Aitken-Schwarz Heterogeneous Domain Decomposition for EMT-TS Simulation

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1 Introduction

The introduction of renewable energies into the power grid leads to the use of more components based on power electronics which have to be well dimensioned in order not to be damaged by electrical disturbances. These components imply faster dynamics, for power system safety simulations, which cannot be handled by traditional Transient Simulations (TS) with dynamic phasors. Nevertheless, for large power grids, it can be expected that the need of high level details requiring Electro-Magnetic Transient (EMT) modeling will be localized close to disturbances, as other parts of the network still use TS modeling. This paper deals with a proof of concept to develop heterogeneous Schwarz domain decomposition with different modeling (EMT-TS) between the sub-domains. Hybrid (Jacobi type) EMT-TS co-simulation has to face several locks [4]: EMT and TS do not use the same time step size, the transmission of values is also a problem as the solutions do not have the same representation and are subject to some information loss. Our approach don't use waveform relaxation [5], and the domain partitioning is not based on cutting the transmission lines [2, 6, 7] as we want to be able to define an overlap between the two representations. On the contrary, we want to use the traditional Schwarz DDM but also where the transmission conditions can lead to divergent DDM. The pure linear convergence/divergence of the linearized problems is then used to accelerate the convergence to the solution by the Aitken's technique. In Section 2, we describe the EMT and TS modeling and perform homogeneous Schwarz DDM accelerated by the Aitken's acceleration of the convergence technique. Section 3 gives behavior results obtained for each modeling. Section 4 describes the heterogeneous EMT-TS DDM and gives first results obtained before concluding in section 5

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2 EMT and TS modeling

Simulation of power grid consists in solving a system of differential algebraic equations (DAE) where the unknowns are currents and voltages. This system is built using the Modified Augmented Nodal Analysis [8] where each component of the grid contributes through relations between currents and voltages and the Kirshoff's laws give the algebraical constraints. Let x (respectively y) be the differential (respectively algebraical) unknowns. For the EMT modeling, we have to solve the DAE:

$$F(t, x(t), \dot{x}(t), y(t)) = 0, \text{ with Initial Conditions.} \tag{1}$$

The linearized BDF time discretization of (1) (Backward Euler here) leads to solve the linear system (2) to integrate the state space representation of the DAE from time step t^n to time step t^{n+1} (operator $\mathbb{I}$ represents the difference between two potentials or the identity for intensity variables, G represents the voltage/intensity sources) :

$$\underbrace{\begin{pmatrix} \mathbb{I} - \Delta t A & B \\ C & D \end{pmatrix}}_{\mathbf{H_{\Delta t}}} \begin{pmatrix} x^{n+1} \\ y^{n+1} \end{pmatrix} = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x^n \\ y^n \end{pmatrix} + G^{n+1}. \tag{2}$$

For TS modeling the variables are assumed to oscillate with a specific angular frequency $\omega_0 = \dfrac{2\pi}{T}$ (where T is the period) and its selected harmonics taken from a subset $I = \{\ldots, -1, 0, 1, \ldots\}$:

$$z(t) = \sum_{k \in I} z_k(t) e^{ik\omega_0 t}, \; z = \{x, y\}. \tag{3}$$

Introducing (3) into (1) leads after simplification (i.e orthogonality of the functions $e^{ik\omega_0 t}$ with respect to the dot product $[f, g] = \dfrac{1}{T}\int_t^{t+T} f(z)g(z)dz$) to another DAE system that takes into account the differential property of the dynamic phasor . The resulting DAE system has smoother dynamics. The number of TS variables is then multiplied by the number of harmonics chosen, and the number of equations must be multiplied accordingly.

