



# 78° North: DD24 Svalbard Norway 2017

**February 6 – 10 2017, Longyearbyen, Norway**

**The local Organizing Committee:**

*Petter E. Bjørstad*


*Talal Rahman*

*Sushmita Gupta*

*Liv Rebecca A. Aae*



**simula**

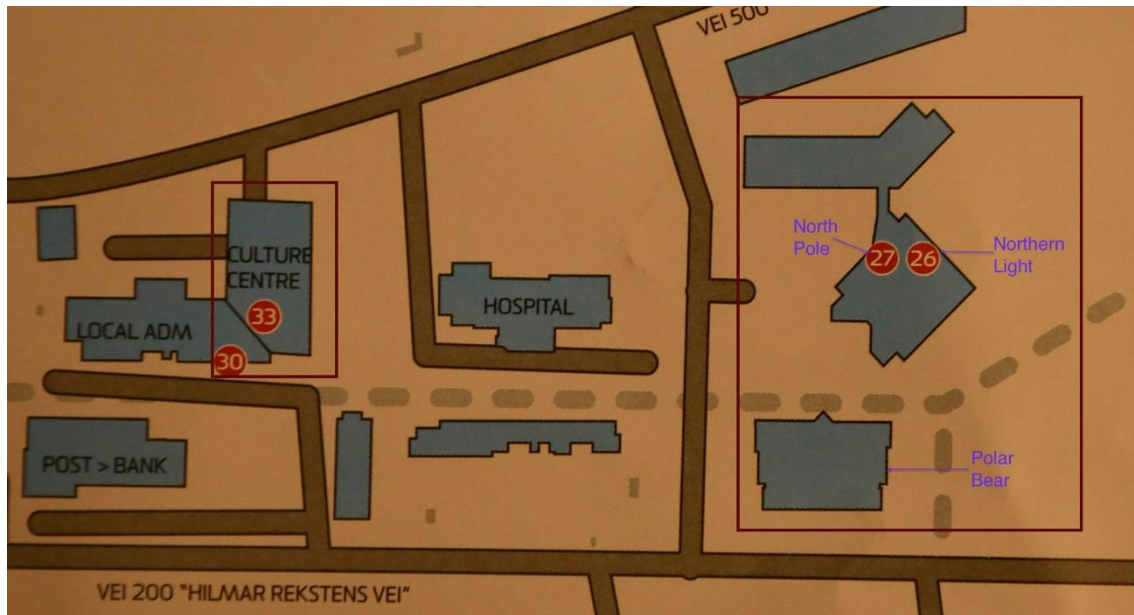
 SparebankenVest

# Map of Longyearbyen



Radisson SAS Polar Hotel, Culture Centre and Coal Miners' Cabin are shown in the map.

## Detailed Map of the Conference Area



**33 Culture Centre**

**27 North Pole**

**26 Northern Light**

**Polar Bear**

**All plenary sessions will be held in the Culture Centre/House.**

## International Scientific Committee

- **Bjørstad, Petter** (University of Bergen, Norway)
- **Brenner, Susanne** (Louisiana State University, USA)
- **Cai, Xiao-Chuan** (CU Boulder, USA)
- **Gander, Martin J.** (University of Geneva, Switzerland, CHAIR)
- **Halpern, Laurence** (Paris 13, France)
- **Keyes, David** (KAUST, Saudi Arabia)
- **Kim, Hyea Hyun** (Kyung Hee University, Republic of Korea)
- **Klawonn, Axel** (Universität zu Köln, Germany)
- **Kornhuber, Ralf** (Freie Universität Berlin, Germany)
- **Langer, Ulrich** (University of Linz, Austria)
- **Quarteroni, Alfio** (EPFL, Switzerland)
- **Widlund, Olof** (Courant Institute, USA)
- **Xu, Jinchao** (Penn State, USA)
- **Zou, Jun** (Chinese University of Hong Kong, Hong Kong)

## Local Organizing Committee

- **Aae, Liv Rebecca** (University of Bergen, Norway)
- **Bjørstad, Petter** (University of Bergen, Norway)
- **Gupta, Sushmita** (University of Bergen, Norway)
- **Rahman, Talal** (Western Norway Univ. of Appl. Sci., Norway)

## Plenary Lectures

**An additive Schwarz analysis of multiplicative Schwarz methods**

*Brenner, Susanne* (Louisiana State University, USA)

**On nonlinear adaptivity with heterogeneity**

*Brown, Jed* (University of Colorado, USA)

**Overlapping methods for high-contrast multiscale problems**

*Galvis, Juan* (Universidad Nacional de Colombia, Colombia)

**Domain Decomposition for high frequency Helmholtz problems**

*Graham, Ivan* (University of Bath, UK)

**Communication avoiding iterative solvers and preconditioners**

*Grigori, Laura* (INRIA, France)

**PDE based mesh generation: domain decomposition approaches**

*Haynes, Ronald* (Memorial University of Newfoundland, Canada)

**Robust preconditioners for coupled problems**

*Hu, Xiaozhe* (Tufts University, USA)

**Modeling and discretization of thin inclusions for flow in deformable porous media**

*Nordbotten, Jan* (University of Bergen, Norway)

**Domain decomposition based methods for multiphysics problems**

*Quarteroni, Alfio* (EPFL, Switzerland)

**Impact of high abstraction/high performance finite element software in biomedical computing**

*Rognes, Marie* (SIMULA, Norway)

**Scalable multilevel preconditioners for cardiac electro-mechanics**

*Scacchi, Simone* (University of Milan, Italy)

**Recent advances on adaptive multilevel BDDC methods for div- and curl-conforming spaces**

*Zampini, Stefano* (KAUST, Saudi Arabia)

## Minisymposia

**MS01: Achieving robustness with new coarse spaces and enlarged Krylov subspaces**

*Organizers: Martin J. Gander and Nicole Spillane*

**MS02: Fast solvers for discontinuous Galerkin methods**

*Organizers: Paul Houston, Iain Smears, and Paola Antonietti*

**MS03: Domain decomposition methods in biomedical modelling and simulation**

*Organizers: Ralf Kornhuber, Rolf Krause, and Alfio Quarteroni*

**MS04: Optimized transmission conditions in domain decomposition methods**

*Organizers: Martin J. Gander and Yingxiang Xu*

**MS05: Parallel solvers for isogeometric analysis**

*Organizers: Luca F. Pavarino and Simone Scacchi*

**MS06: Novel discretizations and solvers for high-contrast and multiscale problems**

*Organizers: Jan Nordbotten, Eduardo Abreu, Marcus Sarkis, and Juan Galvis*

**MS07: Recent progress on Trefftz methods**

*Organizer: Bruno Despres*

**MS08: DD-based control and control-based DD**

*Organizers: Alfio Quarteroni, Paola Gervasio, and Marco Discacciati*

**MS09: Parallel approaches for PDE based mesh generation**

*Organizer: Ronald D. Haynes*

**MS10: Domain decomposition for frequency domain wave problems**

*Organizers: Victorita Dolean and Ivan Graham*

**MS11: Domain decomposition methods for optimal control and PDE constrained optimization**

*Organizers: Martin J. Gander, Felix Kwok, and Julien Salomon*

**MS12: Robust solvers for multiphysics problems**

*Organizers: Xiaozhe Hu, Shuonan Wu, and Jinchao Xu*

**MS13: Domain decomposition methods for nonlinear problems**

*Organizers: Xiao-Chuan Cai, Leszek Marcinkowski, and Talal Rahman*

**MS14: Highly parallel domain decomposition methods and their applications**

*Organizers: Axel Klawonn and Oliver Rheinbach*

**MS15: Heterogeneous domain decomposition methods**

*Organizers: Martin J. Gander and Marco Discacciati*

**MS16: Nonsmooth and nonlinear problems**

*Organizer: Carsten Gräser and Oliver Sander*

**MS17: Time-parallel time integration methods**

*Organizers: Martin J. Gander and Rolf Krause*

**MS18: Fast solvers for space-time discretizations**

*Organizers: Ulrich Langer and Olaf Steinbach*

**MS19: Space-time parallel methods based on waveform relaxation techniques**

*Organizers: Martin J. Gander and Shu-Lin Wu*

## Contributed Talks

**An iterative domain decomposition method for eddy current problems with consideration for the gauge condition**

*Daisuke Tagami*

**An optimised domain decomposition method for large scale eigenvalue problems taking advantage of a contour integral approach**

*Nicolas Marsic*

**CFD Codes on multicore and manycore architectures**

*David Keyes*

**Parallelisation and scalability of a linear advection diffusion code**

*Emanuele Ragnoli*

**Multigrid methods for  $H(\text{div})$  with nonoverlapping domain decomposition smoothers**

*Duk-Soon Oh*

**A three-level BDDC method for incompressible Navier-Stokes equations**

*Martin Hanek*

**Solving large sparse linear systems with a variable s-step GMRES preconditioned by DD**

*Jocelyne Erhel*

**Design of small coarse spaces for two level overlapping Schwarz algorithms for problems with irregular subdomains**

*Olof B. Widlund*

**Non-local transmission operators for non-overlapping DDM with exponential convergence for Helmholtz equation**

*Matthieu Lecouvez*

**Acceleration Techniques for Multilevel FETI Methods**

*Lubomir Riha*

**An efficient and reliable stopping criterion for the solution of symmetric saddle point problems with FETI**

*Ange B. Toulougoussou*

**Scalable TFETI based domain decomposition for contact problems with variationally consistent discretization of non-penetration**

*Zdeněk Dostál*

**Spectral properties of the FETI constraint matrices based on graph theory**

*Václav Hapla*

## Posters

**Implementation of handling multiple subdomains per one computational core into PERMON toolbox to exploit fully TFETI numerical scalability**

*Radim Sojka*

**Asynchronous optimized Schwarz method for Poisson equation on rectangular domains**

*Jose C. Garay*

**A dual iterative substructuring method with a modified penalty term to decouple the inner problems**

*Jongho Park*

**A fast iterative method for multigroup radiation diffusion problems**

*Jose Pablo Lucero Lorca*

**Multigrid method for a staggered discontinuous Galerkin approximation**

*Lina Zhao*



## Overview of Schedule

### Monday, February 6th 2017

	Culture Centre	North Pole	Northern Light	Polar Bear
08:00	Breakfast served			
09:30	Bus departs from Coal Miners' Cabin			
10:00	Welcome/Opening			
10:30	PL01 Ivan Graham			Culture Centre
11:15	PL02 Jan Nordbotten			Culture Centre
12:00	Lunch at Radisson			
13:30	MS06: 1-4	MS10: 1-4	MS14: 1-4	MS17: 1-4
15:10	Coffee at Culture Centre and Radisson			
15:40	MS06: 5-8	MS10: 5:8	MS14: 5-8	MS17: 5-8
17:20	Moving/Stretching legs			
17:30	PL03 Ronald Haynes			Culture Centre
18:15	PL04 Juan Galvis			Culture Centre
19:00	End of Talks			
19:10	Bus departs for Coal Miners' Cabin			

