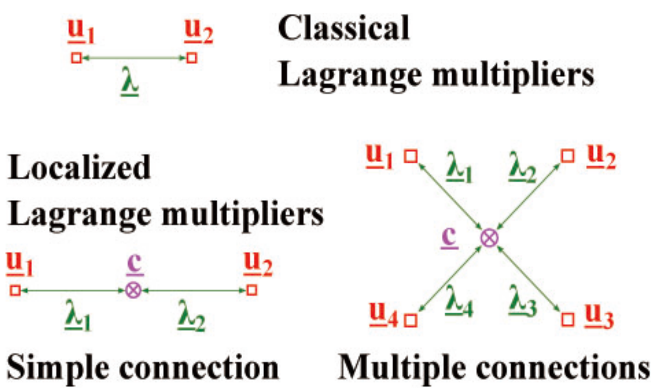
- **All the developments presented here are applicable to general, three-dimensional problems.**

- **Application of the localized Lagrange multipliers technique to enforce the continuity of the displacement field.**
  - Each constraint and corresponding Lagrange multipliers are associated with a single sub- domain unambiguously.

- **All the constraints are assumed to be local.**
  - The interface node is defined along the entire interface.

Proposed FETI approach

❖ Formulation of the proposed FETI approach (1)



▲ Classical and localized Lagrange multipliers

Classical Lagrange multiplier:

$$V_c = \underline{\lambda}^T \underline{C}\,, \qquad \underline{C} = \underline{u}_1 - \underline{u}_2 = 0$$

Localized Lagrange multiplier:

$$V_c = \underline{\lambda}^{[1]T} \underline{C}^{[1]} + \underline{\lambda}^{[2]T} \underline{C}^{[2]}$$

$$\underline{C}^{[1]} = \underline{u}_1 - \underline{c} = \underline{0}\,, \quad \underline{C}^{[2]} = \underline{u}_2 - \underline{c} = \underline{0}$$

- **All the constraints are assumed to be local.**
  - The interface node is defined along the entire interface.

- **No direct constraint is written between the DOF's of the sub-domains.**
  - Lagrange multipliers become "localized."

Proposed FETI approach

❖ Formulation of the proposed FETI approach (2)

Total potential energy of the system:

$$\Pi = A + \Phi + V_c$$

A : **strain energy**

Φ : **potential of the externally applied loads**

V_c : **potential of the constraints**

Strain energy in terms:

$$A = \frac{1}{2}\underline{\breve{u}}^T diag\left(\underline{\underline{\breve{K}}}^{(\alpha)}\right)\underline{\breve{u}}$$

Sub-domain stiffness matrix:

$$\underline{\underline{\breve{K}}}^{(i)} = \begin{bmatrix} \underline{\underline{K}}^{(i)} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{0}} \end{bmatrix}$$

Total potential of the applied loads:

$$\Phi = \sum_{i=1}^{N_s} \Phi^{(i)} = -\sum_{i=1}^{N_s} \underline{u}^{(i)T} \underline{Q}^{(i)} = -\underline{u}^T \underline{Q} = -\underline{\breve{u}}^T \underline{\breve{Q}}$$

$$\underline{\breve{Q}}^T = \left\{\underline{Q}^{(1)T}, \underline{0}^T, \underline{Q}^{(2)T}, \underline{0}^T, \cdots, \underline{Q}^{(N_s)T}, \underline{0}^T\right\}$$

Proposed FETI approach

❖ Formulation of the proposed FETI approach (3)

Kinematic constraint:

$$\underline{C}^{[j]} = \underline{u}_b^{[j]} - \underline{c}^{[j]} = \underline{0}$$

Potential of constraints: (localized Lagrange multiplier technique + penalty method)

$$V_c^{[j]} = s\underline{\lambda}^{[j]T}\underline{C}^{[j]} + \underbrace{\frac{p}{2}\underline{C}^{[j]T}\underline{C}^{[j]}}_{\text{Penalty terms}}$$

$$\delta V_c^{[j]} = \delta\underline{u}_b^{[j]T}\left[s\underline{\lambda}^{[j]} + p\underline{C}^{[j]}\right] + \delta\underline{\lambda}^{[j]T}\left[s\underline{C}^{[j]}\right] + \delta\underline{c}^{[j]T}\left[-s\underline{\lambda}^{[j]} - p\underline{C}^{[j]}\right]$$

Generalized forces of constraint:

$$\underline{f}^{[j]} = \begin{Bmatrix} s\underline{\lambda}^{[j]} + p\underline{C}^{[j]} \\ s\underline{C}^{[j]} \\ -s\underline{\lambda}^{[j]} - p\underline{C}^{[j]} \end{Bmatrix}$$

Stiffness matrix of the constraint:

$$\underline{\underline{k}}^{[j]} = \begin{bmatrix} p\underline{\underline{I}} & s\underline{\underline{I}} & -p\underline{\underline{I}} \\ s\underline{\underline{I}} & 0 & -s\underline{\underline{I}} \\ -p\underline{\underline{I}} & -s\underline{\underline{I}} & p\underline{\underline{I}} \end{bmatrix}$$

Proposed FETI approach

❖ Algorithm of the proposed FETI approach

Constraint forces are partitioned to:

$$\underline{f}^{[j]} = \begin{Bmatrix} \underline{f}_b^{[j]} \\ \underline{f}_c^{[j]} \end{Bmatrix}$$

$$\underline{f}_b^{[j]} = \begin{Bmatrix} s\underline{\lambda}^{[j]} + p\underline{C}^{[j]} \\ s\underline{C}^{[j]} \end{Bmatrix}$$

$$\underline{f}_c^{[j]} = -\left\{ s\underline{\lambda}^{[j]} + p\underline{C}^{[j]} \right\}$$

Constraint stiffness matrix are partitioned to:

$$\underline{\underline{k}}^{[j]} = \begin{bmatrix} \underline{\underline{k}}_{bb}^{[j]} & \underline{\underline{k}}_{bc}^{[j]} \\ \underline{\underline{k}}_{bc}^{[j]T} & \underline{\underline{k}}_{cc}^{[j]} \end{bmatrix}$$

$$\underline{\underline{k}}_{bc}^{[j]} = \begin{bmatrix} -p\underline{\underline{I}} \\ -s\underline{\underline{I}} \end{bmatrix} \quad \underline{\underline{k}}_{bb}^{[j]} = \begin{bmatrix} p\underline{\underline{I}} & s\underline{\underline{I}} \\ s\underline{\underline{I}} & \underline{\underline{0}} \end{bmatrix}$$

$$\underline{\underline{k}}_{cc}^{[j]} = \left[p\underline{\underline{I}} \right]$$

Assembly procedure:

$$\underline{\breve{F}}_b^{(i)} = \sum_{j=1}^{N_b^{(i)}} \underline{\underline{B}}_b^{[j]T} \underline{f}_b^{[j]} \qquad \underline{\underline{\breve{K}}}_{bb}^{(i)} = \sum_{j=1}^{N_b^{(i)}} \underline{\underline{B}}_b^{[j]T} \underline{\underline{k}}_{bb}^{[j]} \underline{\underline{B}}_b^{[j]}$$

$$\underline{F}_c^{(i)} = \sum_{j=1}^{N_b^{(i)}} \underline{\underline{B}}_c^{[j]T} \underline{f}_c^{[j]} \qquad \underline{\underline{K}}_{cc}^{(i)} = \sum_{j=1}^{N_b^{(i)}} \underline{\underline{B}}_c^{[j]T} \underline{\underline{k}}_{cc}^{[j]} \underline{\underline{B}}_c^{[j]}$$

$$\underline{\underline{K}}_{bc}^{(i)} = \sum_{j=1}^{N_b^{(i)}} \underline{\underline{B}}_b^{[j]T} \underline{\underline{k}}_{bc}^{[j]} \underline{\underline{B}}_c^{[j]}$$

Governing equations:

$$\begin{bmatrix} diag\left(\underline{\underline{\breve{K}}}^{(\alpha)} + \underline{\underline{\breve{K}}}_{bb}^{(\alpha)} \right) & \underline{\underline{K}}_{bc} \\ \underline{\underline{K}}_{bc}^T & \underline{\underline{K}}_{cc} \end{bmatrix} \begin{Bmatrix} \underline{\breve{u}} \\ \underline{c} \end{Bmatrix} = \begin{Bmatrix} \underline{\breve{Q}} - \underline{\breve{F}}_b \\ -\underline{F}_c \end{Bmatrix}$$

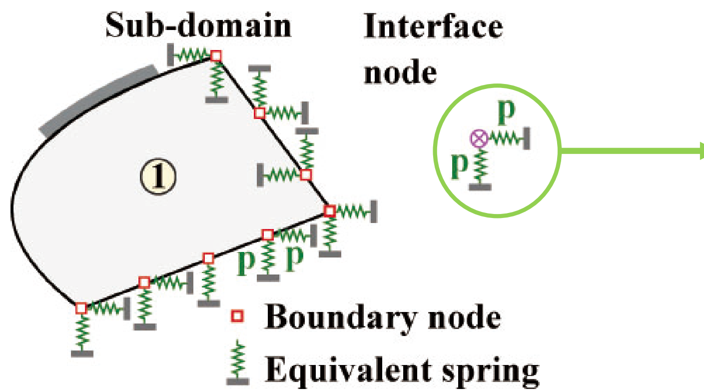
- **The potential of kinematic constraint involves two types of DOF's.**
  - Sub-domain DOF's / Interface DOF's
- **Each kinematic constraint generates an array of constraint forces and a stiffness matrix.**
  - Each kinematic constraint can be viewed as finite element.
- **The assembly procedure can be performed in parallel for all sub-domains.**

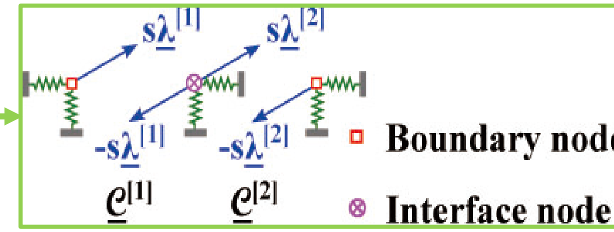
Proposed FETI approach

❖ Penalty method in the proposed FETI approach

$$\underline{\underline{k}}^{[j]} = \begin{bmatrix} \boxed{\begin{matrix} p\underline{\underline{I}} & s\underline{\underline{I}} \\ s\underline{\underline{I}} & \underline{\underline{0}} \end{matrix}} & \begin{matrix} -p\underline{\underline{I}} \\ -s\underline{\underline{I}} \end{matrix} \\ \begin{matrix} -p\underline{\underline{I}} & -s\underline{\underline{I}} \end{matrix} & \boxed{p\underline{\underline{I}}} \end{bmatrix}$$

$\underline{\underline{k}}_{bb}^{[j]}$, $\underline{\underline{k}}_{cc}^{[j]}$





$\pm s\underline{\lambda}^{[j]}$ Newton's third law

- **The leading entry of matrix $\underline{\underline{K}}_{bb}^{[j]}$ is a diagonal matrix, $p\underline{\underline{I}}$, which is added to the diagonal entries of stiffness matrix $\underline{\underline{K}}^{(i)}$ associated with the boundary nodes.**
  - Physically, this corresponds to adding springs of stiffness constant p connected to the ground at each boundary node of sub-domain i .
  - $\underline{\underline{\breve{K}}}^{(i)}$ is singular for any floating sub-domain, $\underline{\underline{\breve{K}}}^{(i)} + \underline{\underline{\breve{K}}}_{bb}^{(i)}$ is not.