For example, on the right is the structure of the matrix H_{TS} by choosing two harmonics $k = a$ and $k = c$ and by solving the imaginary and real part separately and with $\mathbf{S}$ the matrix taking into account the differential property of the dynamic phasor modeling.

$$\mathbf{H}_{TS} = \left(\begin{array}{c|c} \begin{array}{c|c} \mathbf{H}_{\Delta T} & -a\,\omega_0\,\mathbf{S} \\ \hline a\,\omega_0\,\mathbf{S} & \mathbf{H}_{\Delta T} \end{array} & 0 \\ \hline 0 & \begin{array}{c|c} \mathbf{H}_{\Delta T} & -c\,\omega_0\,\mathbf{S} \\ \hline c\,\omega_0\,\mathbf{S} & \mathbf{H}_{\Delta T} \end{array} \end{array}\right).$$

Let x_T^{n+1} (respectively x_E^{n+1}) be the algebraic and differential unknowns of TS (respectively EMT) modeling associated to the linear system $H_{TS}x_T^{n+1} = b_T^n$ (respectively $H_E x_E^{n+1} = b_E^n$).

3 EMT and TS Schwarz homogeneous DDM

We consider a linear RLC circuit of Figure 1 to develop the proof of concept of the the Schwarz DDM on TS and EMT models.

By adapting the notations of [1], we consider a non-singular matrix $H \in \mathbb{R}^{n\times n}$ having a non-zero pattern and the associated directed graph $G = (W, F)$, where the set of vertices $\Omega = \{1, , n\}$ represents the n unknowns and the set of edges $F = \left\{(i, j)|a_{i,j} \neq 0\right\}$ represents the pairs of vertices that are coupled by a non-zero element in H. Next, we assume that a graph partitioning was applied and resulted in N non-overlapping subsets Ω_i^0 whose union is Ω. Let Ω_i^p be the p-overlap partition of Ω, obtained by including all the vertices immediately neighboring the vertices of Ω_i^{p-1}. Let $R_i^p \in \mathbb{R}^{n_i\times n}$ be the operator which restricts $x \in \mathbb{R}^n$ to the components of x belonging to Ω_i^p. Let $\tilde{R}_i^0 \in \mathbb{R}^{n\times n}$ be the operator which restricts $x \in \mathbb{R}^n$ to the components of x belonging to Ω_i^0 and 0 otherwise. Let $\Omega_{i,e}^p = \Omega_i^{p+1}\backslash\Omega_i^p$ and $R_{i,e}^p \in \mathbb{R}^{n_{i,e}\times n}$ the restriction operator which restricts $x \in \mathbb{R}^n$ to the components of x belonging to $W_{i,e}^p$. By defining $H_i = R_i^p H R_i^{pT}$, $F_i = R_i^p H (R_{i,e}^p)^T$, $x_i = R_i^p x$ and $\mathrm{b}_i = R_i^p b$, $x_{i,e} = R_{i,e}^p x$, then the Restrictive Additive Schwarz (RAS) iteration $k + 1$ to solve $Hx^\infty = b \in \mathbb{R}^n$ is written locally for the Ω_i^p partition :

$$x_i^{k+1} = H_i^{-1}(b_i - F_i x_{i,e}^k). \tag{4}$$

The previous paragraph presents the general way of proceeding and among other things to set up the overlap. However, in this work we have chosen another overlap for optimization reasons, because of the small size of our circuit.

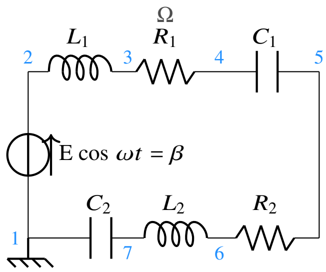
The small linear system associated with the RLC circuit is partitioned into two subdomains using graph partitioning without overlap (Figure 2 top) and with an overlap of 1 (Figure 2 bottom). Each subdomain needs two values from the other to solve its equations.