### Tuesday, February 7th 2017

	Culture Centre	North Pole	Northern Light	Polar Bear
07:00	Breakfast served			
08:10	Bus departs Coal Miners' Cabin			
08:30	PL05 Alfio Quarteroni			Culture Centre
09:15	PL06 Marie Rognes			Culture Centre
10:00	Coffee at Culture Centre and Radisson & Introduction to Poster session			
10:30	MS06: 9-11	MS13: 1-4	MS02: 1-4	CT01: 1-4
12:10	Lunch at Radisson			
13:40	PL07 Xiaozhe Hu			Culture Centre
14:25	Moving/Stretching legs			

14:35	MS12: 1-4	MS13: 5-8	MS02: 5-7	MS15: 1-4
16:15	Coffee at Culture Centre and Radisson			
16:45	MS03: 1-5	MS09: 1-5	CT02: 1-5	MS15: 5-8
18:50	End of talks			
19:00	Bus departs for Coal Miners' Cabin			
19:15	DDM Business meeting at Hjalmar Johansen			
21:00	DDM Dinner with invited speakers at Spitsbergen Hotel 20:45 Bus from Radisson			

### Wednesday, February 8th 2017

	Culture Centre	North Pole	Northern Light	Polar Bear
06:00	Breakfast served			
07:10	Bus departs from Coal Miners' Cabin			
07:30	PL08 Susanne Brenner			Culture Centre
08:15	Moving/Stretching legs			
08:25	MS12: 5-8	MS11: 1-4	MS19: 1-4	MS07: 1-4
10:05	Coffee at Culture Centre and Radisson			
10:35	MS12: 9-11	MS11: 5-7	MS19: 5-7	MS07: 5-7
11:50	Lunch at Radisson			
	Excursion			

### Thursday, February 9th 2017

	Culture Centre	North Pole	Northern Light	Polar Bear
07:00	Breakfast served			
08:10	Bus departs from Coal Miners' Cabin			
08:30	PL09 Simone Scacchi			Culture Centre
09:15	PL10 Jed Brown			Culture Centre
10:00	Coffee at Culture Centre and Radisson			
10:30	MS01: 1-4	MS04: 1-4	MS05: 1-4	MS18: 1-4
12:10	Lunch at Radisson			

13:40	PL11 Stefano Zampini			Culture Centre
14:25	Moving/Stretching legs			
14:35	MS01: 5-8	MS04: 5 MS16: 1	MS05: 5-7	CT03: 1-4
16:15	Coffee at Culture Centre and Radisson			
16:45	MS01: 9-12	MS16: 2-5	MS08: 1-4	
18:50	End of talks			
19:00	Bus departs for Coal Miners' Cabin			
20:00	Conference Banquet 19:45 Bus from Radisson & Coal Miners' Cabin. Attention! Warm clothes			

**Friday, February 10th 2017**

	Culture Centre	North Pole	Northern Light	Polar Bear
07:00	Breakfast served			
08:40	Bus departs from Coal Miners' Cabin Attention Coal Miners! Check out before boarding the bus, take your luggage with you			
09:00	PL12 Laura Grigori			Culture Centre
09:45	Conference closing remarks at Culture Centre			
11:00	Check out from hotels			
	Bus to airport from Radisson: 11:20 (Norwegian) 12:00 (SAS)			

## Monday, February 6<sup>th</sup> 2017

	Culture Centre	North Pole	Northern Light	Polar Bear
08:00	Breakfast served			
09:30	Bus departs from Coal Miners' Cabin			
10:00	Welcome/Opening			
10:30	PL01 Ivan Graham			Culture Centre
11:15	PL02 Jan Nordbotten			Culture Centre
12:00	Lunch at Radisson			
13:30	MS06: 1-4	MS10: 1-4	MS14: 1-4	MS17: 1-4
15:10	Coffee at Culture Centre and Radisson			
15:40	MS06: 5-8	MS10: 5:8	MS14: 5-8	MS17: 5-8
17:20	Moving/Stretching legs			
17:30	PL03 Ronald Haynes			Culture Centre
18:15	PL04 Juan Galvis			Culture Centre
19:00	End of Talks			
19:10	Bus departs for Coal Miners' Cabin			

### 10:00~10:30 Opening Ceremony

Chair: *Martin J. Gander*

### 10:30~11:15 PL01 Ivan Graham Culture Centre

#### **Domain decomposition for high frequency Helmholtz problems**

*Ivan Graham\**

We present a new theory of additive Schwarz-type preconditioners for solving finite element approximations of the high-frequency Helmholtz equation with absorbing boundary conditions. The subdomain and coarse grid problems in the preconditioner are obtained as low-dimensional reductions of a nearby problem with added artificial absorption. This set-up allows rigorous estimates of the norm and field of values of preconditioned matrices which are explicit in the level of absorption and also in the wavenumber. The analysis covers both left and right preconditioning via a duality argument. The flexibility of domain decomposition methods allows the insertion of local solvers which are suitable for high-frequency problems. The analysis depends in part on frequency-explicit estimates for the underlying continuous PDE problem. The talk will include joint work with Eric Chung, Euan Spence, Eero Vainikko and Jun Zou.

### **Modelling and discretization of thin inclusions for flow in deformable porous media**

*Jan Nordbotten\**

We consider the problem of model reduction and discretization for deformable porous media. In particular, we have in mind applications such as fluid injection into thin geological layers, as well as fractured or reinforced materials.

We represent thin layers through a model reduction procedure in order to include them as  $d-1$  dimensional manifolds within the original domain. Intersections (and intersections of intersections) are handled recursively in a unified manner. Our interest is in the case of fully coupled flow and deformation, and we allow for thin layers which are both more permeable (and mechanically weaker) or less permeable (and mechanically reinforcing) than the surrounding materials.

The model problem is represented using a mortar-type approach, in which fluid fluxes and mechanical tractions are chosen as mortar variables. This allows for construction of locally conservative schemes both in terms of flow and momentum balance. Resulting schemes are proved to be well-posed and convergent. We include numerical validation of both the model reduction procedure as well as performance of the numerical discretization relative to standard finite volume approaches for fractured porous media.

This presentation includes joint work with I. Berre, T. I. Bjørnarå, W. M. Boon, M. A. Celia, I. Stefansson and I. Yotov.

Chair: MS-organizer

### **Overlapping Schwarz with a spectrally enriched coarse space in 3D**

*Leszek Marcinkowski\*, Erik Eikeland, Talal Rahman*

In this talk we present an overlapping additive Schwarz method for the second order elliptic problem in three dimensions, with highly varying coefficients. We propose variants of the adaptively built multiscale coarse space each containing local spaces spanned by functions constructed through solving specially defined eigenvalue problems over the 2D structures. The methods are easy to construct, inherently parallel, and overall effective. We present theoretical bounds for the condition number of the two systems, showing they are independent of the contrast in the coefficients when enough local eigenfunctions are added to the coarse space.

### **Primal hybrid discretizations based on BDDC and adaptive selection of primal constraints, and on localized orthogonal decomposition method**

*Marcus Sarkis\*, Alexandre Madureira*

Major progress has been made recently to make FETI-DP and BDDC preconditioners robust with respect to any variation of coefficients inside and across the subdomains. A reason for this success is the adaptive selection of primal constraints technique based on local generalized eigenvalue problems. In this talk, we transfer this solver technology to the field of discretizations. We design discretizations and establish a priori energy error estimate of  $O(H)$  with hidden constants independently of the coefficients. The number of degrees of freedom is the number of primal constraints. The only requirements for the analysis is the uniform ellipticity and that the right-hand side be in  $L^2$ . We note that regularity of the solution is not required for the analysis. We note that the resulting multiscale basis functions are nonlocal however decay

exponentially.

---

**OS-ACMS: a multiscale coarse spaces for overlapping Schwarz method based on the ACMS space in two dimensions**

---

*Alexander Heinlein\*, Axel Klawonn, Jascha Knepper, Oliver Rheinbach*

Robust domain decomposition methods for solving second order elliptic problems with large variations in the coefficient rely on the construction of a suitable coarse space. We propose two-level overlapping Schwarz methods using coarse spaces constructed from the Approximate Component Mode Synthesis (ACMS) multiscale discretization. In particular, we make use of eigenvalue problems using local Schur complements on subdomain edges. The convergence of a corresponding preconditioned Krylov method depends only on a user-specified tolerance and is therefore independent of variations of the coefficient function. The proposed methods also benefit from the low dimension of the local eigenvalue problems and from the fact that they can be solved approximately at the cost of only one or two iterations of LobPCG.

---

**Degenerate approximation of Green's function in the presence of high-contrast**

---

*Mario Bebendorf\**

Hierarchical matrices are well suited for treating non-local operators with logarithmic-linear complexity. In particular, the inverse and the factors of the LU decomposition of finite element discretizations of elliptic boundary value problems can be approximated with such structures. However, a proof for this shows a strong dependence of the local rank on the ratio of the largest and smallest coefficient in the differential operator with respect to the  $L^2$ -norm. Nevertheless, this kind of dependence cannot be observed in practice. The aim of this talk is to show that the above dependence can be avoided also theoretically with respect to a suitable norm. From this, a logarithmic dependence with respect to the  $L^2$ -norm can be deduced.

Chair: MS-organizer

13:30~15:10	<b>MS10-1 domain decomposition for frequency domain wave problems</b>	<b>North Pole</b>
-------------	---	-------------------

---

**A numerical study on the compressibility of Schur complements of discretized Helmholtz equations**

---

*Martin J Gander\**

The compressibility of Schur complement matrices is the essential ingredient for H-matrix techniques, and is well understood for Laplace type problems. The Helmholtz case however is more difficult, and there are several theoretical results which indicate that under certain specific circumstances good compression is possible. We investigate the compressibility with extensive numerical experiments, using only algebraic techniques, namely the singular value decomposition. These experiments indicate that with growing wave number  $k$ , the epsilon-rank is growing like  $k^{0.75}$  for  $2d$  problems, and for  $3d$  problems the growth is about  $k^{1.25}$ .