- **The Lagrange multipliers can be interpreted as the forces that interconnect the various parts of the structure.**
  - At convergence, all kinematic constraints will be satisfied. $\underline{C}^{[j]} = \underline{0}$
  - Constraint forces reduce to equal and opposite forces. (boundary/interface node)

Proposed FETI approach

❖ Computational method in the proposed FETI approach

Step I

- Assemble Boolean matrices
- Assemble sub-domain stiffness matrix
- Adopted ALF
- Define sub-domain coarse mesh stiffness matrix using sparse linear solver (PARDISO library)
- Define sub-domain coarse mesh load array

Step II

- Assemble interface problem stiffness matrix
- Assemble interface problem load arrays
- Solve the interface problem using sparse linear solver (PARDISO library)

Decomposition

- Read geometry data
- Decompose the domain into a number of sub-domains
  → parallelization for corresponding processors

Step III

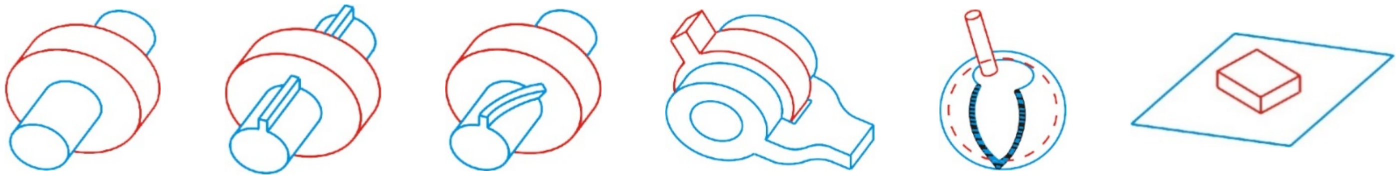
- Sends the interface degrees of freedoms array to all other processes.
- Recover the solution of each sub-domain using sparse linear solver (PARDISO library)
  → parallelization for corresponding processors

- **Proposed FETI-local approach proceeds in three computational steps.**
- **Step I sets up the structural interface problem (possible to parallelize).**
- **Step II obtains the solution of the structural interface problem.**
- **Step III recovers the solution in each sub-domain (possible to parallelize).**

Flexible multi-body dynamics simulation

❖ DYMORE, Simulation tools for flexible multibody systems

- **An FEM-based multibody dynamics analysis**
  - ✓ Features beam and shell elements capable of dealing with composite materials
  - ✓ Capable of modeling complex configuration including mechanical joints



▲ Six lower pairs of joint in DYMORE

- **Finite element-based multibody dynamics approaches**
  - ✓ Yields accurate predictions for complex systems, but at high computational costs
  - ✓ Use the constraints via Lagrange multiplier technique to enforce nonlinear kinematic constraints
  - ✓ **Solve the resulting Differential Algebraic Equations using direct solvers.**

    → **ill-conditioned system matrices involving large condition number** are generally induced.

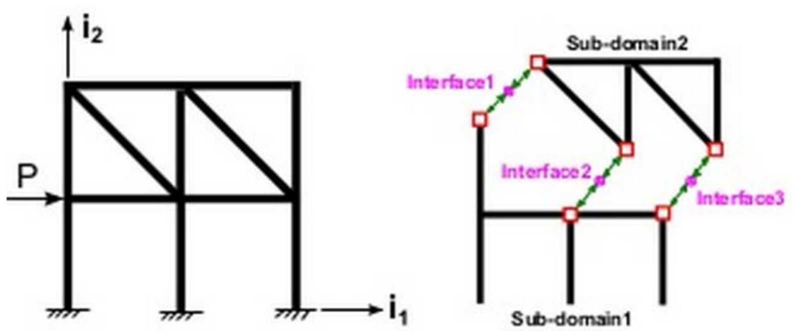
    → **Significant increase of a number of DOFs and computational time** due to the multi-connected structure or multi-disciplinary analysis including aerodynamic loads**.**

Parallel processing of MBD simulation

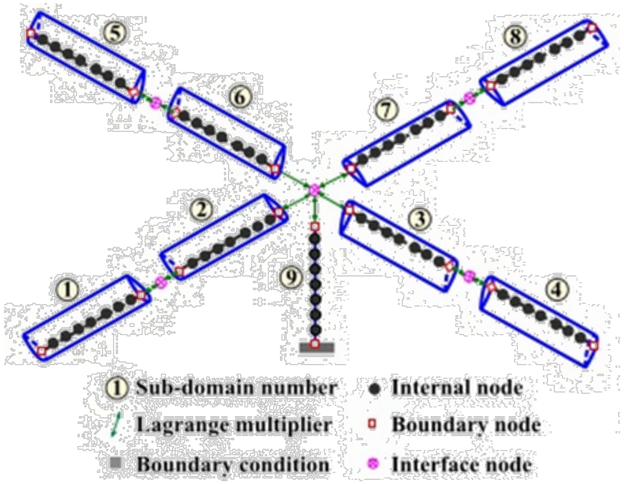
❖ Parallelization of DYMORE

- **FETI method with localized Lagrange multiplier is implemented (Heo, 2014).**
  - The comprehensive analysis of multibody system must satisfy contradictory requirements.
    - ✓ Increasingly accurate predictions and Faster execution times
    - ✓ Advanced modeling techniques require an exponential increase in computational resources

- **Overall solution procedure in parallelized DYMORE**
  ① Factorize sub-domain stiffness matrices (Parallel)
  ② Factorize interface stiffness matrix
  ③ Forward-substitute sub-domains (Parallel)
  ④ Solve for interface displacements by forward- and back-substitution
  ⑤ Solve for sub-domain displacements by back-substitution (Parallel)



▲ Grids of a beam (Heo, 2014)
– partitioned into two sub-domain with multiple interfaces



▲ Dynamics of the helicopter rotor (Heo, 2014)

Nonlinear analysis using the proposed approach

❖ Flexible multibody system including the motion with large amplitude

- **Helicopter or wind turbine blades, missiles, high altitude long endurance aircraft, flapping wings**

Ex) Flapping wings = Vein + Wing membrane

▲ **Insect wing**

✓ Modularized fashion
✓ Simple mathematics but sophisticated
✓ **Capable for <u>the vein and wing membrane</u> and <u>their angular motion</u>**

→

Vein
CR beam
with warping DOF
Wing membrane
CR shell

- **Co-rotational (CR) FEs can be useful for various structural analysis accommodating the motion with large amplitude.**
  → modularized and unified algorithm

✓ **Need to improve the proposed FETI algorithm**

- For static analysis
(Load incremental Newton-Raphson method + Proposed FETI algorithm)

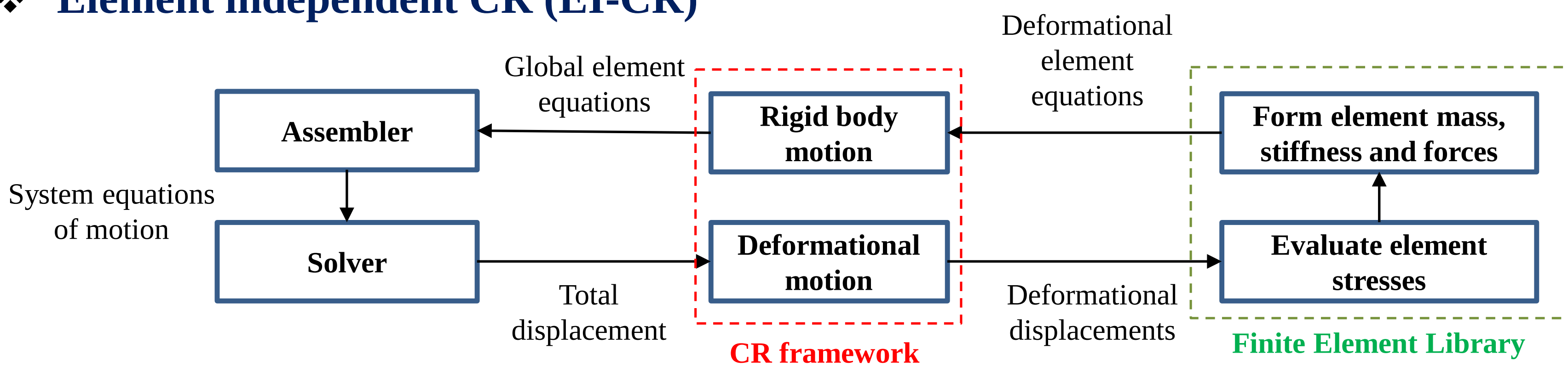
- For time-transient analysis
(Hilbert Hughes Taylor α method + Proposed FETI algorithm)

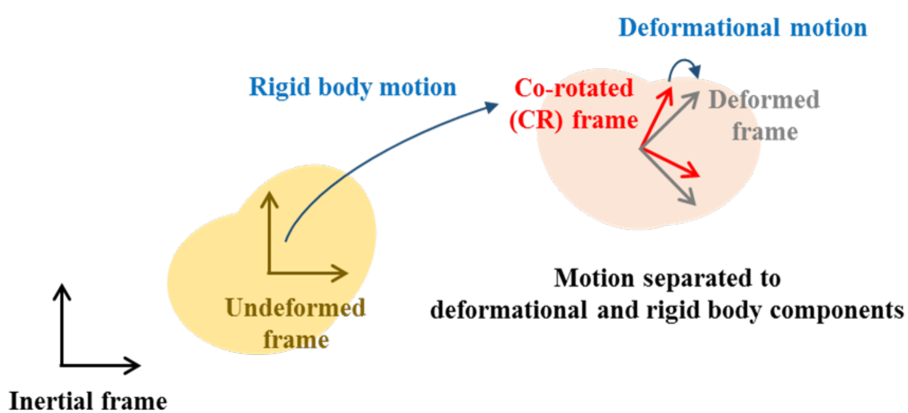
Nonlinear analysis using the proposed approach