The RAS applied to each time step has a pure linear convergence i.e. the error operator P does not depend on the RAS iteration.

$$x^{m+1,p+1} - x^{m+1,\infty} = P(x^{m+1,p} - x^{m+1,\infty}). \tag{19}$$

Thus, if it does not stagnate, it can be accelerated with the Aitken's acceleration of the convergence, using (19), to obtain the true solution regardless of its convergence or divergence [3] :

$$x^{m+1,\infty} = (I_d - P)^{-1}(x^{m+1,1} - Px^{m+1,0}). \tag{20}$$



$$v_1 = 0, \tag{5}$$
$$v_2 - v_1 - E - Z_s i_{12} = 0, \tag{6}$$
$$v_3 - v_2 - L_1 \frac{di_{23}}{dt} = 0, \tag{7}$$
$$v_4 - v_3 - R_1 i_{34} = 0, \tag{8}$$
$$C_1\left(\frac{dv_5}{dt} - \frac{dv_4}{dt}\right) - i_{45} = 0, \tag{9}$$
$$v_6 - v_5 - R_2 i_{56} = 0, \tag{10}$$
$$v_7 - v_6 - L_2 \frac{di_{67}}{dt} = 0, \tag{11}$$
$$C_2\left(\frac{dv_1}{dt} - \frac{dv_7}{dt}\right) - i_{71} = 0, \tag{12}$$
$$i_{12} - i_{23} = 0, \tag{13}$$
$$i_{23} - i_{34} = 0, \tag{14}$$
$$i_{34} - i_{45} = 0, \tag{15}$$
$$i_{45} - i_{56} = 0, \tag{16}$$
$$i_{56} - i_{67} = 0, \tag{17}$$
$$i_{67} - i_{71} = 0. \tag{18}$$

Fig. 1: Linear RLC circuit and its associated EMT modeling DAE system with $x = \{v_1, i_{23}, v4, v5, i_{67}, v_7\}$ and $y = \{v_2, i_{12}, v_3, i_{34}, i_{45}, i_{56}, v_6, i_{71}\}$. $L_1 = L_2 = 0.7$, $C_1 = C_2 = 1.10^{-6}$, $R_1 = R_2 = 77$, $Z_s = 1.10^{-6}$, $\omega = 2\pi\, 50$, $E = 5$.

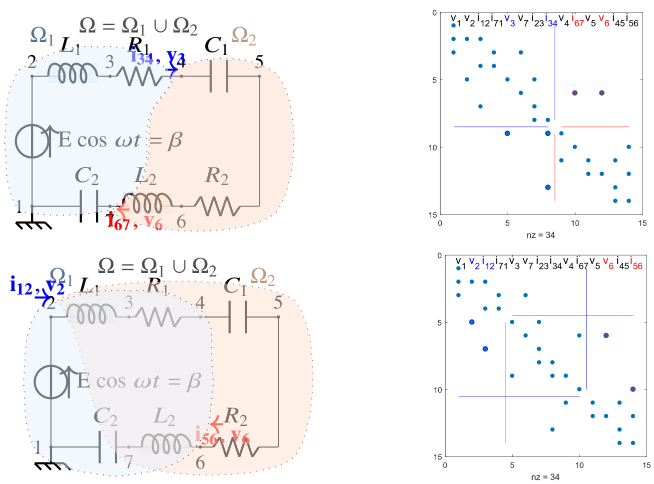


Fig. 2: Graph partitioning of the RLC circuit in two subdomains and the associated matrix partioning without overlap (top) and with overlap of 1 (bottom).

For this small problem it can be directly computed working on the matrix partitioning.

$$P = -[(\tilde{R}_1)^t A_1^{-1} E_{1,e} R_{1,e} + (\tilde{R}_2)^t A_2^{-1} E_{2,e} R_{2,e}]. \tag{21}$$

Table 1 gives the larger eigenvalue in modulus for the P RAS (Restricted Additive Schwarz) error operator for the EMT modeling and for the P RAS error operator for the TS modeling harmonics $k = 0, 1$ applied to the RLC circuit. In