---

**A two-level preconditioner for the Maxwell's equations in conductive media**

---

*Marcella Bonazzoli, Victorita Dolean\*, Ivan Graham, Euan Spence*

Solving Maxwell's equations by numerical methods has several aspects and difficulties: first, it's very similar nature with the Helmholtz equations and secondly the properties of the underlying solution making difficult a coherent discrete and compatible characterisation. In order to circumvent in part some of the problems in [Gander et al., Numer. Math., 131(3):567--614, 2015] the authors have introduced the idea of preconditioning of the Helmholtz equation by a shifted Laplacian with a well-

chosen absorption coefficient followed by the preconditioning of the shifted Laplacian by a two-level domain decomposition method [Graham et. al., arXiv:1606.07172, 2016] using a coarse space correction constrained by the wave number. Our aim is to extend these ideas to the Maxwell's equations in conductive media, both from the theoretical and numerical point of view.

---

**What is the minimal shift to use in the shifted Helmholtz preconditioner for its effective inversion by multigrid?**

---

*Pierre-Henri Cocquet\*, Martin J. Gander*

The Helmholtz equation involves the continuous operator  $H_k = (\Delta + k^2)$ , where  $k > 0$  is the wave number, and can be used as a model problem for time-harmonic wave propagation. Solving this problem with iterative methods is very challenging (see e.g. [Ernst and Gander, Numerical Analysis of Multiscale Problems, I. Graham et al., eds., Springer Verlag, 325--363, 2012]) for instance due to the lack of coercivity of the operator  $H$  or highly oscillatory solutions. Krylov subspaces methods like GMRES or BiCGStab are still used in regards of their robustness. Nevertheless the latter are not fast enough without a good preconditioner. Among many proposed preconditioners like Incomplete LU, Analytic ILU or domain decomposition based preconditioner, the shifted Helmholtz preconditioner (SHP) [Erlangga et al., Applied Numerical Mathematics, 409--425, 2004; Sheikh et al., Numerical Linear Algebra with Applications, 20, 645--662, 2013] has received a lot of attention over the last decade thanks to its simplicity and its relevance to heterogeneous media. The SHP is defined as a discretization of the operator  $H_{\tilde{k}}$  for  $\tilde{k}^2 = k^2 + i\varepsilon$  where  $\varepsilon > 0$  is the so-called shift. It has been proved recently in [Gander et al., Numerische Mathematik, 53(1), 573--579, 2015] that  $\varepsilon$  can be at most  $O(k)$  to get wave number independent convergence of the preconditioned system solved with GMRES, provided the preconditioner is inverted exactly. However, in practice, one only inverts the preconditioner approximately and, for large enough shift, this can be done effectively by standard multigrid methods. In this talk, we will give some theoretical and numerical answers to the question: What is the minimal shift to use in the SHP for its effective inversion by multigrid? At the theoretical level, we will show that, for a given wave number  $k$ , taking  $\varepsilon = O(k^2)$  with a large enough constant always ensures convergence of the multigrid method applied to the SHP. Our proofs are based on a general convergence result from [Reusken, Multiscale Simulation Methods in Molecular Sciences, 467--506, 2009], and also on sharp  $k$ -dependent estimates for the solution to the shifted Helmholtz equation. Since the preconditioner is usually not inverted exactly in practice but rather approximated with a few iterations of multigrid, the question of what is the best shift to use cannot be given by our theoretical analysis. We will then also present in this talk some numerical simulations on a wave-guide type square domain to study the performance of preconditioned GMRES, for shifts ranging from  $O(\sqrt{k})$  to  $O(k^2)$ , where the preconditioner is inverted only with some V-cycle iterations. This will show that a shift  $\varepsilon = O(k^{3/2})$  can yield lower iteration counts than the one for which we proved convergence from our analysis, namely  $\varepsilon = O(k^2)$ .

---

**Two-level preconditioners for the Helmholtz equation**

---

*Marcella Bonazzoli\*, Victorita Dolean, Pierre-Henri Tournier*

Solving the Helmholtz equation  $-\Delta u - k^2 u = 0$  is a challenging task because of its indefinite nature and its highly oscillatory solution when the wavenumber  $k$  is high. Although there have been different attempts to solve it efficiently, we believe that there is no established and robust preconditioner whose behavior is independent of  $k$ . In [Conen et al., J. Comput. Appl. Math., 271:83--99, 2014] a two-level preconditioner was introduced, where the coarse correction involves local eigenproblems of DtN maps. This method proved to be very robust w.r.t. the reference preconditioner based on plane waves. Another method was developed in [Gander et

al., Numer. Math., 131(3):567—614, 2015], whose idea is preconditioning the equation with a shifted Laplacian with a well-chosen absorption coefficient, followed by the preconditioning of this shifted Laplacian by a two-level DDM [Graham et al., arXiv:1606.07172, 2016] with a coarse correction constrained by  $k$ . Our purpose is to compare numerically the robustness of the latter with the two-level method based on DtN maps.

Chair: MS-organizer

**13:30~15:10**    **MS14-1 Highly parallel domain decomposition methods and their applications**    **Northern Light**

---

### **FE2TI - An extremely scalable computational homogenization approach**

*Axel Klawonn, Martin Lanser, Oliver Rheinbach\**

We use the  $FE^2$  computational multiscale approach well known in engineering to incorporate micromechanics in macroscopic simulations, e.g., of modern multiphase steels. In such approaches, a microscopic boundary value problem based on the definition of a representative volume element (RVE) is solved at each macroscopic Gauss integration point. Linear or nonlinear nonoverlapping domain decomposition methods of the FETI-DP type are used to solve the nonlinear multiscale problems, including plasticity. We show parallel scalability to millions of MPI ranks and up to 40 billion d.o.f.

---

### **Nonlinear BDDC**

*Axel Klawonn, Martin Lanser\*, Oliver Rheinbach*

The numerical solution of nonlinear partial differential equations, e.g., nonlinear elasticity or elastoplasticity problems, on modern and future supercomputers requires fast and highly scalable parallel solvers. Domain decomposition methods such as FETI-DP (Finite Element Tearing and Interconnecting - Dual Primal) and BDDC (Balancing Domain Decomposition by Constraints) are such robust and efficient methods, and they are widely used to solve problems arising in the field of structural mechanics, e.g., the simulation of elastic or plastic deformations. In this talk, we will present our highly efficient BDDC implementation, realized in PETSc, and provide a comparison of the classical Newton-Krylov-BDDC approach with our recent Nonlinear-BDDC approach [Klawonn, Lanser and Rheinbach, SIAM J. Sci. Comput., 36(2):737--765, 2014]. In Nonlinear-BDDC, local nonlinear problems are solved on the interior part of each subdomain, which, in many cases, has the effect of a nonlinear globalization strategy. It can be observed that Nonlinear-BDDC often allows the application of larger loads in nonlinear elasticity or elastoplasticity problems. Our BDDC implementation can also use inexact subdomain solvers as well as an inexact solve for the BDDC coarse problem (see also [Li and Widlund, Comput. Methods Appl. Mech. Engrg., 196(8), 2007] for inexact linear BDDC), and is scalable to half a million parallel tasks on JUQUEEN (Jülich Supercomputing Center, Germany) supercomputer.

---

### **An advanced OO software framework for the development of MLBDDC preconditioners**

*Santiago Badia\*, Alberto F. Martín*

In this work, we present an object-oriented (OO) software framework for the development of highly-scalable multilevel domain decomposition methods. Our motivation here is not only to show the excellent scalability results of the preconditioners resulting from the framework, but also to define a software engineered architecture based on software abstractions that be can easily be customized by the user, in order to tackle new problems and challenges. The software design presented in this work has been implemented in FEMPAR, an open source FORTRAN2008



scientific computing library, which provides software tools for the finite element integration of complex multiscale and multiphysics PDEs, but also a numerical linear algebra subsystem ground in the framework. The framework can therefore be designed such that it permits a very tight coupling among the integration and solver steps of the simulation pipeline. We consider that such integration is basic for the development of efficient and scalable solvers for numerical PDEs. The framework we propose starts with a cell-based partition of the mesh, computed by means of, e.g., automatic graph partitioning. As a result of this process, we create a cell aggregation into subdomains. Proceeding recursively, we can create as many levels in a mesh hierarchy as convenient, in which subdomains in one level are cells at the next level. In our framework, the computations to be performed at each level of the hierarchy is mapped to MPI tasks such that each of the latter set have only duties at one level, in order to boost performance and scalability. We provide two customizable building blocks in the BDDC method, using the Strategy OO design pattern: (1) Definition of coarse DoFs: the user can define the type of constraints to be imposed between subdomains, based on the definition of the subdomain interface; (2) Definition of weights: At this level, the user can define its own operator. The library provides all the finite element integration tools for its computation.

---

### **Scalability study of overlapping Schwarz for 3D unstructured meshes**

---

*Xiao-Chuan Cai\**

Overlapping Schwarz has been proven to be a highly scalable preconditioner for ‘elliptic’ problems, assuming that ‘a good subdomain solver’ and ‘several coarse meshes’ are available, but for many practical problems some of the conditions (i.e., ellipticity, good subdomain solver, and multiple coarse meshes) may not be easy to satisfy due to the nature of the physical system or the complicated geometry of the computational domain, etc. In this talk, we discuss some recent experiences with the application of overlapping Schwarz in a few engineering problems defined on 3D unstructured meshes involving turbulent flows, non-Newtonian flows, and nonlinear materials.

Chair: MS-organizer

**13:30~15:10 MS17-1 Time-parallel time integration methods**

**Polar Bear**

---

### **Space-time multigrid methods for parabolic problems**

---

*Martin Neumüller\**

In this talk we will consider two different space-time multigrid techniques for parabolic problems. We will show their advantages and disadvantages by looking at the Fourier mode analysis. Moreover we will combine the two approaches to obtain a method which allows an efficient parallel implementation with respect to space and time. We confirm the theoretical results with several numerical tests.