❖ Nonlinear analysis based on co-rotational (CR) framework

- The most recent of the Lagrangian kinematic descriptions **(Total-Lagrangian, Updated-Lagrangian, Co-rotational)**
- Kinematic assumptions: arbitrarily large displacements and rotations, but small deformations

❖ Element independent CR (EI-CR)

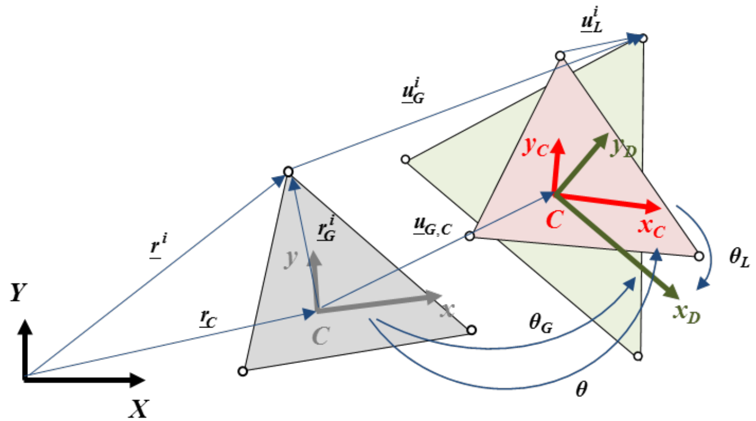




▲ **Coordinate in the CR framework**

Nonlinear analysis using the proposed approach

❖ Co-rotational formulation for planar element



▲ **Coordinate in triangular planar elements**

- **Coordinate systems and element kinematics**
  - The motion of the element is split in rigid translation and rotation and local deformation with respect to the local frame.
  - Element rigid rotation obtained by using translation behavior

$$\tan\theta = \frac{\sum_{i=1}^{N}\left[x_i\left(Y_i + V_i - Y_C - V_C\right) - y_i\left(X_i + U_i - X_C - U_C\right)\right]}{\sum_{i=1}^{N}\left[y_i\left(Y_i + V_i - Y_C - V_C\right) + x_i\left(X_i + U_i - X_C - U_C\right)\right]}$$

- **Local element rotation obtained by using global rotation dof**

$$\mathbf{R}_G^i = \begin{bmatrix} \cos(\theta_{Gi}) & -\sin(\theta_{Gi}) \\ \sin(\theta_{Gi}) & \cos(\theta_{Gi}) \end{bmatrix} \qquad \mathbf{R}_L^i = \mathbf{R}^T\mathbf{R}_G^i \qquad \tan\theta_{Li} = \frac{\mathbf{R}_L^i(2,1)}{\mathbf{R}_L^i(1,1)}$$

Nonlinear analysis using the proposed approach

- **The local system → with respect to the existing finite element hypothesis**
  - Internal force vector and stiffness matrix in the local frame

$$f=\begin{Bmatrix}\dfrac{\partial\Phi}{\partial u_L}\\[2ex]\dfrac{\partial\Phi}{\partial\theta_L}\end{Bmatrix}\qquad k=\begin{bmatrix}\dfrac{\partial^2\Phi}{\partial u_{L,i}\partial u^e_{L,j}} & \dfrac{\partial^2\Phi}{\partial u_{L,i}\partial\theta_{L,j}}\\[2ex]\dfrac{\partial^2\Phi}{\partial\theta_{L,i}\partial u_{L,j}} & \dfrac{\partial^2\Phi}{\partial\theta_{L,i}\partial\theta_{L,j}}\end{bmatrix}$$

strain energy

u_L pure nodal translation DOF in the deformed frame

θ_L pure nodal rotation DOF in the deformed frame

- **Internal forces and stiffness matrices along changes of rotation variables**
  - Global internal force vector and stiffness matrix

$$\mathbf{f}_G=\mathbf{B}^T f$$

$$\mathbf{K}_G=\mathbf{B}^T k\mathbf{B}+\mathbf{K}_h$$

Tangent stiffness matrix

$$\mathbf{K}_h=\mathbf{E}\left[-\mathbf{F}_2^{\,T}\mathbf{G}-\mathbf{G}^T\mathbf{F}_1\mathbf{P}\right]\mathbf{E}^T$$

where

$$\mathbf{F}_{1,i}=\begin{bmatrix}n_{i,1} & -n_{i,2} & 0\end{bmatrix}$$

$$\mathbf{F}_{2,i}=\begin{bmatrix}n_{i,1} & -n_{i,2} & -n_{i,3}\end{bmatrix}$$

  - Transformation matrices, E and B

$$\mathbf{E}=\mathbf{diag}\begin{bmatrix}\mathbf{R} & \cdots & \mathbf{R}\end{bmatrix}$$

where $\mathbf{B}=\mathbf{P}\mathbf{E}^T$

$$\mathbf{P}=\mathbf{I}-\mathbf{A}\mathbf{G}\qquad \mathbf{A}_i=\{-y_{di}\quad x_{di}\quad 1\}^T$$

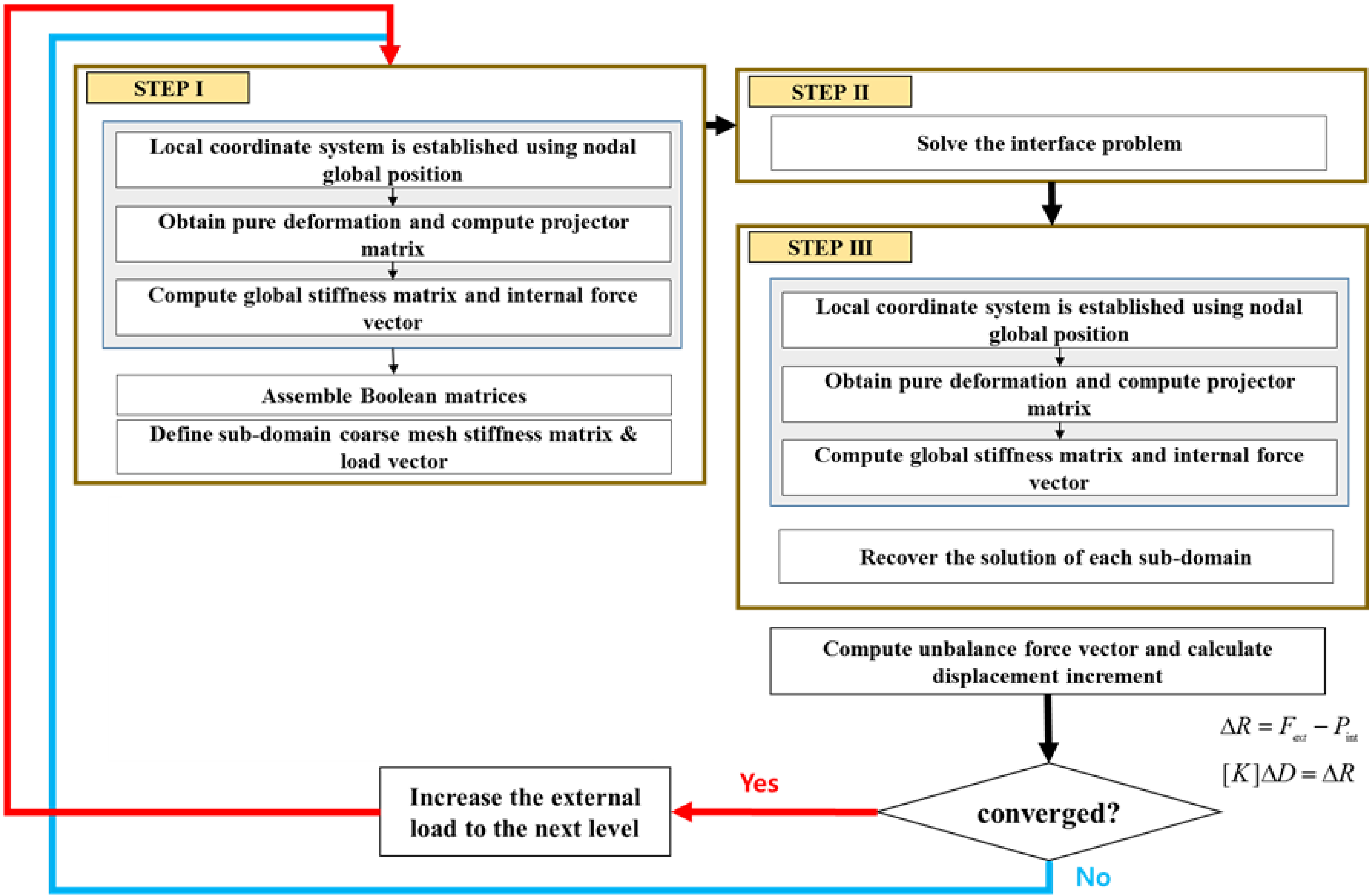
$$\mathbf{G}_i=\frac{1}{\sum_{i=1}^{N}\left(x_i x_{di}+y_i y_{di}\right)}\{-y_{di}\quad x_{di}\quad 0\}$$

projector matrix extracting the deformational component from the total motion

$$\mathbf{R}=\begin{bmatrix}\cos(\theta) & -\sin(\theta) & 0\\ \sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{bmatrix}$$