$\lambda(P)$	without overlap	with overlap	Schwarz	time step
EMT	± 6.0638i	± 6.0638i	RAS	2.10^{-4}
TS $_{k=1}$	±0.3667 ± 6.0635i	±0.3667 ± 6.0635i	RAS	2.10^{-4}
TS $_{k=0}$	±6.0638i	±6.0638i	RAS	2.10^{-4}
TS $_{k=1}$	± 0.2114 ±1.1548i	± 0.2114 ±1.1548i	RAS	2.10^{-3}
TS $_{k=0}$	±1.1946i	±1.1946i	RAS	2.10^{-3}

Table 1: Larger eigenvalue for P error operator for RAS and EMT modeling ($\Delta t = 2.10^{-4}$), and for RAS and TS $k = 0, 1$ ($\Delta T = 2.10^{-4}$, $\Delta T = 2.10^{-3}$) modeling.

both cases EMT and TS modeling the eigenvalue modulus is greater than one, so the method diverges. We can observe that the overlap does not impact the divergence of the method. The time step increasing from $\Delta T = 2.10^{-4}$ to $\Delta T = 2.10^{-3}$ has a beneficial effect on the TS-TS DDM divergence. Nevertheless, the divergence is purely linear and the Aitken's acceleration (20) can be performed after the first iteration if P is known (here by Eq(20)). If P is unknown, the pure linear convergence property also hold for the solution iterated at the global artificial interface $\Gamma = \left\{ y \in \mathbb{R}^{n_\Gamma} | y = (x_{1,e}^T, x_{2,e}^T)^T \right\}$. Let R_Γ be the restriction operator from $\mathbb{R}^n$ to Γ, $y^{m+1,p} = R_\Gamma x^{m+1,p}$ be the RAS iterated solution restricted to Γ and $e^{p+1} = y^{m+1,p+1} - y^{m+1,p}$ be the error between two consecutive iterations. Then, from $e^{p+1} = P_\Gamma e^p$, one can build $P_\Gamma = [e^{n_\Gamma+1}, \ldots, e^2][e^{n_\Gamma}, \ldots, e^1]^{-1}$ with $n_\Gamma + 1$ RAS iterations and the true solution at interface $y^{m+1,\infty}$ is obtained with $y^{m+1,\infty} = (I_d - P_\Gamma)^{-1}(y^{m+1,n_\Gamma+1} - P_\Gamma y^{m+1,n_\Gamma})$. Then one local solve gives $x^{m+1,\infty}$.

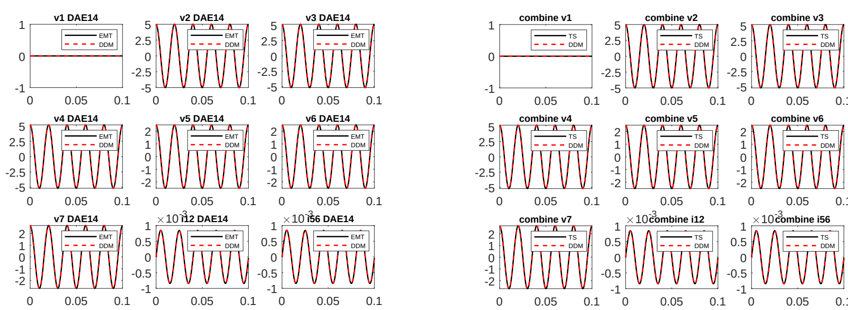


Fig. 3: Homogeneous DDM results comparison with DAE monodomain: (Left) RAS for EMT modeling with $\Delta t_E = 1.10^{-4}$ and (right) RAS for TS modeling with $\Delta t_T = 2.10^{-3}$.

4 Heterogeneous DDM EMT-TS

Our goal is to simulate, using heterogeneous RAS DDM, the electrical network with one part with a TS modeling which can use large time steps ΔT and the other part with the EMT modeling which requires smaller time steps Δt as the high oscillations remain.