---

### **Time parallelization of waveform relaxation methods**

---

*Felix Kwok\**

In this talk, we consider waveform relaxation (WR) methods for solving time-dependent PDEs. WR methods are distinctive in that a typical subdomain problem is posed in both space and time; each iteration requires the parallel solution of these space-time subproblems, followed by an exchange of interface data defined over the whole time window. An often cited advantage of WR methods is that they allow each subdomain to use a different spatial and temporal grid that is adapted to the dynamics of the local subproblem.

Here, we show that WR methods have another advantage, namely that they allow additional parallelism in time. We observe that in most cases, one has enough initial

and interface data to start a WR iteration even before the previous iteration has completed; as a result, several iterations can run simultaneously. Based on this observation, we propose two ways of parallelizing WR methods in time. The first one uses a fixed time window and is mathematically equivalent to the original WR method. The second one chooses the time -window size dynamically based on how many free processors are available; this leads to a method with a different convergence behaviour. We demonstrate the effectiveness of both approaches by comparing their running times against those obtained from classical time-stepping methods, where the same number of processors is used to parallelize in space only.

### **A multigrid perspective on the parallel full approximation scheme in space and time**

*Matthias Bolten\**

For the numerical solution of time-dependent partial differential equations, time-parallel methods have recently shown to provide a promising way to extend prevailing strong-scaling limits of numerical codes. One of the most complex methods in this field is the ‘Parallel Full Approximation Scheme in Space and Time’ (PFASST). PFASST already shows promising results for many practical cases, and many more work are in progress. However, a solid and reliable mathematical foundation is still missing. We show that under certain assumptions the PFASST algorithm can be conveniently and rigorously described as a multigrid-in-time method. Following this equivalence, first steps towards a comprehensive analysis of PFASST using block-wise local Fourier analysis are taken. The theoretical results are applied to examples of diffusive and advective type.

### **Parallel in time methods based on PFASST for PDE constrained optimal control problems**

*Michael Minion\*, Sebastian Götschel*

For adjoint based gradient descent methods for PDE constrained optimal control problems, in addition to solving the forward problem in time at each step, one must also solve an equation for the adjoint which evolves ‘backward’ in time. We will discuss methods to apply parallel in time techniques based on the PFASST algorithm to both the forward and adjoint problems concurrently. For longer time integration windows, we rely on additional domain decomposition in the time direction to break the adjoint equation into smaller subproblems. We will also discuss re-using both forward and adjoint solutions in the optimization loop.

## **Coffee Break at Radisson and Culture Centre**

Chair: MS-organizer

<b>15:40~17:20</b>	<b>MS06-2 Novel discretizations and solvers for high-contrast and multiscale problems</b>	<b>Culture Centre</b>
--------------------	---	-----------------------

### **A multiscale model reduction method for nonlinear monotone elliptic equations in heterogeneous media**

*Eric T. Chung\*, Yalchin Efendiev, Ke Shi, Shuai Ye*

In this talk, we present a multiscale model reduction framework within Generalized Multiscale Finite Element Method (GMsFEM) for nonlinear elliptic problems. We consider an exemplary problem, which consists of nonlinear p-Laplacian with heterogeneous coefficients. The main challenging feature of this problem is that local subgrid models are nonlinear involving the gradient of the solution (e.g., in the case of scale separation, when using homogenization). Our main objective is to develop

snapshots and local spectral problems, which are the main ingredients of GMsFEM, for these problems. Our contributions can be summarized as follows. (1) We recast the multiscale model reduction problem onto the boundaries of coarse cells. This is important and allows capturing separable scales as discussed. (2) We introduce nonlinear eigenvalue problems in the snapshot space for these nonlinear "harmonic" functions. (3) We present convergence analysis and numerical results, which show that our approaches can recover the fine-scale solution with a few degrees of freedom. The proposed methods can, in general, be used for more general nonlinear problems, where one needs nonlinear local spectral decomposition.

---

### **An adaptive GMsFEM for high-contrast flow problems**

*Guanglian Li\*, Eric T. Chung, Yalchin Efendiev*

In this paper, we derive an a-posteriori error indicator for the Generalized Multiscale Finite Element Method (GMsFEM) framework. This error indicator is further used to develop an adaptive enrichment algorithm for the linear elliptic equation with multiscale high-contrast coefficients. We consider two kinds of error indicators where one is based on the  $L^2$ -norm of the local residual and the other is based on the weighted  $H^1$ -norm of the local residual where the weight is related to the coefficient of the elliptic equation. We show that the use of weighted  $H^1$ -norm residual gives a more robust error indicator which works well for cases with high contrast media. The convergence analysis of the method is given.

---

### **Nonoverlapping additive Schwarz for hp-DGFEM with discontinuous coefficients**

*Piotr Krzyzanowski\**

We discuss a nonoverlapping additive Schwarz method for an  $h$ - $p$  DGFEM discretization of an elliptic PDE with discontinuous coefficients, where the fine grid is decomposed into subdomains of size  $H$  and the coarse grid consists of cell size  $\mathcal{H}$  such that  $h \leq H \leq \mathcal{H}$ . We prove the condition number is  $O(\frac{p^2}{q}) \cdot O(\frac{\mathcal{H}^2}{Hh})$  and is independent of the jumps of the coefficient if the discontinuities are aligned with the coarse grid.

---

### **Finite volume discretizations of poro-elasticity in high-contrast media**

*Eirik Keilegavlen\*, Eren Ucar, Jan Nordbotten, Inga Berre*

Simulations of fluid flow through deformable porous media are of increasing importance in subsurface applications such as energy recovery and carbon sequestration. A particular challenge is the dynamics of fractures, which are essentially lower-dimensional objects in the simulation domain that can have decisive impact on the overall system dynamics. In this work, we present a novel simulation approach for dynamics in deformable and fractured porous media. While the common practice is to discretize elasticity equations with finite elements, our approach is to apply finite volumes both to flow and deformation. The methodology has been proven stable for elasticity and poro-elastic problems, and it is in particular robust for parameter heterogeneities and complex grids. We discuss the extension of the methodology to fractured media, and show simulation results for two and three dimensional problems motivated by geothermal energy recovery.

Chair: MS-organizer

15:40~17:20	<b>MS10-2 Domain decomposition for frequency domain wave problems</b>	<b>North Pole</b>
-------------	---	-------------------

---

### **New stability results for the Helmholtz equation with variable coefficients**

*Ivan Graham\*, S. A. Sauter*

The stability analysis of the high frequency Helmholtz equation in the case of variable wave speed is a topic of considerable practical importance and contemporary interest. Such stability estimates are a key ingredient for the derivation of finite element error estimates and the analysis of solvers. Successful application of the classical Rellich or Morawetz identities to this problem imposes substantial constraints on the wave speed function essentially requiring it to be non-oscillatory. This should be compared to the diffusion equation where the case of highly variable and oscillatory coefficients is well understood. We present new sharp estimates for the interior impedance Helmholtz problem (in a restricted range of geometries) which show that the energy of the solution can blow up exponentially in the variation of the wave speed. We also discuss briefly the implications for discretization methods and solvers.

---

**Block-Jacobi multi-trace formulations and optimized Schwarz methods**

---

*Xavier Claeys\*, Victorita Dolean, Martin J. Gander, Pierre Marchand*

Considering time harmonic wave propagation in piecewise constant media, we are interested in the solution to such problems by means of local multi-trace boundary integral formulations (local MTF). In boundary integral approaches, the unknowns are traces of the volumic wave field taken at interfaces. In the local multi-trace framework, traces are doubled at each interface, and each subdomain is coupled with its neighbours by means of a local transmission operator. Nonlocal operators only come into play subdomain-wise, so that this approach is appealing from a domain decomposition perspective. In the present talk, we will discuss the close relationship between a block-Jacobi strategy applied to local MTF on the one hand, and Optimized Schwarz Methods applied to volumic equations of the wave propagation problem on the other hand.

---

**Non-overlapping Schwarz domain decomposition methods with Padé-localized transmission operators for time-harmonic acoustic, electromagnetic and elastodynamic waves**

---

*Vanessa Mattesi\*, Christophe Geuzaine*

In this talk we will present recent developments in the design and implementation of non-overlapping Schwarz domain decomposition methods for high-frequency wave problems, that lead to quasi-optimal convergence properties, i.e., with a convergence that is optimal for the evanescent modes and significantly improved compared to competing approaches for the remaining modes. These improved properties result from a combination of an appropriate choice of transmission conditions and a suitable localization of the Dirichlet-to-Neumann operator using Padé approximants. We will detail the construction of the resulting algorithms for time-harmonic acoustic, electromagnetic and elastodynamic wave problems.

---

**Domain substructuring using IGA and FEM with application to radio frequency cavity simulation**

---

*J. Corno\*, A. Buffa, C. de Falco, S. Schps, R. Vazquez*

The use of isogeometric analysis (IGA) in cavity simulation has been proven to be beneficial both in terms of accuracy and of overall reduction of the computational cost [Corno et al., Computational Physics Communication, 201, 1--7, 2016; Vazquez et al., IEEE Transactions on Magnetics, 46(8):3305--3308, 2010]. However, the simulation of large and complex structures remains an overwhelming task. Particularly if small geometric features like high order mode couplers have to be resolved. In this work we propose an IGA/FEM- framework for domain decomposition that we see as a promising tool for reducing the complexity of those simulations. In particular we present two instances of a method fitting into this framework. The first one is inspired by the State Space Concatenation (SSC) method recently introduced by Flisgen et al. [Flisgen, Heller and van Rienen, IPAC, 2014], while the second one is a Mortar method that exploits the inherent properties of the IGA basis to naturally define the

approximation space for the multipliers. Both approaches are cast in the general framework of the Three Fields Method [Brezzi et al., Contemporary Mathematics 157, 1994]. Results for IGA-IGA and IGA-FEM coupling are presented.

Chair: MS-organizer

15:40~17:20 **MS14-2 Highly parallel domain decomposition methods and their applications** Northern Light

---

**Projector-less highly scalable variant of the TFETI method implemented in the PERMON toolbox**

*David Horak\*, Zdeněk Dostál, Václav Hapla, Radim Sojka, Jakub Kruzik*

The original FETI-1 method became numerically scalable after introducing the projectors to the natural coarse space. Some problems with stable implementations were resolved by TFETI. The FETI methods turned out to be very efficient, in particular their modification for variational inequalities, but their parallel scalability deteriorated fast. The reason is the increasing cost of implementation of the projectors containing coarse problem (CP) solution. Quadratic programming (QP) problems employing FETI methods are often solved by means of the MPRGP and SMALBE algorithms with the Hessian matrix, which contains three projector applications, which can be implemented using two CP solutions. These QP algorithms and FETI methods are implanted in our new software package PERMON toolbox based on PETSc. The presentation deals with the modification of the TFETI method eliminating projectors application including CP solution while the same numerical scalability of the solver is preserved. The same effect as that achieved by the projectors can be achieved by using the Moore-Penrose pseudoinverse reachable by projecting a suitable pseudoinverse onto the range of the stiffness matrix. This operation is purely local and very cheap. The CP contained in the penalized term of the Hessian could be omitted or approximated by purely local action as well. Numerical experiments demonstrating performance of this new approach will be presented.