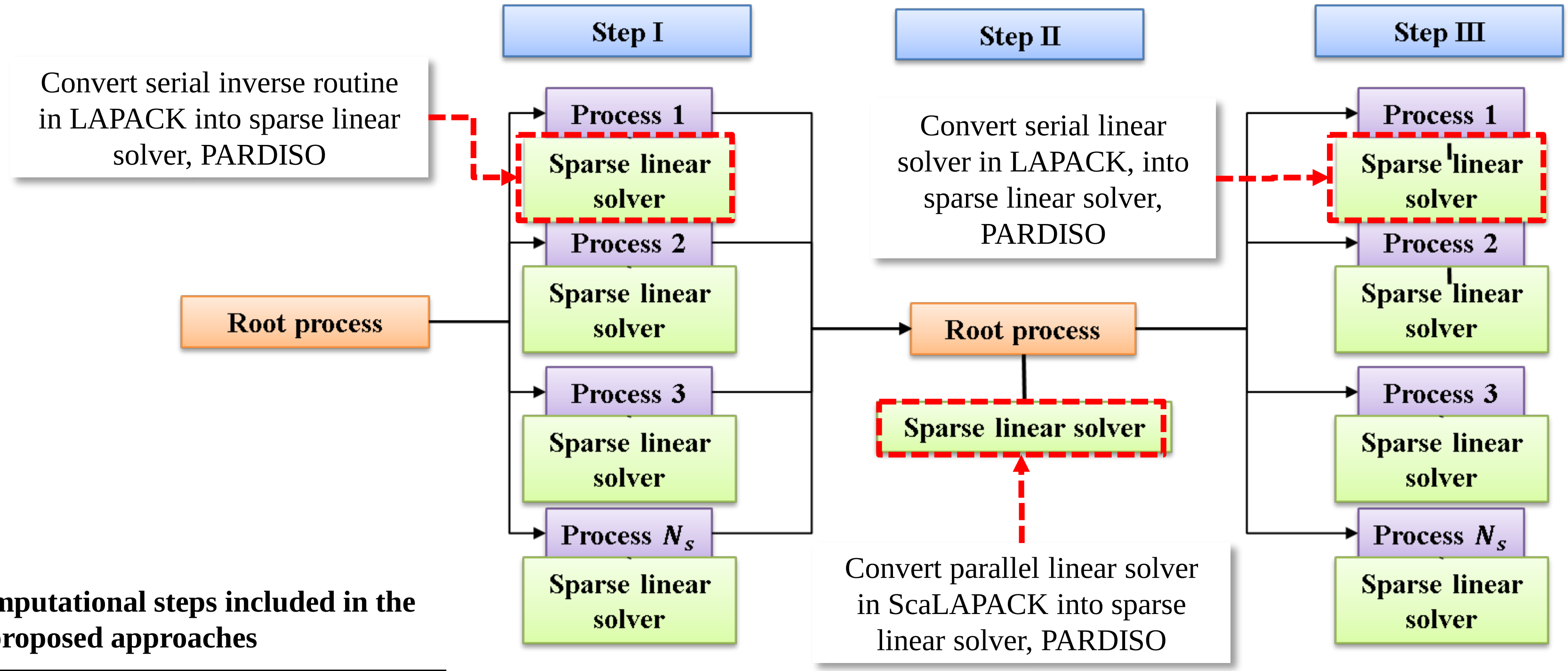
Nonlinear analysis using the proposed approach

❖ **Load incremental Newton-Raphson scheme**



Parallel algorithm of the proposed FETI approach

❖ Parallelization of the proposed FETI approach



▼ **Computational steps included in the proposed approaches**

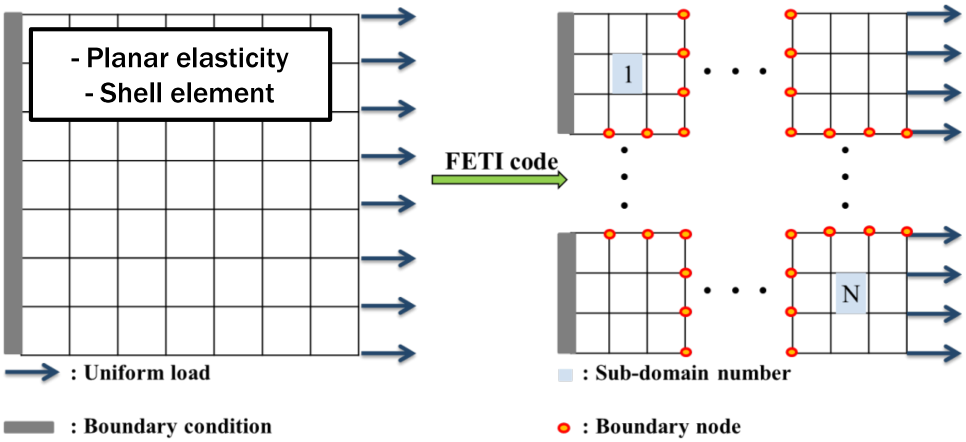
Calculation procedure	Proposed FETI-local
Step I	Inverse routine
	PARDISO library
Step II	Linear solver routine
	PARDISO library
Step III	Linear solver routine
	PARDISO library

- **Sparse matrix library, PARDISO, is employed to handle the sparsity of the defined matrices, efficiently.**
- **Message passing interface (MPI) is implemented.**
- **Collective communication algorithm is applied. (MPI_REDUCE, MPI_BCAST).**

- Introduction
- Formulations
- **Numerical results**
- Conclusions and Future works

Numerical results

- Static analysis with the two-dimensional problems were conducted to examine the computational costs and the scalability.



Method	Serial algorithm	Parallel algorithm	FSI
Original FETI	O	-	-
FETI-DP	O	O	-
Proposed FETI-local	O	O	O

- Parallel computations are conducted on a TACHYON system.

- Time transient analysis are conducted by applying the constant and sinusoidal tip loads.

- FSI analysis regarding an axisymmetric engine configuration is conducted.

Condition number

❖ Comparisons of the condition number

▼ **Comparison on the condition number**

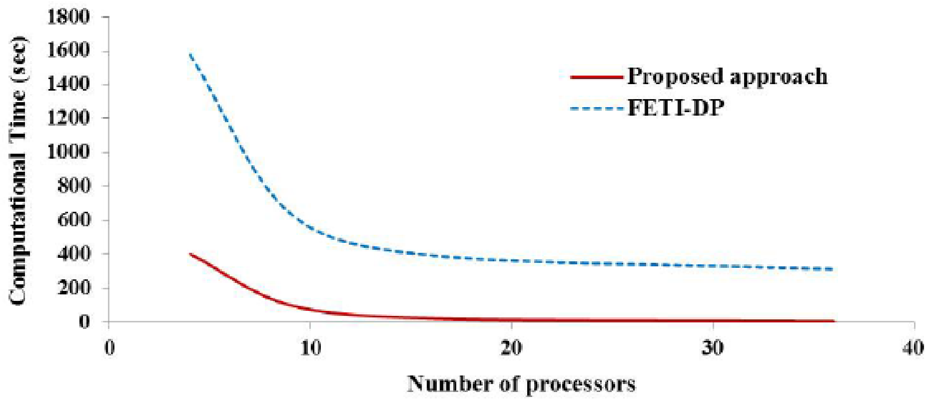
Analysis	Condition number	Displacement (m)
Original FETI	4.70×10^{16}	1.6×10^{-3}
FETI-DP	6.98	1.6×10^{-3}
FETI-local	1.17	1.6×10^{-3}

- **Condition number of the original FETI is relatively large.**
- **Condition number of the flexibility matrix of the proposed methods approaches unity.**
- **The four methods give the same numerical values for the displacement.**

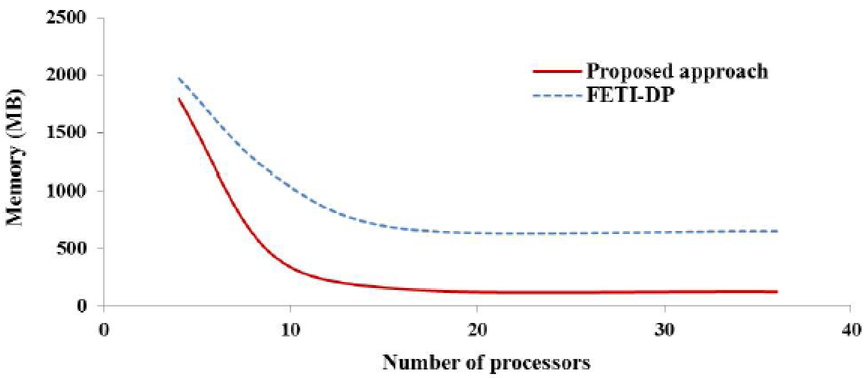
Advantages : Excellent conditioning of the interface problem

Computational efficiency test

❖ Comparisons of the computational costs with FETI-DP



▲ **Computational time and trend of the FETI-local approach**

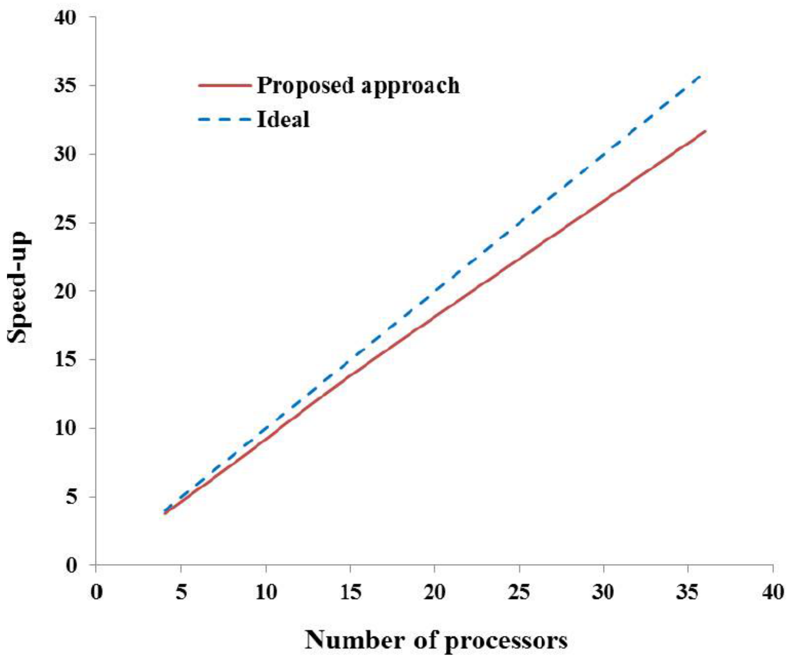


▲ **Memory usage trend of the FETI-local approach**

- The number of the sub-domains is increased from 4 to 36, but the number of DOFs is kept to a total of 35,378.
- Behavior of the proposed FETI-local is similar to that of the FETI-DP.
- Proposed FETI-local features the smaller computational time and memory usages than FETI-DP does.

Computational efficiency test

❖ Scalability test with the speed-up capability



▲ Speed-up result by the FETI-local in parallel computing environment

- Figure shows the scalability of the proposed FETI-local, and it is estimated by the speed-up capability.

$$S_{ps} = \frac{\text{Time for sequential processing with one processor}}{\text{Time for parallel processing with } p \text{ processors}}$$

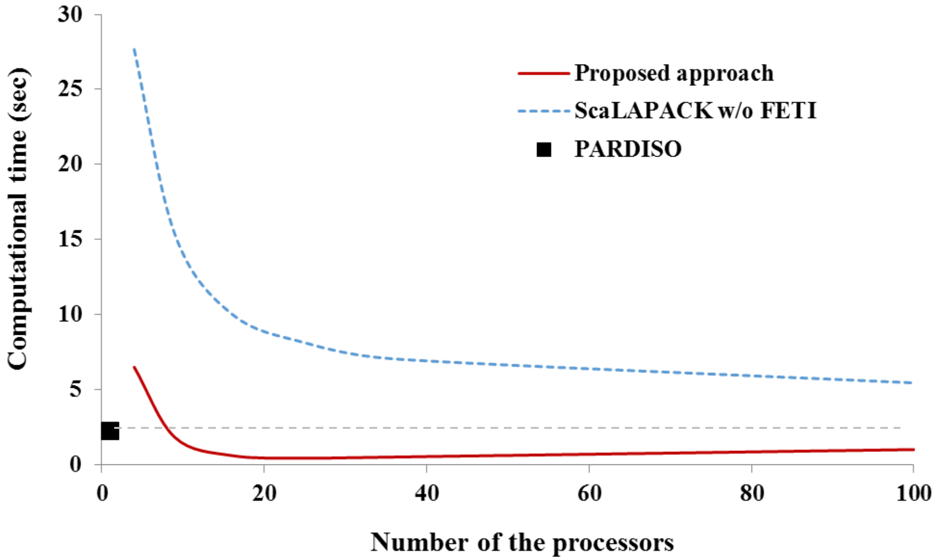
- Proposed FETI-local approach reinforced with the parallel linear solver improves the computational efficiency.