These two representations TS and EMT of the solution imply having some operators E_{emt}^{TS} (respectively E_{TS}^{emt}) to transfer the solution from the subdomain EMT (respectively TS) to the other TS (respectively EMT). The E_{TS}^{emt} operator needs to compute the fundamental harmonic and other harmonics chosen of the solution from the history of the EMT solution. The history time length is one period. This is performed by the FFT of the solution over the time period and keeping the mode corresponding to the chosen harmonics.

The E_{emt}^{TS} operator is more simple as it consists in recombining the TS modes of the solution with the appropriate Fourier basis modes.

Let us consider a linear electrical network with the TS modeling. The time discretisation of the DAE to integrate from T^N to T^{N+1}, assuming that $\Delta T = m\Delta t$ can be witten as:

$$\underbrace{\begin{pmatrix} \mathbb{I} - \Delta T A_{TS} & B_{TS} \\ C_{TS} & D_{TS} \end{pmatrix}}_{H_{TS}} \underbrace{\begin{pmatrix} x_{TS}^{N+1} \\ y_{TS}^{N+1} \end{pmatrix}}_{w_{TS}^{N+1}} = \underbrace{\begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix}}_{\Theta_{TS}} \begin{pmatrix} x_{TS}^{N} \\ y_{TS}^{N} \end{pmatrix} + \underbrace{\begin{pmatrix} E_{TS}^{A} & E_{TS}^{B} \\ E_{TS}^{C} & E_{TS}^{D} \end{pmatrix}}_{E_{TS}^{emt}} \begin{pmatrix} x_{emt}^{m} \\ y_{emt}^{m} \end{pmatrix} + G_{TS}^{N+1}.$$

Similarly one time step for the EMT side to integrate from t^n to t^{n+1} can be witten as:

$$\underbrace{\begin{pmatrix} \mathbb{I} - \Delta t A_{emt} & B_{emt} \\ C_{emt} & D_{emt} \end{pmatrix}}_{H_{emt}} \underbrace{\begin{pmatrix} x_{emt}^{n+1} \\ y_{emt}^{n+1} \end{pmatrix}}_{w^{n+1}} = \underbrace{\begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix}}_{\Theta_{emt}} \begin{pmatrix} x_{emt}^{n} \\ y_{emt}^{n} \end{pmatrix} + \underbrace{\begin{pmatrix} E_{emt}^{A} & E_{emt}^{B} \\ E_{emt}^{C} & E_{emt}^{D} \end{pmatrix}}_{E_{emt}^{TS}} \underbrace{\begin{pmatrix} x_{TS}^{N+1}(t^{n+1}) \\ y_{TS}^{N+1}(t^{n+1}) \end{pmatrix}}_{W^{N+1}(t^{n+1})} + G_{emt}^{n+1}.$$

The m time steps can be gathered in one larger system considering $t^n = T^N$:

$$\underbrace{\begin{pmatrix} I & & & & \\ -\Theta_{emt} & H_{emt} & & & \\ & \ddots & \ddots & & \\ & & -\Theta_{emt} & H_{emt} & \\ & & & -\Theta_{emt} & H_{emt} \end{pmatrix}}_{\mathbb{H}_{emt}} \underbrace{\begin{pmatrix} w_{emt}^{n} \\ w_{emt}^{n+1} \\ \vdots \\ w_{emt}^{n+m-1} \\ w_{emt}^{n+m} \end{pmatrix}}_{\mathbb{W}_{emt}} =$$