---

**Advanced techniques for designing scalable multilevel preconditioners**

*Pierre Jolivet\**

In this talk, I will present some recent algorithmic advances to improve the scalability of existing domain decomposition methods that use Galerkin coarse grid operators. The three main parts of the presentation will deal with: 1) multilevel overlapping Schwarz methods, 2) coarse grid preconditioners with mixed-precision arithmetic, and 3) domain decomposition methods with enlarged Krylov subspaces, i.e., with multiple search directions. I will show numerical results on a variety of examples (elliptic, saddle point, and frequency domain problems) and present some technical details of the implementation in the open-source framework HPDDM (<https://github.com/hpddm/hpddm>).

---

**Lean and Mean: Finite elements solvers beyond a trillion degrees of freedom**

*Ulrich Rüde\**

Large supercomputers currently have up to a petabyte of memory, enough to store 125 double precision vectors each with a dimension of a trillion ( $10^{12}$ ) elements. With advanced data structures and clever algorithms the storage of the stiffness and the mass matrix can often be avoided and so it becomes possible to set up finite element systems with in excess of a trillion degrees of freedom. For problems of such size, the algorithms must have asymptotically optimal or almost optimal complexity. Going beyond scalability, care must be taken to select the algorithms that offer the lowest absolute cost and to implement them such that they can exploit modern processor architectures.

## **Combining adaptive mesh refinement with a parallel multilevel BDDC solver**

---

*Jakub Šístek\*, Pavel Kůs*

Adaptive mesh refinement (AMR) is a well-established technique for the finite element method (FEM). However, its use in a massively parallel setting is complicated and still rather limited. In our contribution, we present an implementation of a parallel FEM code employing AMR. Parallel load re-balancing during the adaptivity process is based on splitting a space-filling curve by the p4est library. The drawback of the use of space filling curves for mesh partitioning is the poor shape of arising subdomains, which are typically composed of several components. The main focus is studying the effect of AMR on the solution of the arising systems of equations by the Balancing Domain Decomposition based on Constraints (BDDC). In particular, we use the parallel implementation of multilevel BDDC available in the BDDCML library. We describe several ingredients required for this combination, including hanging nodes on subdomain interfaces and accommodating disconnected substructures composed of several components. Our largest numerical results use highly adapted meshes with over 1 billion unknowns solved using 2048 CPU cores.

Chair: MS-organizer

15:40~17:20 MS17-2 Time-parallel time integration methods

Polar Bear

## **An iterative approach for time-parallel time integration based on discontinuous Galerkin methods**

---

*Xiaozhou Li\*, Pietro Benedusi, Rolf Krause*

We present a new class of iterative schemes for solving initial value problems (IVP) based on discontinuous Galerkin (DG) methods. Starting from the weak DG formulation of an IVP, we derive an iterative method based on a preconditioned Picard iteration. Using this new approach, we can systematically construct explicit, implicit and semi-implicit schemes with arbitrary order of accuracy. We also show that the same schemes can be constructed by solving a series of correction equations based on the DG weak formulation. The accuracy of the schemes is proven to be  $\min\{2p+1, K\}$  with  $p$  the degree of the DG polynomial basis and  $K$  the number of iterations. The stability is explored numerically; we show that the implicit schemes are  $A$ -stable at least for  $0 \leq p \leq 9$ .

Furthermore, we combine the methods with a multilevel strategy to accelerate their convergence speed. The new multilevel scheme is intended to provide a flexible framework for high order space-time discretizations and to be coupled with space-time multigrid techniques for solving partial differential equations (PDEs). We present numerical examples for ODEs and PDEs to analyze the performance of the new methods. Moreover, the newly proposed class of methods, due to its structure, is also a competitive and promising candidate for parallel in time algorithms such as Parareal, PFASST [Emmett and Minion, Comm. in App. Math. and Comp. Sci., vol. 7, pp. 105–132, 2012], multigrid in time, etc.

## **Lossy compression of finite element coefficients for reducing communication in time-parallel simulations**

---

*Sebastian Götschel\*, Martin Weiser, Lisa Fischer, Thomas Steinke, Florian Wende, Alexander Kammeyer*

Communication bandwidth between nodes of parallel machines grows slower than the nodes' computational performance and increasingly becomes a bottleneck of PDE solvers. We present lossy compression techniques for finite element solutions based on multilevel prediction in hierarchical grids and entropy coding as a means to reduce the amount of data to be communicated between nodes in SDC-based time-parallel

simulations. Compression rate and impact on convergence depending on the quantization error are investigated both theoretically and on some numerical examples realized in the finite element code Kaskade 7, which leads to an adaptive choice of the quantization threshold.

### **Matching algorithms and their use in time-parallel molecular dynamics simulations**

---

*Frederic Legoll, Tony Lelievre, Keith Myerscough, Giovanni Samaey\**

Time-parallel methods (such as parareal) invoke a coarse-scale and cheap propagator in conjunction with a fine-scale and expensive one to perform simulations with fine-scale accuracy at a fraction of the wall-clock time of a naive fine-scale simulation. These methods are typically iterative: in every iteration one performs a number of fine-scale simulations in parallel and processes the discrepancies between the fine-scale and coarse-scale models. In this talk, we go into the additional problems that arise when the fine-scale solver is a stochastic molecular dynamics model, namely overdamped Langevin dynamics, and the coarse-scale solver is the corresponding Fokker-Planck equation for only the slow degrees of freedom at the fine-scale. In this situation, the absence of the fast degrees of freedom in the coarse propagator, as well as positivity requirements on the computed densities, induce additional complications in the parareal algorithm. In this talk, we propose algorithms that deal with these complications and study convergence of the resulting methods in a number of standard test cases.

This talk is based on joint work with Frederic Legoll, Tony Lelievre and Keith Myerscough.

### **On scalable space-time balancing domain-decomposition solvers**

---

*Santiago Badia, Marc Olm\**

The usual approach to transient problems is to exploit sequentiality in time, and solve one space problem every time step. This sequential approach has a clear problem, parallelization cannot be exploited in time. Many key computational engineering problems, e.g., in additive manufacturing simulations or turbulent flow simulations, involve even millions of time steps, and a scalable parallel solver in space leads to unacceptable computation times in these problems. On the other hand, space parallelization always saturates at some point (it has no sense, e.g., to consider more processors than finite elements in the mesh) and to efficiently exploit the forthcoming exascale platforms one needs to exhibit further concurrency. In this situation, the development of space-time solvers is an excellent approach because deals with all time steps at once and exhibits much more concurrency than space-only solvers. In this presentation, we will introduce a new type of (non)linear space-time solvers for the solution of finite element systems arising from the discretization of transient problems, based on the novel extension of balancing domain decomposition methods to space-time, and show numerically the scalability of the proposed schemes on a set of academic problems, based on a highly scalable implementation based on overlapped multilevel task implementation.

Chair: *Susanne Brenner*

**17:30~18:15 PL03 Ronald Haynes**

**Culture Centre**

### **PDE based mesh generation: domain decomposition approaches**

---

*Ronald Haynes\**

An approach to solve PDEs whose solutions evolve on disparate space and time scales is introduce adaptive and dynamic meshes. In this talk we will review a class of PDE based mesh generators in 1D and 2D. Here a PDE for the mesh is formulated and

coupled with the physical PDE of interest. The hope is that the cost of computing the mesh should not substantially increase the total computational burden and ideally will fit within the framework being used to solve the physical PDE. Here we consider parallel solution strategies for the mesh PDE and the coupled system using various domain decomposition strategies. The analysis of the algorithms reminds us of several classical tools from the 1950s and 1960s including Peaceman-Rachford iterations and monotone convergence using the theory of M-functions.

18:15~19:00 PL04 Juan Galvis

Culture Centre

---

### **Overlapping methods for high-contrast multiscale problems**

---

*Juan Galvis\**

In this talk, we review the design and analysis of robust overlapping domain decomposition algorithms for high-contrast multiscale problems. In particular, we review the construction of coarse spaces using spectral information of local bilinear forms. We present several alternatives to incorporate this spectral information into the coarse problem in order to obtain minimal coarse space dimension. We also show numerically that several, rather rough, approximations of the eigenvectors can also be used with good results. This allows us to lower the cost of the construction of the coarse space.



## Tuesday, February 7<sup>th</sup> 2017

	Culture Centre	North Pole	Northern Light	Polar Bear
07:00	Breakfast served			
08:10	Bus departs from Coal Miners' Cabin			
08:30	PL05 Alfio Quarteroni			Culture Centre
09:15	PL06 Marie Rognes			Culture Centre
10:00	Coffee at Culture Centre and Radisson & Introduction to Poster session			
10:30	MS06: 9-11	MS13: 1-4	MS02: 1-4	CT01: 1-4
12:10	Lunch at Radisson			
13:40	PL07 Xiaozhe Hu			Culture Centre
14:25	Moving/Stretching legs			
14:35	MS12: 1-4	MS13: 5-8	MS02: 5-6	MS15: 1-4
16:15	Coffee at Culture Centre and Radisson			
16:45	MS03: 1-5	MS09: 1-5	CT02: 1-5	MS15: 5-8
18:50	End of talks			
19:00	Bus departs for Coal Miners' Cabin			
19:15	DDM Business meeting at Hjalmar Johansen			
21:00	DDM Dinner with invited speakers at Spitsbergen Hotel 20:45 Bus from Radisson			

Chair: *Olof Widlund*

**8:30~9:15 PL05 Alfio Quarteroni**

**Culture Centre**

### Domain decomposition based methods for multiphysics problems

*Alfio Quarteroni\**

In this presentation I will discuss about the use of Interface Control Domain Decomposition (ICDD) to address partial differential equations (PDEs) on overlapping subdomains, the use of INTERNODES, a method for nonconforming approximations on neighbouring subdomains, and on FaCSI, a domain decomposition based preconditioner for multiphysics problems. Applications will be given in the context of the mathematical modelling of the cardiocirculatory system and the coupling of surface-subsurface flows.