Computational efficiency test

❖ **Comparisons of the computational costs with existing numerical libraries**

▼ **Comparisons of computational time**

Number of sub-domains	Proposed approach [s]	Number of CPUs	Parallel ScaLAPACK [s]	Number of CPUs	Serial PARDISO [s]	Serial LAPACK [s]
4	6.52	4	27.63	1	2.25	99.42
9	1.84	9	15.37			
16	0.64	16	10.07			
25	0.45	25	8.17			
36	0.53	36	7.09			
100	1.03	100	5.49			

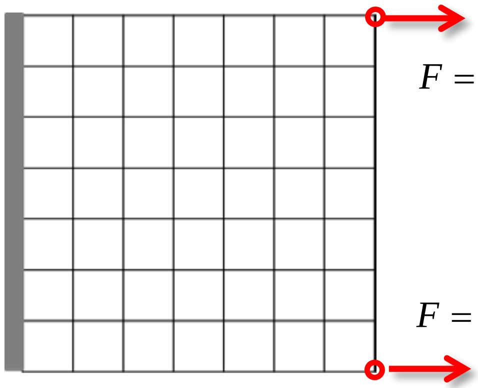


▲ **Computational time and trend of the FETI-local approach**

- The number of the sub-domains is increased from 4 to 100, but the number of DOF's is kept to a total of 7,442.
- Structural analyses consisting of the same DOF's are conducted by the existing numerical libraries, LAPACK, ScaLAPACK and PARDISO, respectively.
- Proposed FETI-local features the smaller computational time usages than other existing numerical libraries do.

Time transient analysis results

❖ Time transient analysis of the proposed FETI and FETI-DP method



$$F = F_0 \sin(40\pi t)$$

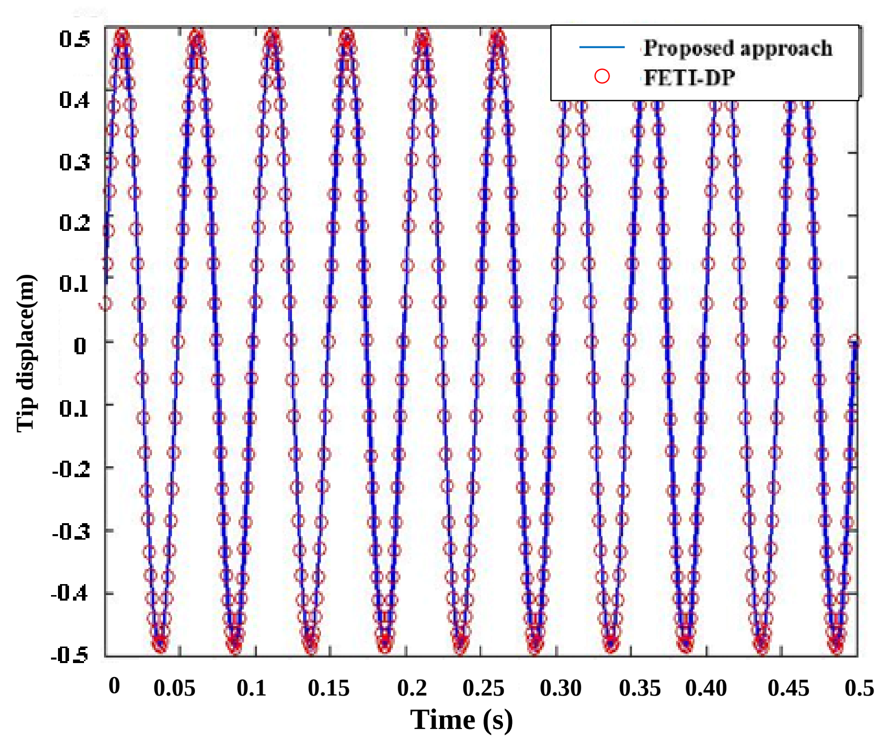
$$F = F_0 \sin(40\pi t)$$

▲ **Time transient analysis condition**

▼ **Analysis condition**

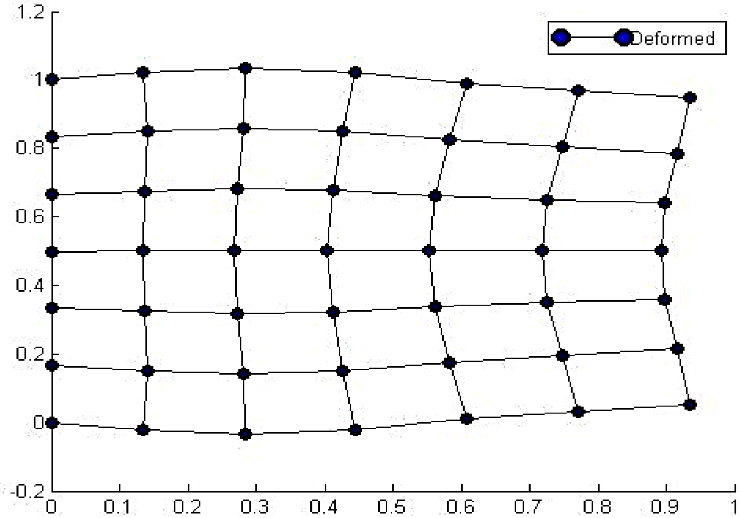
Time step size (s)	0.001
Mass density(kg/)	4430
Elastic modulus (GPa)	114
Poisson's ratio	0.33
Input load (N)	sinusoidal

- Standard Newmark method is employed for time transient analysis,
- The plot shows the time transient structural analysis results for proposed time transient FETI and Dual-primal FETI methods.
- The tip deflection shows an oscillation with respect to the static deflection.
- During the 500 time steps, both analyses show good agreement with a difference smaller than 0.01%.



Time transient analysis results

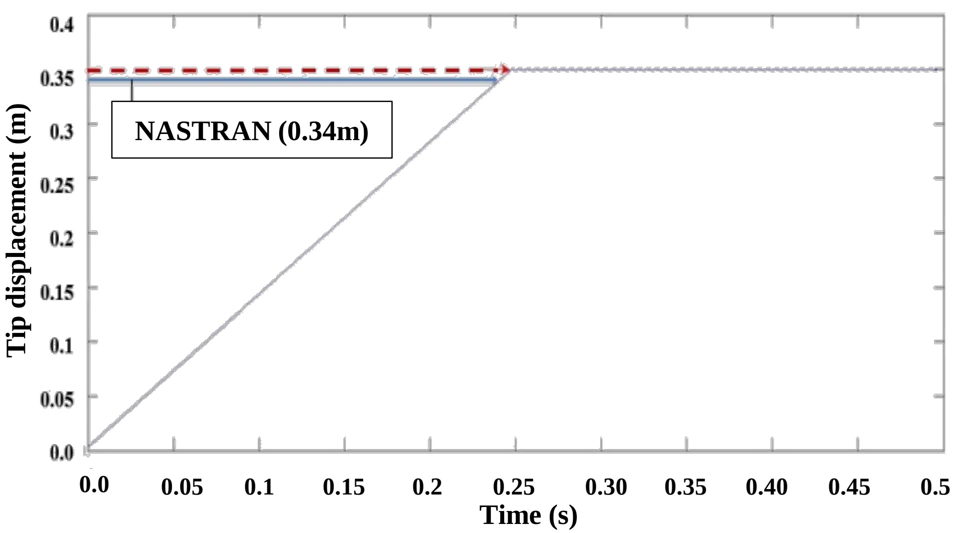
❖ **Validation upon the time transient analysis of the proposed FETI method**



▲ **Deformed configuration**

▼ **Analysis condition**

Time step size(s)	0.001
Mass density(kg/)	4430
External forcing frequency(Hz)	20
Elastic modulus(	114
Poisson's ratio	0.33
Input load()	

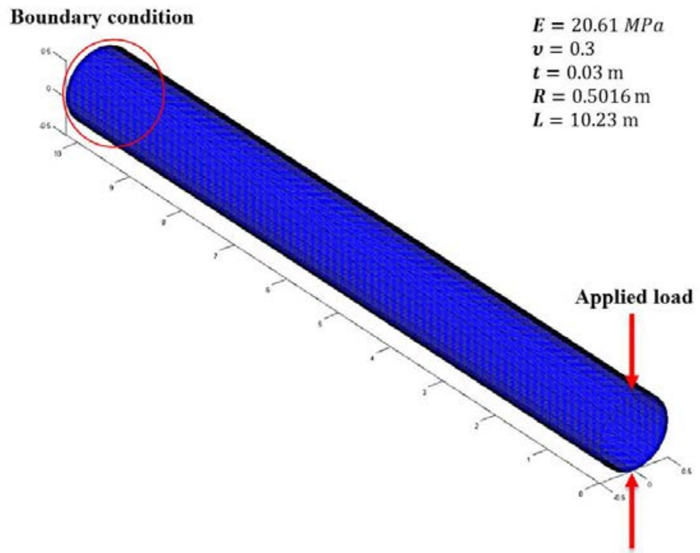


▲ **Response of tip displacement**

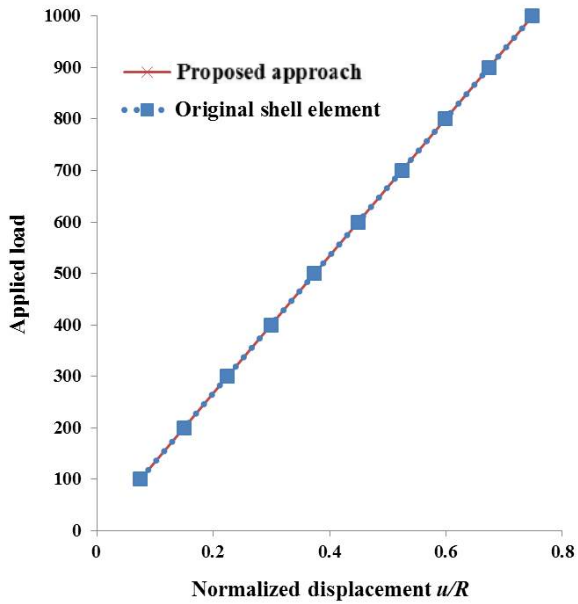
- The proposed time transient FETI method is applied to the solution of a two dimensional time transient plane strain problem.
- The present result is compared with those obtained by NASTRAN static analysis result.
- The result shows good agreement with that from NASTRAN.

