# Parallel Domain Decomposition Algorithms for Blood Flows in Patient-specific Cerebral Arteries

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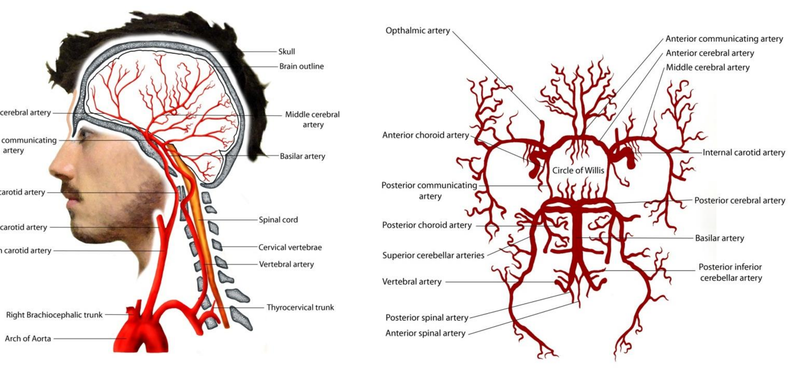
## Abstract

We develop parallel domain decomposition algorithms to simulate blood flows in the patient-specific cerebral artery using geometric information obtained by standard medical imaging techniques. The complicated patient-specific geometry in the human brain makes the problem challenging. We use a Galerkin/least-squares(GLS) finite element method to discretize the 3D incompressible Navier-Stokes equations, which are employed to model the blood flow, and the resulting large sparse nonlinear system of equations is solved by a Newton-Krylov-Schwarz algorithm. From the computed flow fields, we are able to understand some of the behavior of the blood flow in the dysfunctional territories.

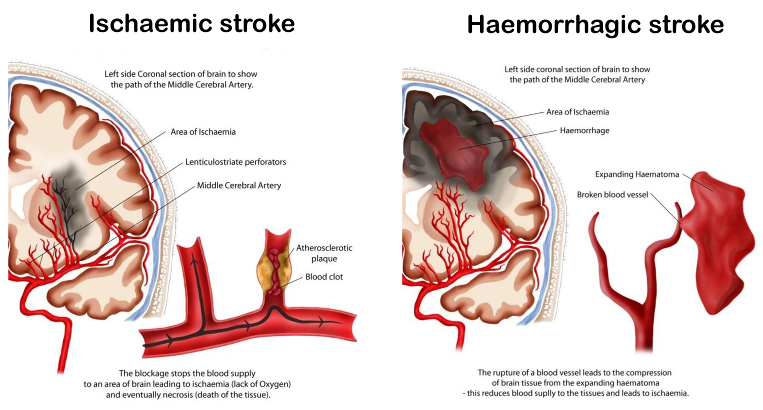
## Introduction

The cerebrovascular system

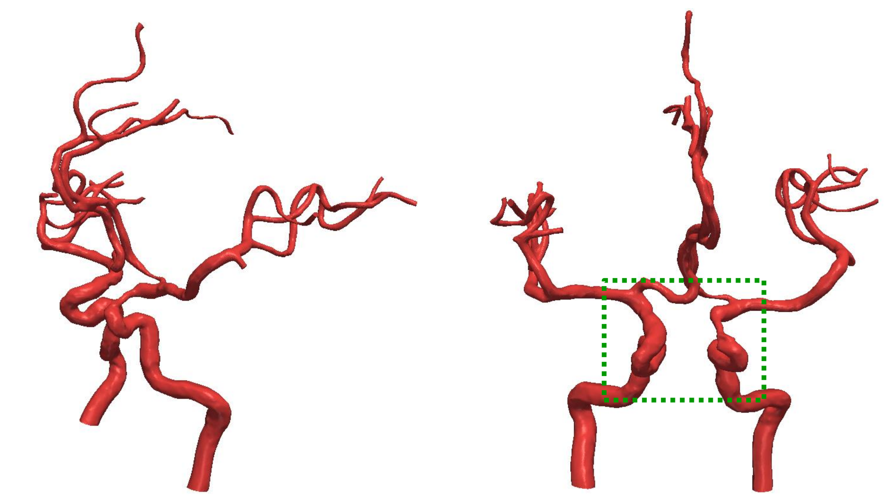


Source: https://neuro4students.wordpress.com/pathophysiology/

Cerebrovascular accident

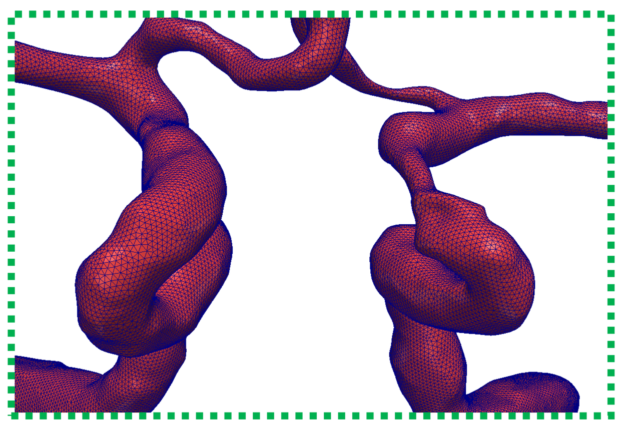



### 3D reconstruction of patient-specific cerebrovascular geometry



- Data from Beijing Tian Tan Hospital

Unstructured gird



- with 964674 tetrahedral elements and 210652 nodes

## Computational Techniques

Consider the 3D time-dependent incompressible Navier-Stokes equations:

$$\begin{cases} \rho\left(\frac{\partial u}{\partial t} + u\cdot\nabla u\right) - \nabla\cdot\sigma = 0 & \text{in } \Omega\times(0,T) \\ \nabla\cdot u = 0 & \text{in } \Omega\times(0,T) \\ u = g & \text{on } \Gamma_{in}\times(0,T) \\ u = 0 & \text{on } \Gamma_{wall}\times(0,T) \\ \sigma\cdot n = 0 & \text{on } \Gamma_{out}\times(0,T) \\ u = u_0 & \text{in } \Omega \text{ at } t=0 \end{cases}$$

where $\sigma$ is the Cauchy stress tensor defined as

$$\sigma = -pI + 2\mu D$$

here $D$ is the deformation tensor defined as

$$D = \frac{1}{2}[\nabla u + (\nabla u)^T]$$

**Discretization**

- **the temporal domain:** implicit backward Euler finite difference method
- **the spatial domain:** P1-P1 stabilized GLS finite element method

Find $u_h^{n+1} \in V_h^g$ and $p_h^{n+1} \in P_h$ such that

$$B(u_h^{n+1}, p_h^{n+1}; v, q) = 0 \quad \forall(v,q) \in V_h^0 \times P_h$$

with

$$B(u,p;v,q) = \left(\rho\left(\frac{u-u^n}{\Delta t}\right), v\right) + ((\rho\nabla u)\cdot u, v) + (\mu\nabla u, \nabla v) - (\nabla\cdot v, p) - (\nabla\cdot u, q)$$
$$+ \sum_{K\in\mathcal{T}^h}\left(\rho\left(\frac{u-u^n}{\Delta t}\right) + (\nabla u)\cdot u + \nabla p - 2\mu\nabla\cdot\nabla u, \tau\left((\nabla v)\cdot v + \nabla q - 2\mu\nabla\cdot\nabla v\right)\right)_K$$
$$+(\nabla\cdot u, \delta\,\nabla\cdot v).$$

Note: $\tau$ and $\delta$ are the stabilization parameters suggested in (Franca and Frey, 1992)

The GLS formulation can be written as a large, sparse, nonlinear algebraic system

$$F(x) = 0,$$

where the vector $x$ corresponds to both the nodal velocity $u_h$ and pressure $p_h$ at time $t = (n+1)\Delta t$.

**Newton-Krylov-Schwarz algorithm**

- **Newton:** Inexact Newton's method

$$x^{(k+1)} = x^{(k)} + \lambda^{(k)} s^{(k)}, \quad \text{where } \lambda^{(k)} \in (0,1]$$

- **Krylov:** Krylov subspace method such as GMRES

$$J_k s^{(k)} = -F(x^{(k)}), \quad \text{with } s^{(k)} = M_k^{-1} y$$

- **Schwarz:** One-level additive Schwarz preconditioner

$$M_k^{-1} = \sum_{i=1}^{N} \left(R_i^h\right)^T J_i^{-1} R_i^h$$

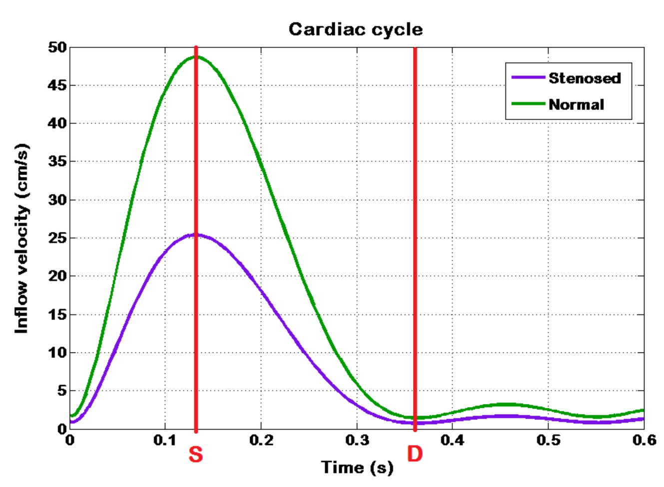
In this work,

- Domain decomposition with 24 subdomains, each subdomain is assigned to a core.
- Overlapping size for the Schwarz preconditioner is set to be 2.
- Subdomain linear system is solved by a sparse ILU(1) decomposition method.
- The Jacobian system is solved inexactly by using an additive Schwarz preconditioned GMRES with forcing term $10^{-4}$.
- The convergence of the NKS algorithm when the absolute tolerance $10^{-10}$ or the relative tolerance $10^{-6}$ is satisfied.