### **Impact of high abstraction/high performance finite element software in biomedical computing**

---

*Marie Rognes\**

The development of numerical software in general and finite element software in particular is recognized as a challenging and error-prone process -- traditionally requiring in-depth expertise from a number of scientific fields. The FEniCS and Dolfin-adjoint projects target this challenge by developing generic algorithms and open source software for the automated solution of partial differential equations using finite element methods. The FEniCS Project is described by in The FEniCS book [Logg et al., Springer, 2012, doi:10.1007/978-3-642-23099-8], and in a number of research papers, see e.g. [Alnæs et al., Archive of Numerical Software, 3(100):9--23, 2015]. The related Dolfin-adjoint software, winner of the 2015 Wilkinson Prize for Numerical Software, automatically derives discrete adjoint and tangent linear models from a forward FEniCS model [Farrell et al., SIAM Journal on Scientific Computing, 35(4):C369--C393]. These adjoint and tangent linear models are key ingredients in many important algorithms, such as data assimilation, optimal control, sensitivity analysis, design optimisation, and error estimation.

In this presentation, I will give an overview of the FEniCS and Dolfin-adjoint projects focusing on current developments and applications in biomedical computing.

### **Coffee Break at Radisson and Culture Centre**

Chair: MS-organizer

**10:30~12:10 MS06-3 Novel discretizations and solvers for high-contrast and multiscale problems**

Culture Centre

### **MsFEM accelerated topology optimization of linear elastic and heat transfer systems**

---

*Boyan S. Lazarov\**

Topology optimization is an iterative process which distributes a given amount of material in a design domain by minimizing an objective function and fulfilling a set of constraints. It is based on repetitive finite element analyses, regularization and design update steps. The finite element solutions are the most time consuming part, thus, requiring the utilization of large high performance computing platforms and iterative solution techniques. The convergence speed of the solution process is affected significantly by the high contrast in the material distribution, the requirement for fine resolution in order to represent clearly the boundaries of the optimized designs, and the manufacturability constraints. Therefore the focus of this work is on accelerating the optimization process of linear elastic and heat transfer structures by multi-scale finite element techniques for high contrast media. The development results in manufacturable designs with fully resolved micro-structural details accounting for localized functional requirements, loading and boundary effects.

### **Robust discretization of flow in fractured porous media**

---

*Wietse Boon\*, Jan Nordbotten, Ivan Yotov*

Fractures are ubiquitous in natural rocks, and in many cases have a leading order impact on the structure of fluid flow. Since fractures frequently have high aspect ratios, it is appealing to consider them as lower-dimensional features.

We present a modelling approach based on mixed finite element methods and the mortar method which fully couples the physics in domains with different dimensions.

In particular, we apply the approach to Darcy flow in fractured media and show how abrupt fracture tips as well as fracture intersections are naturally handled. The proposed discretization is applicable to both two and three spatial dimensions and is capable of handling conductive as well as blocking fractures. Furthermore, the method respects mass conservation and handles non-matching grids. We establish both theoretically and through numerical examples that our method is convergent in all relevant physical limits.

---

### **Conservative properties of high-order finite element approximations**

*Juan Galvis\*, Marcus Sarkis, Eduardo Abreu, Ciro Daz*

In this talk we explore a method for the construction of locally conservative flux fields. The flux values are obtained through the use of a Ritz formulation in which we augment the resulting linear system of the continuous Galerkin (CG) formulation in a higher-order approximation space. To test the performance of the method we consider equations with heterogeneous permeability coefficients that have high-variation and discontinuities, and couple the resulting fluxes to a two-phase flow model. The increase in accuracy associated with the high order approximation of the pressure solutions is inherited by the flux fields and saturation solutions. A variety of numerical examples are offered to validate the performance of the method. A proposal for the analysis is presented for the case of homogeneous coefficient. In this case we can obtain optimal  $H^1$  and  $L^2$  errors by using standard arguments of saddle point problem.

Chair: MS-organizer

<b>10:30~12:10</b>	<b>MS13-1 Domain decomposition methods for nonlinear problems</b>	<b>North Pole</b>
--------------------	---	-------------------

---

### **Nonlinear FETI-DP methods for nonlinear problems**

*Axel Klawonn\*, Martin Lanser, Oliver Rheinbach, Matthias Uran*

Parallel Newton-Krylov FETI-DP domain decomposition methods are fast and robust solvers, e.g., for nonlinear implicit problems in structural mechanics. In these methods, the nonlinear problem is first linearized and then decomposed. By changing the order of these operations, new parallel nonlinear FETI-DP methods with increased locality and reduced communication can be designed. We discuss different nonlinear FETI-DP methods which are equivalent when applied to linear problems but which show a different performance for nonlinear problems. We then describe these methods as different right- preconditioned Newton methods applied to a nonlinear master system.

---

### **Additive and restricted additive Schwarz methods for quasilinear inequalities**

*Lori Badea\**

In this talk, we study the convergence of the additive Schwarz (AS) and restricted additive Schwarz (RAS) methods for quasilinear inequalities. In this case, we have to solve at each iteration a number of smaller nonlinear inequalities corresponding to the subdomains in the decomposition of the domain. First, we prove the convergence of the two methods for the usual variational inequalities (of the first kind) perturbed by a Lipschitz operator. Then, by taking the Lipschitz operator of a particular form, we prove the convergence and give error estimates of the AS and RAS methods for quasilinear inequalities, provided that some convergence conditions are satisfied.

---

### **Domain decomposition methods for a class of non-smooth and non-additive convex variational problems**

*Andreas Langer\**

We present overlapping and non-overlapping domain decomposition approaches for the minimization of non-smooth and non-additive functionals. A prominent example of

such problems is the minimization of the total variation. An estimate of the distance of the limit point obtained from the proposed domain decomposition methods to the true global minimizer is established. With the help of this estimate, it is demonstrated by numerical experiments that the sequence generated by the domain decomposition algorithm indeed approaches the global minimizer of the objective functional. Moreover, we introduce non-overlapping domain decomposition methods for non-smooth and non-additive problems which guarantee to converge to a global minimizer in a discrete setting. We provide several numerical experiments, showing the successful application of the algorithms.

---

### **Nonlinear field-split preconditioners and applications**

*Lulu Liu\**

Nonlinear field-split preconditioners are very effective in improving the convergence of solution methods such as Newton's method or ASPIN for systems with unbalanced nonlinearities. They are naturally based on partitioning the degrees of freedom of a nonlinear PDE system by field type rather than by subdomain. In the context of field splitting, a natural convection cavity flow PDE system is solved using multiplicative Schwarz preconditioned inexact Newton (left nonlinear preconditioners) resulting from different groupings and orderings of the PDEs and their associated fields, and convergence results are reported over a range of Rayleigh numbers; we also demonstrate the application of nonlinear elimination (right nonlinear preconditioners) in 1D flame sheet model.

Chair: MS-organizer

<b>10:30~12:10</b>	<b>MS02-1 Fast solvers for discontinuous Galerkin methods</b>	<b>Northern Light</b>
--------------------	---	-----------------------

---

### **Adaptive aggregations on graphs and applications**

*Ludmil Zikatanov\**

We generalize some of the functional (hyper-circle) a posteriori estimates from finite element settings to general graphs or Hilbert space settings. We provide several theoretical results in regard to the generalized a posteriori error estimators. We use these estimates to construct aggregation based coarse spaces and multilevel solvers for graph Laplacians or discretizations of elliptic problems with non-standard shape elements. The estimator is used to assess the quality of an aggregation adaptively. Furthermore, a reshaping algorithm based is tested on several numerical examples.

---

### **Auxiliary space preconditioners for discontinuous Galerkin discretizations of $H(\text{curl})$ -elliptic problems with discontinuous coefficients**

*Blanca Ayuso de Dios\*, Ralf Hiptmair, Cecilia Pagliantini*

We propose a family of preconditioners for linear systems of equations ensuing from a piecewise polynomial symmetric Interior Penalty discontinuous Galerkin discretization of  $H(\text{curl}; \Omega)$ -elliptic boundary value problems arising in eddy current models. The design and analysis of the proposed solvers relies on the auxiliary space method. For conforming simplicial meshes, we show optimality of the proposed preconditioners with respect to mesh refinement. We also address the influence on the asymptotic performance of the preconditioners of possible discontinuities in the coefficients  $\nu$  and  $\beta$  in the second and zero-th order parts of the operator respectively. We validate the theory with extensive 2D numerical experiments.

---

### **A nonoverlapping DD preconditioner for heterogeneous elliptic problems**

*Susanne Brenner, Eun-Hee Park\*, Li-yeng Sung*

In this talk we will discuss a nonoverlapping domain decomposition preconditioner for a symmetric interior penalty Galerkin method. There are two key ingredients in the proposed preconditioner: one is a subspace decomposition of the discontinuous finite element space and the other is a procedure based on the FETI-DP approach. Theoretical results on the condition number estimate of the preconditioned system will be presented along with numerical results.

### **Robust and efficient preconditioners for the discontinuous Galerkin time-stepping method**

---

*Iain Smears\**

The discontinuous Galerkin time-stepping method has received significant interest in solving parabolic equations due to its variational-in-time formulation, its unconditional stability properties, and its potential for arbitrarily high-order approximations. Moreover, it coincides with the classical RADAU IIA time-integration scheme. In computations, it leads to the task of solving a large non-symmetric system at each time-step. We present a fully robust and efficient preconditioning strategy for solving these systems. Drawing on parabolic inf-sup theory, we first construct a left preconditioner that transforms the linear system to a symmetric positive definite problem to be solved by the preconditioned conjugate gradient algorithm. We then prove that the transformed system can be further preconditioned by an ideal block diagonal preconditioner, leading to a condition number bounded by 4 for any time-step size, any approximation order and any positive-definite self-adjoint spatial operators. Numerical experiments demonstrate the low condition numbers and fast convergence of the algorithm for both ideal and approximate preconditioners, and show the feasibility of the high-order solution of large problems.