Application for three-dimensional problem

❖ Validation of the present shell analysis



▲ Configuration of the three-dimensional problem



▲ Load-deflection result comparison between the general shell FEM and the proposed FETI-local

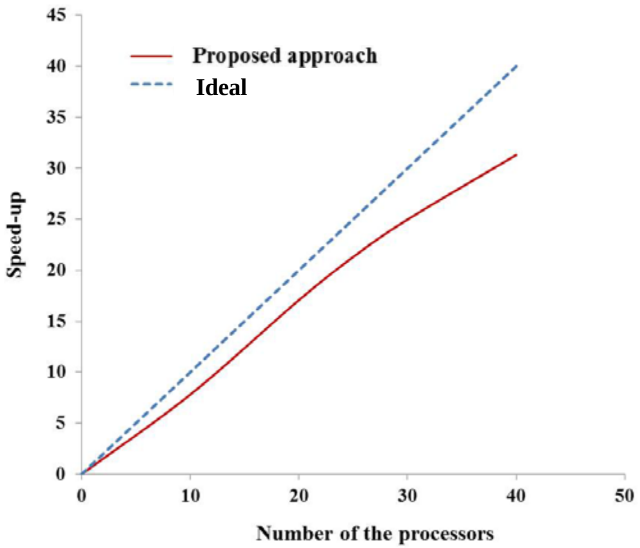
- The number of the sub-domains is increased from 10 to 40, but the number of DOFs is kept to a total of 86,544.
- The static deflection predicted by the proposed FETI-local compares well with that by the general shell FEM analysis.

Application for three-dimensional problem

❖ Computation costs for the present shell problem

▼ **Computational time and memory usage**

Number of sub -domains	Computational time (s)	Memory usage of each process (Mb)
10	466.63	1785.30
20	114.34	271.81
25	74.93	172.12
30	54.07	177.13
40	33.88	177.13

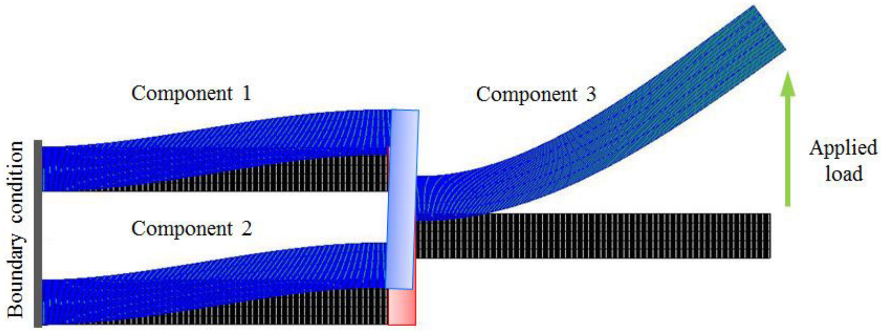


▲ **Speed-up result by Version 2 FETI-local**

- As the number of processors is increased, the computational time is varied from 466.63 to 33.88 (sec), and the maximum memory usage is from 1785 to 179.78 MB per process.
- **Figure shows benign scalability characteristics possessed and exhibited by the proposed FETI-local.**

Application for multi-body analysis

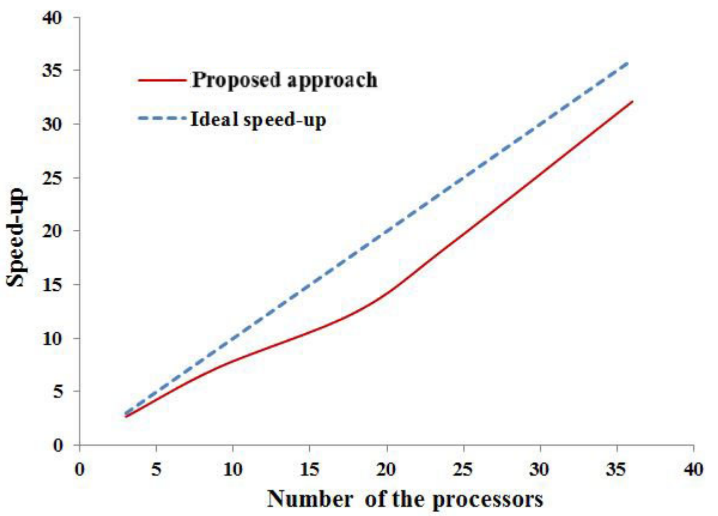
❖ **Parallel implementation for multi-body configuration using linearized planar element**



▲ **Multibody finite element configuration**

▼ **Computational time and memory usage (MB finite element configuration)**

Number of sub-domains	Computational time (s)	Memory usage of each processor (Mb)
6	155.15	778
9	54.45	393
18	8.95	106
24	4.25	69
36	1.65	43

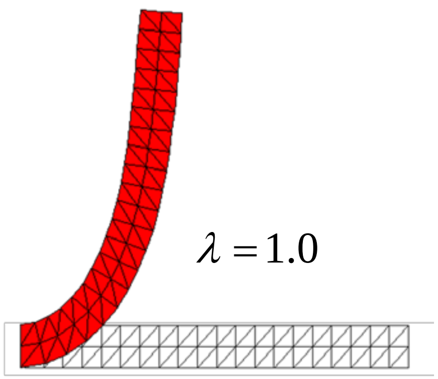


▲ **Speed-up result for the MB finite element configuration by the proposed FETI-local method in a parallel computing environment**

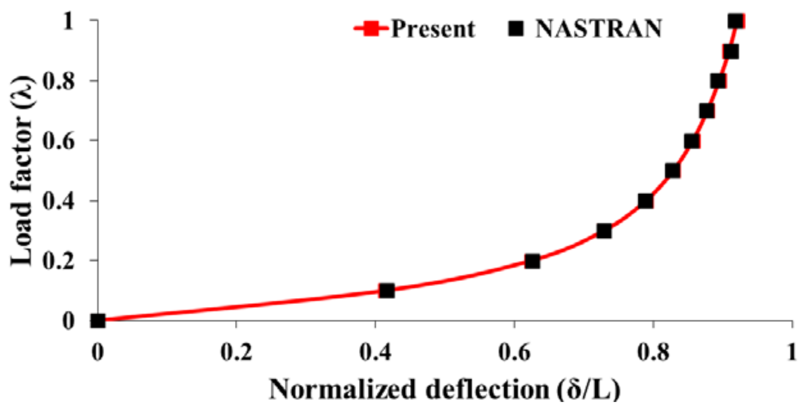
Application for nonlinear analysis

❖ Validation of presently employed CR planar element

▲ Configuration of the nonlinear problem



▲ Deformed configuration

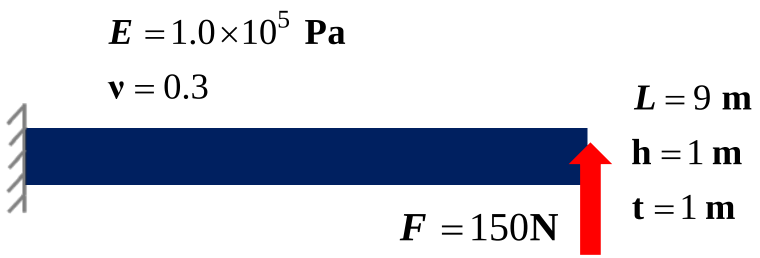


▲ Comparison of tip deflection

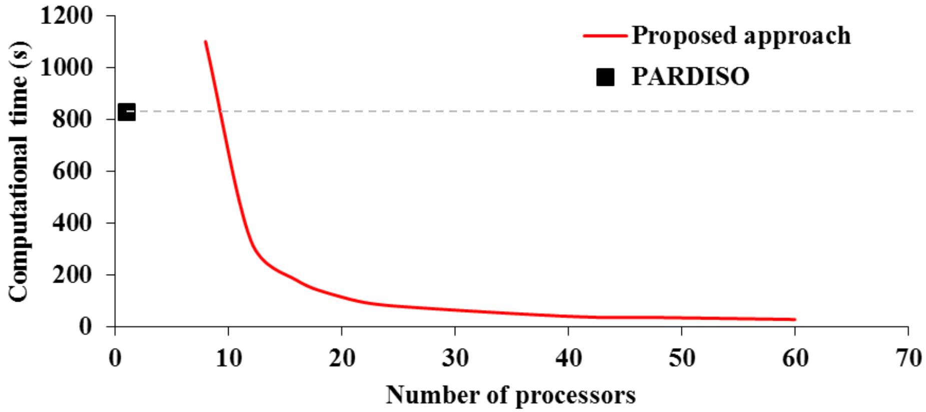
- Deflection of the planar plate is compared by increasing the concentrated tip load.
- Present results shows good correlation with those obtained by NASTRAN prediction and both results show geometrically nonlinear deflection.

Application for nonlinear analysis

❖ Computation costs for nonlinear analysis



▲ **Configuration of the nonlinear problem**

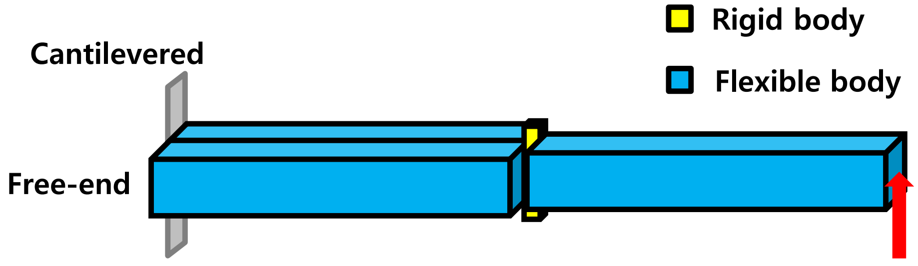


▲ **Computational time and trend of the proposed approach**

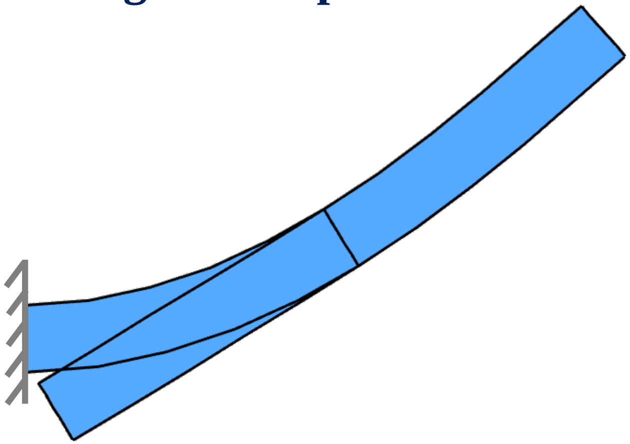
- The number of the sub-domains is increased from 8 to 60, but the number of DOFs is kept to a total of 39,864.
- Figure shows benign scalability characteristics possessed and exhibited by the proposed approach in nonlinear structural analysis.
- **By the parallel computation, the proposed approach shows more efficient characteristics when compared with that by PARDISO.**

Application for nonlinear multi-body analysis

❖ **Parallel implementation for multi-body system using the CR planar element**



▲ **Configuration of multi-body analysis**



▲ **Deformed configuration**

- The number of the sub-domains is increased from 9 to 36, but the number of DOFs is kept to a total of 32,400.