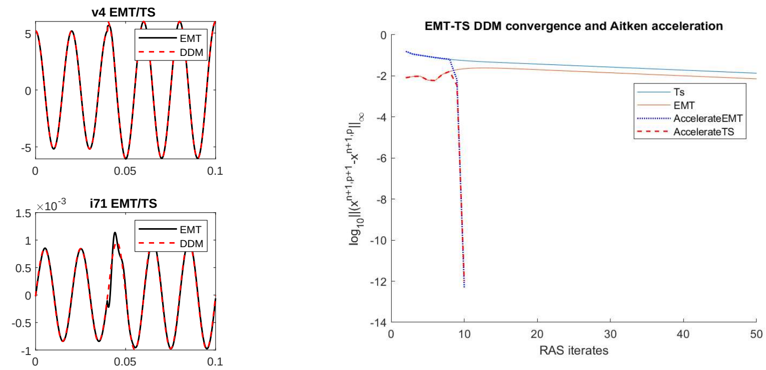


Fig. 4: Heterogeneous EMT ($\Delta t = 2.10^{-4}$)-TS($\Delta T = 2.10^{-2}$) DDM results comparison with DAE monodomain (Left) and RAS convergence error for each subdomain at $t = 0.02$ and its Aitken's acceleration with P_Γ computed numerically from 9 iterates ($n_\Gamma = 8$) (right).

$$\underbrace{\begin{pmatrix} I & & & & \\ & E_{emt}^{TS} & & & \\ & & \ddots & & \\ & & & E_{emt}^{TS} & \\ & & & & E_{emt}^{TS} \end{pmatrix}}_{\mathbb{E}_{emt}^{TS}} \underbrace{\begin{pmatrix} (x^n, y^n)^t \\ W^{N+1}(t^{n+1}) \\ \vdots \\ W^{N+1}(t^{n+m-1}) \\ W^{N+1}(t^{n+m}) \end{pmatrix}}_{\mathbb{W}_{TS}^{N+1}} + \underbrace{\begin{pmatrix} 0 \\ G_{emt}^{n+1} \\ \vdots \\ G_{emt}^{n+m-1} \\ G_{emt}^{n+m} \end{pmatrix}}_{\mathbb{G}_{emt}^{N+1}}. \tag{22}$$

This system needs the values that the TS solution connected to the EMT part has taken on the small time steps. The two domains are connected via the connected or flowing variables. Since these variables should be the solution at time T^{N+1}, we need the Schwarz iterative algorithm to obtain the exact values. We then iterate the iteration $p + 1$ by taking the connected values, at the iteration p, from the other subdomain. We can use the multiplicative form or the additive form as follows:

$$\begin{cases} H_{TS}\, w_{TS}^{N+1,\mathbf{p+1}} &= \Theta_{TS}\, w_{TS}^{N} + E_{TS}^{emt}\, w_{emt}^{m,\mathbf{p}} + G_{TS}^{N+1}, \\ \mathbb{H}_{emt}\, \mathbb{W}_{emt}^{N+1,\mathbf{p+1}} &= \mathbb{E}_{emt}^{TS}\, \mathbb{W}_{TS}^{N+1,\mathbf{p}} + \mathbb{G}_{emt}^{N+1}. \end{cases} \tag{23}$$

Figure 4 (left) shows the solutions v_4 EMT and i_{71} TS of heterogeneous DDM EMT ($\Delta t = 2.10^{-4}$) -TS ($\Delta T = 2.10^{-2}$) with comparison with the DAE solution on monodomain. We proceed to a jump in amplitude at $t = 0.04$ for the voltage source. Figure 4 (right) gives the log_{10} of the error between two consecutive RAS iterates at $t = 0.02$. It shows a linear convergence behavior and can therefore be accelerated by the Aitken's accelerating of the convergence technique after 9 iterates needed to numerically construct the error operator P_Γ.

5 Conclusion

A Schwarz heterogeneous DDM was used to co-simulate an RLC electrical circuit where a part of the domain is modeled with EMT modeling and the other part with TS modeling. We showed the convergence/divergence property of the homogeneous DDM EMT-EMT and TS-TS and of the heterogeneous DDM TS-EMT, with or without overlap and we use the pure linear divergence/convergence of the method to accelerate it toward the true solution with the Aitken's acceleration of the convergence technique. The domain partitioning is only based on connectivity considerations since we want, in the long term, for the electrical network, to take advantage of the two TS and EMT representations on the overlap in order to identify the loss of information between the two models. We would like then to use this knowledge to work on other transmission conditions than Dirichlet to conserve some invariants such as electrical power.

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