Chair: *Ulrich Langer*

<b>10:30~12:10</b>	<b>Contributed Talks Session 1: Domain decomposition algorithms and applications</b>	<b>Polar Bear</b>
--------------------	--	-------------------

### **An iterative domain decomposition method for eddy current problems with consideration for the gauge condition**

---

*Daisuke Tagami\*, Shin-Ichiro Sugimoto*

An iterative Domain Decomposition Method (DDM) is applied into a mixed formulation of eddy current problems with the gauge condition, which is an extension in case of magnetostatic problems; see Tagami [COE Lecture Note, Vol. 39, Kyushu University, pp. 96--101, 2012].

From the engineering point of view, eddy current problems are often formulated by neglecting the gauge conditions, where the magnetic vector potential is only one unknown function; see, for example, [Kameari et al., IEEE Trans. Magn., 33(2):1223--1226, 1997] and [Kanayama, et al., COMPEL, 21(4):554--562, 2002]. The formulation without any gauge conditions enables us to reduce computational costs in case of the conventional one domain problem, and to formally introduce an iterative DDM. However, the formulation without any gauge conditions yields an indeterminate linear system. Therefore, to the best of our knowledge, mathematical justifications of numerical results such as unique solvability and convergence are not available and it is hard to generally apply direct solvers required for the reduction of computational costs of the iterative DDM. Moreover, when the formulation without any gauge conditions applied, iterative procedures diverge in case of large scale computational models whose numbers of degrees of freedom (DOF) are larger than  $10^7$ ; see, for example, [Sugimoto et al., Trans. Japan Soc. Simul. Tech., 7(1):11--17, 2015]. To overcome difficulties mentioned above, a gauge condition is introduced and eddy current problems are formulated by mixed variational problems, which is related with the magnetostatic case in [Kikuchi, Japan J. Appl. Math., 6(2):209--221, 1989] and the

eddy current case in [Alonso and Valli, Eddy current approximation of Maxwell equations: theory, algorithms and applications, Springer-Verlag, 2010]. The mixed formulation regards the magnetic vector potential and the Lagrange multiplier as two unknown functions that are approximated by the Nedelec curl-conforming edge element and by the conventional piecewise linear element, respectively. Moreover, an efficient iterative procedure is established by means of the property of the Lagrange multiplier that vanishes in the whole domain. Finally, some numerical results are shown in case of ultra-large computational models whose numbers of DOF are  $10^7$ -- $10^9$ .

---

### **An optimised domain decomposition method for large scale eigenvalue problems taking advantage of a contour integral approach**

*Nicolas Marsic\*, Christophe Geuzaine, Felix Wolf, Sebastian Schöps, Herbert De Gersem*

In this talk we present a domain decomposition (DD) technique for the treatment of eigenvalue problems, arising from the time-harmonic Maxwell's equations. The proposed solution relies on the contour integral method developed by Beyn in 2012 [W.-J. Beyn, Linear Algebra and its Applications, 436(10), 2012]. This approach requires only the solution of direct problems with different complex wavenumbers. The resulting linear systems can then be easily solved by well-known optimised domain decomposition techniques for electromagnetic waves, opening the door for large scale eigenvalue problems thanks to the excellent memory scaling of DD approaches. Indeed, in commonly used eigenvalue algorithms, this last aspect is often the computational bottleneck. This talk discusses both theoretical aspects and numerical results.

---

### **CFD codes on multicore and manycore architectures**

*David Keyes\*, Bilel Hadri, Alexander Heinecke, Maxwell Hutchinson, Matteo Parsani*

Weak scaling over distributed memory by domain decomposition is well established for structured and unstructured grid CFD simulations, as evidenced by (besides many modeling achievements of the CFD community over the decades) Gordon Bell Prizes for computational performance. Strong scaling over shared memory to exploit multicore and manycore architectures is less satisfactory to date. In this talk, we report on two campaigns to port to Intel multicore and manycore environments a pair of CFD codes that shared the 1999 Gordon Bell Prize, when hybrid distributed-shared programming models were limited to just 2 cores per node, as opposed to the hundreds per node available today. Shared memory parallelization of the flux kernel of PETSc-FUN3D, an unstructured tetrahedral mesh Euler flow code is evaluated on Intel multicore processors through Haswell, and Intel many-core processors through KNL. We document several thread-level optimizations to improve flux kernel and overall performance. In addition, a geometry-simplified fork of a widely employed spectral element Navier–Stokes code, Nek5000, has been co-designed with many algorithmic and implementation innovations for multi-core environments, using very high order (up to 32<sup>nd</sup>-order) elements, resolving duct flow at a record-high Reynolds number (for this particular internal flow) of 100,000, on over 100,000 cores overall. In both simulations, we emphasize feedback to the algorithm coming from the architecture.

---

### **Parallelisation and scalability of a linear advection diffusion code**

*Emanuele Ragnoli\*, Fearghal O'Donncha*

The feasibility and scalability of systems that apply advection-diffusion PDE models is of fundamental importance to applications in a large range of scientific domains. For example, in geophysics, it is of particular importance the real time tracking and forecasting of a density in a fluid, with a prescribed degree of precision, like an oil spill in a marine environment. Those requirements, temporal feasibility and prescribed accuracy, impose extremely high computational costs that cannot be feasibly resolved

except with many-core computing. In this paper we present the porting of a linear advection-diffusion transport code to parallel using domain decomposition (DD) and MPI. The objective of the research is to better elucidate on the performance and scalability of DD approaches and limitations to achieving many core performance.

In this study, the global domain is split into sub-domains and the equations are discretised on each sub-domain using Finite Element Method (FEM). Interface boundary conditions are enforced using an adaptive Dirichlet-Neumann (ADN), a Domain Decomposition iterative procedure to guarantee fidelity of solution. Parallelisation adopts a Message Passing Interface (MPI) between subdomains to communicate solutions of the ADN integrations. Within sub-domains further parallelization is achieved through a fine-grained parallelisation of the ADN solver while optimized multithreaded matrix operations compute the sub-domain solution. The performance of the scheme is then investigated with an idealized test case. Both the case of strong scaling (i.e. the overall spatial domain size remains constant regardless how many processors are used to solve it), and weak scaling (increasing domain size as number of processors increase) are investigated and a detailed profiling of the advection diffusion code with several configurations is presented, in order to find the optimum decomposition strategy that balances computation and communication expenses.

Chair: *Jinchao Xu*

**13:40~14:25 PL07 Xiaozhe Hu**

**Culture Centre**

---

### **Robust preconditioners for coupled problems**

*Xiaozhe Hu\**

Many mathematical models in physics, engineering, biology, and other fields are governed by coupled systems of partial differential equations (PDEs). An essential component, and usually the most time-consuming part of simulating coupled PDEs, is solving the large-scale and ill-conditioned linear systems of equations arising from the linearization and discretization of the PDEs. In this work, we generalize and improve the traditional framework of preconditioners on saddle point systems for several practical applications, including electromagnetism, magnetohydrodynamics, and poromechanics. We show that the new preconditioners are robust with respect to certain physical and discretization parameters and preserve important physical laws if necessary. Preliminary numerical experiments are presented to support the theory and demonstrate the robustness of our algorithms. This is joint work with James Adler (Tufts), Francisco Gaspar and Carmen Rodrigo (University of Zaragoza), Jinchao Xu and Ludmil Zikatanov (Penn State)

Chair: MS-organizer

**14:35~16:15 MS12-1 Robust solvers for multiphysics problems**

**Culture Centre**

---

### **Towards scalable and efficient solution of full Maxwell electromagnetics – multifluid plasma systems**

*John N. Shadid\*, Edward G. Phillips, Eric C. Cyr, Roger P. Pawlowski, Sean A. Miller*

The mathematical basis for the continuum modelling of multi-fluid plasma physics systems is the solution of the governing partial differential equations (PDEs) describing conservation of mass, momentum, and total energy for each fluid species, along with Maxwell's equations for the electromagnetic fields. This PDE system is non-self adjoint, strongly-coupled through source terms and first-order off diagonal operators, highly-nonlinear, and characterized by physical phenomena that span a very large range of length- and time-scales. To enable accurate and stable approximation of

these systems a range of spatial and temporal discretization methods are commonly employed. In the context of finite element spatial discretization these include nodal and discontinuous Galerkin methods of the fluid sub-systems, and structure-preserving (physics-compatible) approaches for the electromagnetics system. For effective time integration of the longer time-scale response of these systems some form of implicitness is required. Two well-structured approaches, of recent interest, are fully-implicit and implicit-explicit (IMEX) type methods. The requirement to accommodate disparate spatial discretizations, and allow the flexible assignment of mechanisms as explicit or implicit operators, implies a wide variation in unknown coupling, ordering, the nonzero block structure, and the conditioning of the implicit sub-system. These characteristics make the scalable and efficient iterative solution of these systems extremely challenging.

In this talk initial results for the stability and efficiency of applying fully-implicit and IMEX time integration to these multiple-time-scale systems will be presented. Additionally an overview of our approach to the development of scalable block preconditioners and initial weak parallel scaling results will also be presented.

---

### **A mixed-method $B$ -field finite-element formulation for incompressible, resistive magnetohydrodynamics**

---

*James Adler \**

Magnetohydrodynamics (MHD) models describe a wide range of plasma physics applications, from thermonuclear fusion in tokamak reactors to astrophysical models. These models are characterized by a nonlinear system of partial differential equations in which the flow of the fluid strongly couples to the evolution of electromagnetic fields. In this talk, we consider the one-fluid, viscoresistive MHD model in two dimensions. There have been numerous finite-element formulations applied to this problem, and we will briefly discuss the applications of two; a least-squares and mixed-method formulation. In the latter, we consider inf-sup stable elements for the incompressible Navier-Stokes portion of the formulation, Ned el ec elements for the magnetic field, and a second Lagrange multiplier added to Faraday’s law to enforce the divergence-free constraint on the magnetic field.

Regardless of the formulation, the discrete linearized systems that arise in the numerical solution of these equations are generally difficult to solve, and require effective preconditioners to be developed. Thus, the final portion of the talk, will involve a discussion of monolithic domain decomposition preconditioners, using an extension of a well-known relaxation scheme from the fluid dynamics literature, Vanka relaxation, to this formulation. To isolate the relaxation scheme from the rest of the domain decomposition method, we utilize structured grids, geometric interpolation operators, and Galerkin coarse grid operators. Numerical results are shown for the Hartmann flow problem, a standard test problem in MHD.