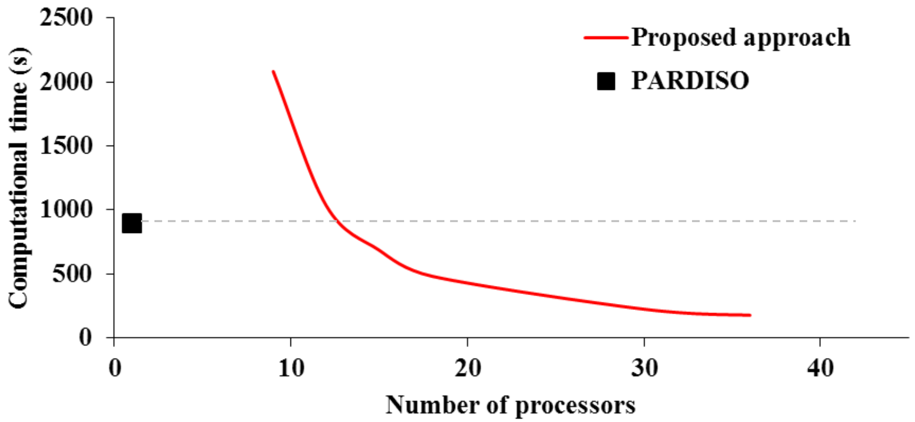
- **To verify an efficiency of the proposed approach, equivalent analysis employing the classical Lagrange multiplier and the sparse linear solver, PARDISO, is conducted and compared.**

Application for nonlinear multi-body analysis

❖ Computation costs for multi-body system using the CR planar element

▼ **Comparison of computational time**

Proposed approach		**PARDISO**	
Number of sub-domains	Computational time (s)	Number of CPUs	Computational time (s)
9	2081.09	1	902.39
12	1033.90		
15	685.28		
18	481.93		
30	224.76		
36	177.03		

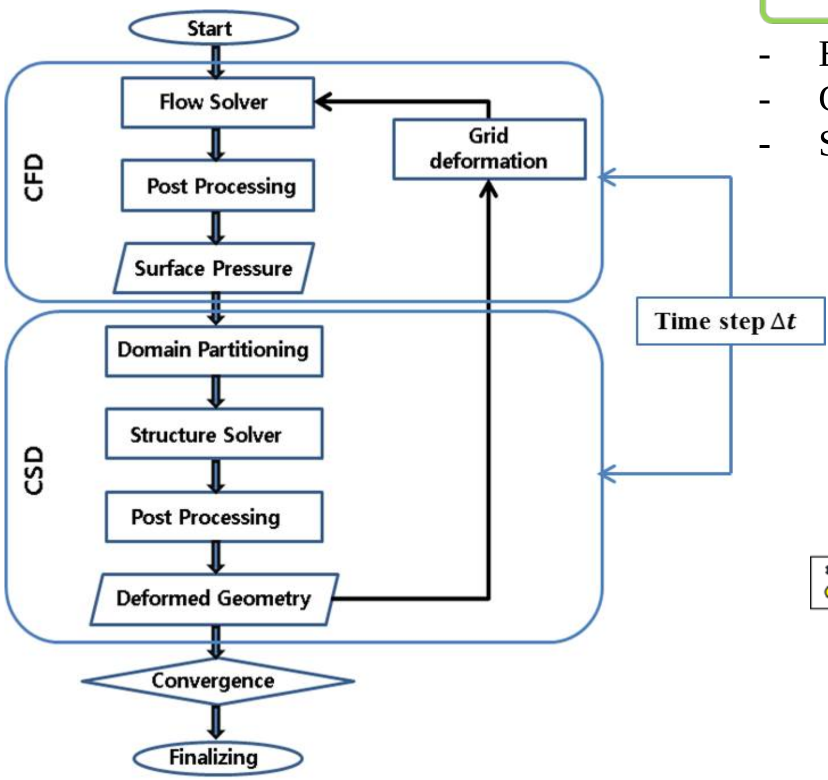


▲ **Computational time and trend of the proposed approach in nonlinear multibody analysis**

- As the number of processors is increased, the computational time is varied from 2081.09 to 177.03 (sec).
- **Figure shows benign scalability characteristics possessed and exhibited by the proposed approach.**
- **The proposed approach shows outstanding efficiency upon the computational time by comparing with that by PARDISO.**

Application for FSI Analysis

❖ CFD/CSD coupling methodology



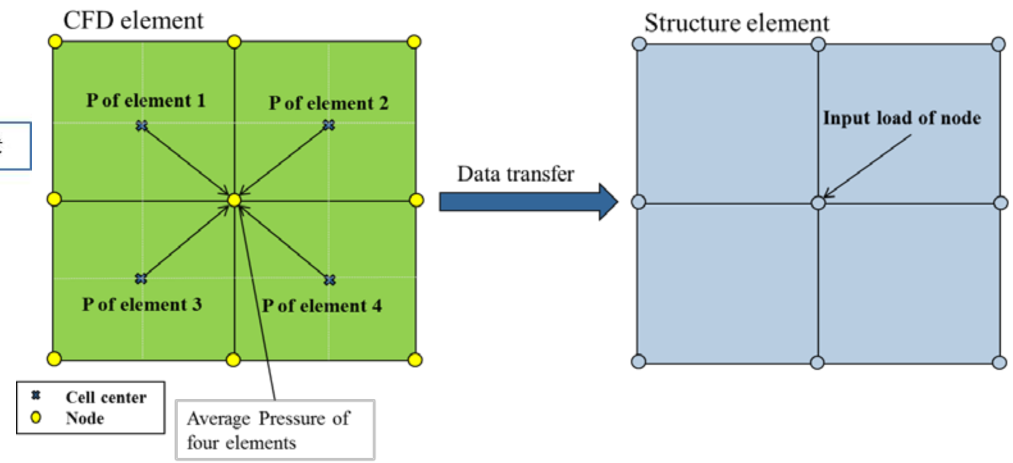
▲ **CFD-CSD interaction program**

CFD code

- Euler equation
- Compressible inviscid flow
- Steady/Unsteady

CSD code

- Parallel FETI-local
- Static/Transient (Newmark)



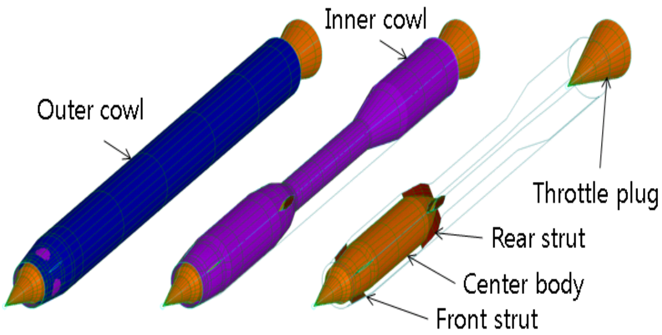
▲ **Matched grid methodology used in the present FSI analysis**

Interface

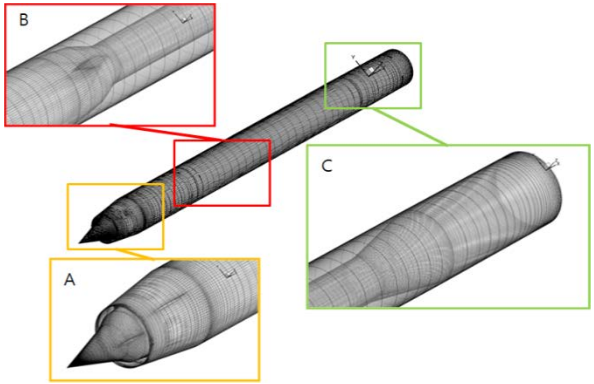
- Matched grid (high stability of interpolation)
- Loosely coupled
- Pressure data: CFD→CSD
- Deformation data: CSD→CFD

Application to the FSI analysis

❖ Analytical model in FSI analysis



▲ Engine geometry and surface model



▲ Engine geometry and surface model

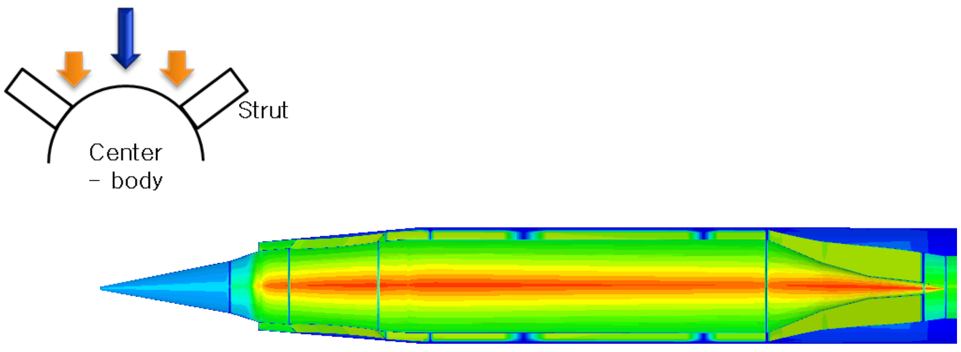
▼ **Input data of the structure**

Thickness (mm)	2
Elastic Modulus (GPa)	116
Poisson Ratio	0.31
Total No. of subdomain	28
Total No. of DOF	222,858

- An axisymmetric engine configuration.
- Free-stream Mach number is 2.0 and the atmospheric pressure is referred to the standard sea level atmosphere.
- 0.31 throttling ratio and zero angle of attack.
- The three-dimensional grid system consists of 100 blocks and about 1 million grids.
  - Physical time step for CFD is 40 μsec.
  - Physical time step for CSD is 400 μsec.