## Simulation results

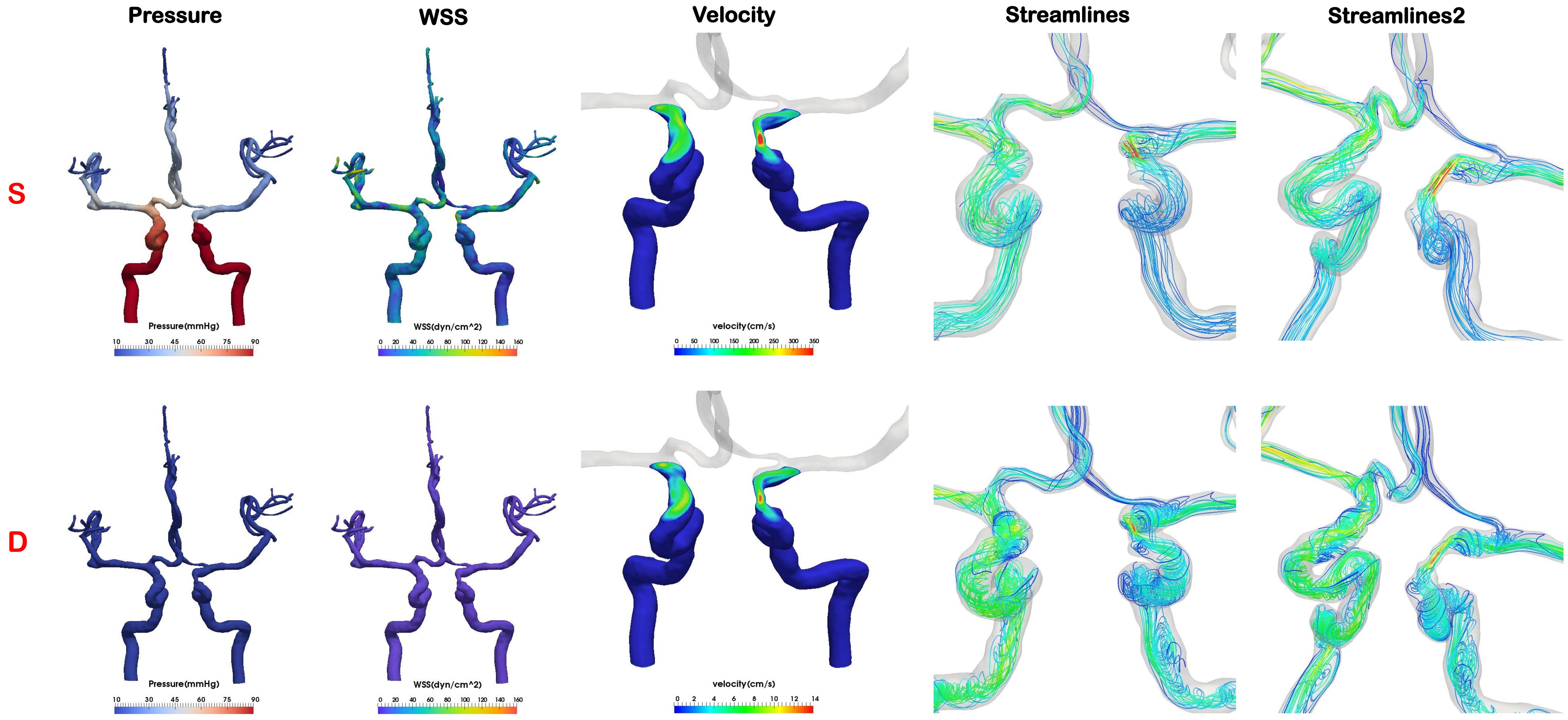
Comparison of the blood behavior of normal artery and stenosed artery during systolic and diastolic periods.

- Viscosity $\mu = 0.0345\ ^{g}/_{cm\,s}$.
- Density $\rho = 1.06\ ^{g}/_{cm^3}$.
- Using 240 timesteps per cycle.
- Inflow velocity profile:




| Time point | Normal | Stenosed |
|---|---|---|
| S (t=0.1325) | 48.66 | 25.39 |
| D (t=0.3625) | 1.42 | 0.74 |

- Take snapshots at the instants of systole (S) and diastole (D).




## Concluding remarks

- At the instant of systole, the stenosed artery has larger pressure ratio between inlet and distal portions than normal artery.
- Stenosed artery segment also has larger wall shear stress, and the maximal speed appears at stenosed artery segment.
- From the streamline plots, blood flow looks more disordered during the diastolic period than the systolic period. As the flow speed decreases to a valley of the velocity profile, the motion of blood flow becomes helical rotation at bending segments.
- Future work: add the non-Newtonian fluid property, and move on to fluid-structure interaction method.