Application to the FSI analysis

❖ Structural results



▲ **Deformation history of the present FSI analysis**

- Maximum average von Mises stress is found to be 42 MPa at the rear center body.
- The magnitude of the maximum von Mises stress is found to be 61 MPa at the front center body (tensile yield stress, 434 MPa).
- The main factor that decides dominant frequency is length of the inlet.

▼ **Dominant frequency**

	n=1	**n=2**	**n=3**	**n=4**	**n=5**
Theory [Newsome, 1984]	33.97	56.61	79.25	101.9	124.5
Fluid	24.51	49.02	73.8	98	122
FSI	25.23	50.46	75.70	100.93	125

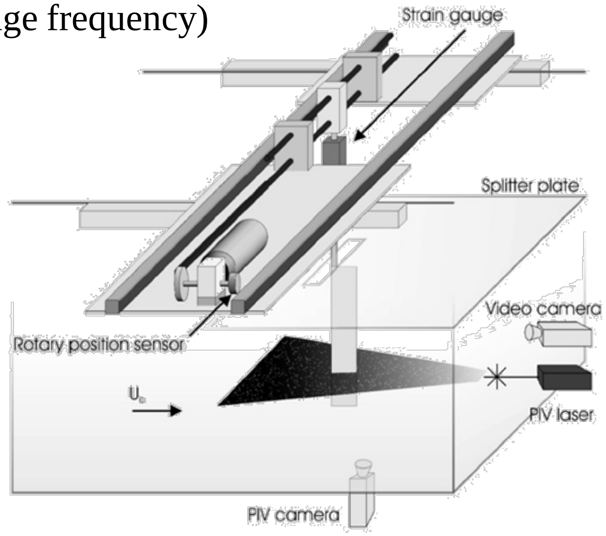
The FSI analysis using CR elements

❖ The FSI analysis for NACA0012 plunge wing

▼ **Operating condition**

Reynolds number	30000
Flow velocity (m/s)	10
Water density (kg/m^3)	1000
Plunge amplitude (m)	0.0175
Reduced frequency	1.82

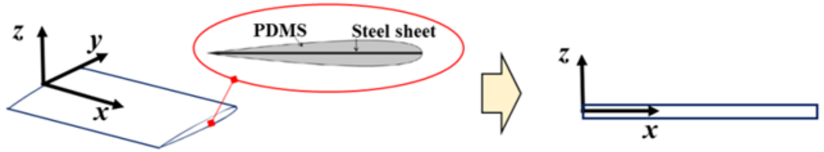
- Reduced frequency, $k_G = \dfrac{\pi f c}{U_\infty}$,
  (f = plunge frequency)



▲ **Experiment of NACA0012 plunge wing [Heathcote, Univ. of Bath (2008)]**

▼ **Wing structural properties**

	Value		**Value**
Semi-span width (m)	0.3	**Poisson's ratio**	0.3
Chord length (m)	0.1	**Material density (kg/m^3)**	7800
Thickness (m)	0.001	**Young's modulus (GPa)**	210

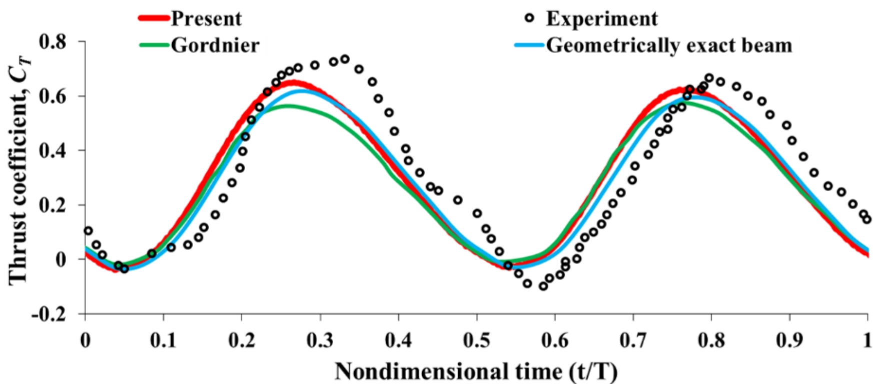


▲ **Schematic of the present flapping wing structural analysis**

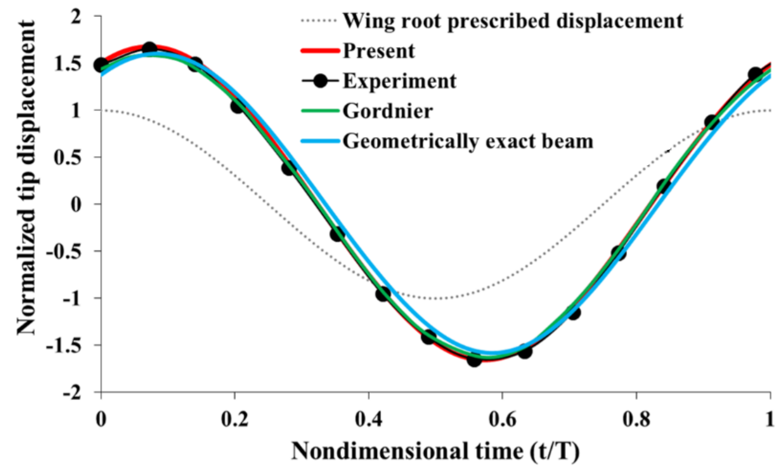
- CR planar element was employed for the present FSI analysis.

The FSI analysis using CR elements

❖ Aerodynamic and structural results



▲ Thrust coefficient response

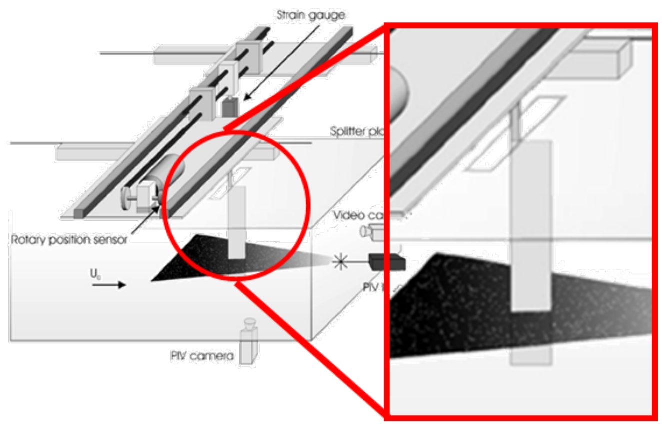


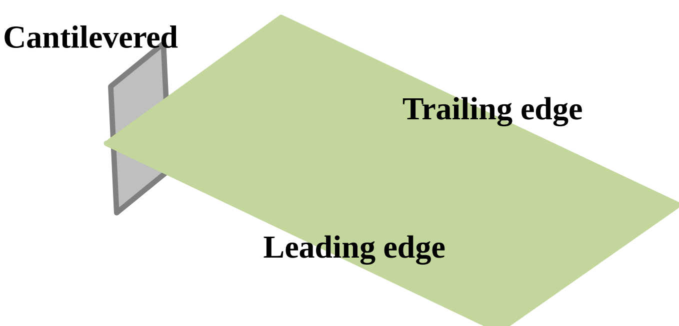
▲ Wing tip displacement response

- Both thrust coefficient and wing tip displacement response show good correlation with experimental results.
- Currently, **CR shell element is developed and it will be applied for the FSI analysis by including the presently improved FETI approach.**

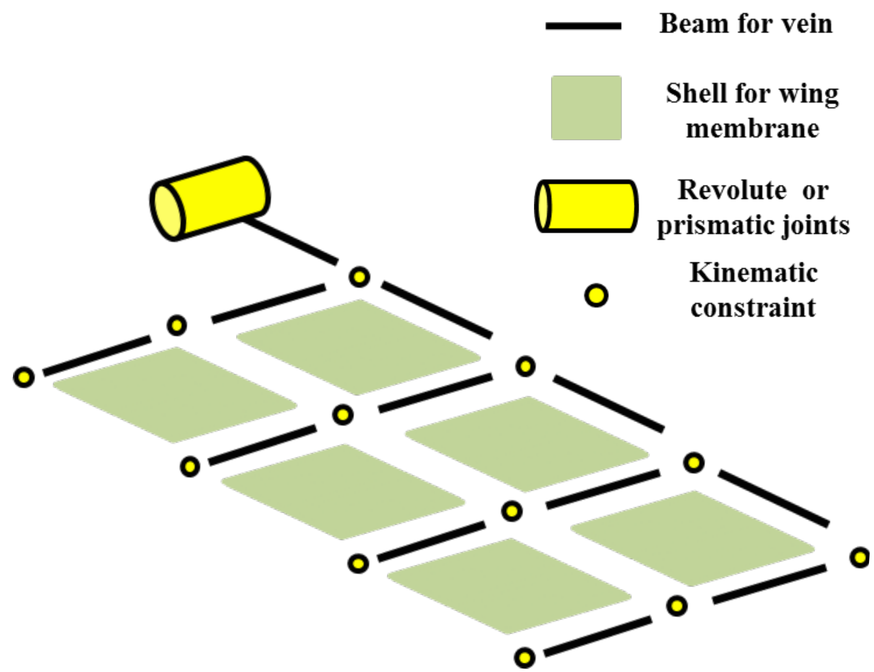
Application of the CR shell and proposed approach

❖ Modeling examples using the CR shell and proposed FETI approach





▲ Consideration of realistic geometrical boundary condition and efficient computation



▲ Multibody approach to modeling the flapping wing

- Introduction
- Formulations
- Numerical results
- **Conclusions and Future works**

Conclusions

- An efficient domain decomposition method capable of large-size structural analysis is developed.
  - The general DDM is performed first, and the ALF is used to enforce continuity of the displacement field at the sub-domain interface.
  - The proposed approach with localized Lagrange multiplier approach is introduced.

- The solution strategy and the computational algorithm of the proposed approach are developed.
  - The proposed approach proceeds in three computational steps.
  - The proposed FETI-local methodology is implemented in a parallel computing hardware using MPI.

- Condition number of the interface system matrix of the proposed methods approached unity.

Conclusions

- The proposed approach is implemented in the parallel hardware.
  - The overall behavior of the proposed parallel algorithm is better than that of the original FETI-DP.
  - The proposed approach has an advantage that a parallel solver for linear equations can be implemented easily for the interface problem.
  - The scalability characteristics of the proposed FETI-local is compared for the various examples. (80-92 % of parallel efficiency is achieved)

- The proposed approach is improved by applying for the nonlinear structural analysis.
  - The scalability characteristics of the proposed approach is examined by various examples.

Future works

- The computational costs can be reduced by using characteristics of a sparse matrix for the proposed FETI-mixed algorithm.
- The solution strategy for the interface problem will be discussed.
- It is expected that the proposed approach will be extended to the time-transient solution of the nonlinear kinematic constraints.

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