---

### **Iterative methods for nonlinear discrete MHD systems**

---

*Kaibo Hu\**

In this talk, we will first present structure-preserving finite element methods for MHD models which preserve the magnetic Gauss’s law strongly in the discretization. Then we present and analyze some iterative methods for the resulting nonlinear algebraic systems based on the well-posedness results. Energy stability is crucial for the well-posedness and convergence of the schemes. This is a joint work with Jinchao Xu.

---

### **A conforming enriched finite element method for elliptic interface problems**

---

*Hua Wang, Jinru Chen\**

A conforming enriched finite element method are proposed for elliptic interface problems with an interface-unfitted mesh. The conforming enriched finite element space is constructed based on the  $P_1$  conforming FE space. The standard conforming Galerkin method is considered without any penalty stabilization term. In our method,



the coefficients of the elliptic interface problem can be any positive broken  $C^1$ -continuous function. The error estimate in  $H^1$ -norm is proved to be optimal. Numerical simulations are given to illustrate the theoretical results.

Chair: MS-organizer

14:35~16:15 **MS13-2 Domain decomposition methods for nonlinear problems** **North Pole**

---

**Domain decomposition method for two-phase flow model using Global Jacobian**

---

*Kundan Kumar\*, Benjamin Ganis, Gergina Pencheva, Mary F. Wheeler, Ivan Yotov*

We consider a fully-implicit formulation for two-phase flow in a porous medium with capillarity, gravity, and compressibility in three dimensions. The method is implicit in time and uses the multiscale mortar mixed finite element method for a spatial discretization in a nonoverlapping domain decomposition context. The interface conditions between subdomains are enforced in terms of Lagrange multiplier variables defined on a mortar space. The novel approach in this work is to linearize the coupled system of subdomain and mortar variables simultaneously to form a global Jacobian. This algorithm is shown to be more efficient and robust compared to previous algorithms that relied on two separate nested linearization of subdomain and interface variables. We also examine various up-winding methods for accurate integration of phase mobility terms near subdomain interfaces. Numerical tests illustrate the computational benefits of this scheme.

---

**Robust iteration for nonlinear flows in highly heterogeneous media**

---

*Juan Galvis\**

We study robust iterative solvers for finite element systems resulting in approximation of steady-state Richards' equation in porous media with highly heterogeneous conductivity fields. It is known that in such cases the contrast, ratio between the highest and lowest values of the conductivity, can adversely affect the performance of the preconditioners and, consequently, a design of robust preconditioners is important for many practical applications. The proposed iterative solvers consist of two kinds of iterations, outer and inner iterations. Outer iterations are designed to handle nonlinearities by linearizing the equation around the previous solution state. As a result of the linearization, a large scale linear system needs to be solved. This linear system is solved iteratively (called inner iterations), and since it can have large variations in the coefficients, a robust preconditioner is needed. First, we show that under some assumptions the number of outer iterations is independent of the contrast. Second, based on the recently developed iterative methods, we construct a class of preconditioners that yields convergence rate that is independent of the contrast. Thus, the proposed iterative solvers are optimal with respect to the large variation in the physical parameters. Since the same preconditioner can be reused in every outer iteration, this provides an additional computational savings in the overall solution process. Numerical tests are presented to confirm the theoretical results.

---

**Linear and nonlinear domain decomposition techniques for the steady-state resolution of electroelastoacoustic problems**

---

*Alexandre Halbach\*, Christophe Geuzaine*

In this talk we compare linearly as well as nonlinearly preconditioned domain decomposition methods with overlap for large arrays of nonlinear, coupled 3D electroelastoacoustic problems. The iteration unknowns for the domain decomposition are either taken only at the subdomain interfaces or in the whole volume. Because of the nonlinearity a pure harmonic excitation leads to multiple harmonics in the displacement field, in the pressure field and in the electric potential. A multi-harmonic

resolution method automatically handles this. Domain decomposition convergence is investigated for the different harmonic frequencies.

### **Scalable domain decomposition preconditioners for cardiac electro-mechanical models**

---

*Luca F. Pavarino\*, P. Colli Franzone, S. Scacchi, S. Zampini*

We present scalable domain decomposition solvers for a 3D strongly coupled electro-mechanical model consisting of four coupled components: a) the quasi-static transversely isotropic finite elasticity equations for the deformation of the cardiac tissue; b) the active tension model for the intracellular calcium dynamics and cross-bridge binding; c) the anisotropic Bidomain model for the electrical current flow through the deforming cardiac tissue; d) the membrane model of ventricular myocytes, including stretch-activated channels. The numerical simulations are based on our finite element parallel solver, which employs Multilevel Additive Schwarz preconditioners for the solution of the discretized Bidomain equations and Newton-Krylov methods with AMG or BDDC preconditioners for the solution of the discretized nonlinear finite elasticity equations. We present the results of several parallel simulations showing the scalability of both linear and nonlinear solvers and their application to the study of re-entrant waves in presence of different mechano-electrical feedbacks

Chair: MS-organizer

**14:35~16:15 MS02-2 Fast solvers for discontinuous Galerkin methods Northern Light**

### **Agglomeration based h-multigrid solution strategies for dG discretizations of the incompressible Navier-Stokes equations**

---

*Francesco Bassi, Lorenzo Botti\*, Alessandro Colombo*

When dealing with incompressible fluid flow computations the efficiency of the solution strategy (that is solving the system of equations resulting from the spatial discretization at hand) is of primary importance for the success of the numerical method [Benzi et al., *Acta Numerica*, 14:1--107, 2005]. In this work we consider agglomeration based h-multigrid strategies [Bassi et al., *Mathematical Models and Methods in Applied Sciences, Special Issue on Recent Techniques for PDE Discretizations on Polyhedral Meshes*, 24(8):1495--1539, 2014; Antonietti et al., <http://arxiv.org/abs/1412.0913>, 2014] for accelerating the numerical solution of coupled variable DG discretizations of the incompressible Navier-Stokes equations.

Thanks to geometrical flexibility of physical frame dG discretizations [Bassi et al., *Journal of Computational Physics*, 231(1):45--65, 2012; Cangiani et al., *Mathematical Models and Methods in Applied Sciences*, 24(10):2009--2041, 2014; Giani and Houston, *Numerical Methods for Partial Differential Equations*, 30(4):1342--1367, 2014] we construct coarse problems relying on coarse grids generated by recursive agglomeration of the fine grid. Interestingly, the sole requirement of the solver strategy is a fine mesh of the computational domain and no restrictions are imposed on the mesh topology.

Both the expense of building coarse grid operators and the performance of the resulting multigrid iteration are investigated. For the sake of efficiency coarse grid operators are inherited through element-by-element  $L^2$  projections, avoiding the cost of numerical integration over agglomerated elements. Using a single iteration of preconditioned FGMRES as smoothing strategy, the multigrid convergence is uniform with respect to the number of levels and the typical multigrid efficiency is closely approached on model problems. The ability to beat direct solvers on arbitrarily unstructured low quality grids and the appealing performance obtained on parallel real-life computations might revert the common belief that discontinuous Galerkin discretizations are more expensive to solve as compared to standard finite element and finite volume formulations.

## **Nonoverlapping Schwarz preconditioners for high-order DG methods: optimal bounds**

---

*Paola F. Antonietti\** (given by *Iain Smears*)

We present the derivation of  $hp$ -optimal spectral bounds for a class of domain decomposition preconditioners based on the Schwarz framework for high-order discontinuous Galerkin finite element approximations of second-order elliptic partial differential equations [Antonietti et al., Int. J. Numer. Anal. Model., 13(4):513--524, 2016]. In particular, we improve the bounds derived in [Antonietti et al., J. Sci. Comput., 46(1):124--149, 2011] in the sense that the resulting estimate on the condition number of the preconditioned system is not only explicit with respect to the coarse and fine mesh sizes  $H$  and  $h$ , respectively, and the fine mesh polynomial degree  $p$ , but now also explicit with respect to the polynomial degree  $q$  employed for the coarse grid solver. We show that the resulting spectral bounds are of order  $p^2H/(qh)$  for the  $hp$ -version of the discontinuous Galerkin method. The key aspect is the derivation of an  $hp$ -optimal approximation property between the coarse and fine finite element spaces. Here, we follow the recent analysis presented in [Smears, Technical report, arXiv:1409.4202, 2015] for problems posed within the  $H^2$  context to deduce analogous results in the present setting.

## **Advances in hybrid discontinuous Galerkin methods**

---

*Eun-Jae Park\**

In this talk we present a priori and a posteriori error analysis for hybrid discontinuous Galerkin (HDG) methods for elliptic equations. First, we present arbitrary-order HDG methods to solve the Poisson problem and propose residual type error estimators. Next, we present guaranteed type error estimators by postprocessing scalar and flux unknowns. Then, we consider diffusion problems with discontinuous coefficients. Several numerical examples are presented to show the performance of the methods.

Chair: MS-organizer

**14:35~16:15 MS15-1 Heterogeneous domain decomposition methods Polar Bear**

## **Domain decomposition partition of unity methods**

---

*Gabriele Ciaramella\*, Martin J. Gander*

In many applications, mathematical and numerical models involve simultaneously more than one single phenomenon. In this situation different equations are used in possibly overlapping subregions of the domain in order to approximate the physical model and obtain an efficient reduction of the computational cost. The coupling between the different equations must be carefully handled to guarantee accurate results. However in many cases, since the geometry of the overlapping subdomains is neither given a-priori nor characterized by coupling equations, a matching relation between the different equations is not available; see, e.g. [Gander and Michaud, DDM in Science and Engineering XXI, 2015] and references therein. To overcome this problem, we introduce a new methodology that interprets the (unknown) decomposition of the domain in a fuzzy sense [Zadeh, Information and Control, 1965]: each subdomain is associated to a partition of unity function. Then, by exploiting the feature of the partition of unity method developed in [Babuska and Melenk, Int. J. Num. Meth. in Eng., 1996] and [Griebel and Schweitzer, SIAM J. on Sci. Comp., 2000], we define a new domain-decomposition strategy that can be easily embedded in infinite-dimensional optimization settings. This allows us to develop a new optimization methodology that is capable to design coupling mechanisms between the different approximate equations. Numerical experiments demonstrate the efficiency of

the proposed framework

---

### **Optimized Schwarz methods in the Stokes-Darcy coupling**

---

*Luca Gerardo-Giorda\*, Marco Discacciati*

The Stokes-Darcy problem describes the filtration of an incompressible fluid through a porous medium. The model couples the Stokes equation in the fluid region with the Darcy equation in the porous medium region through the surface separating the two physical domains. Its multi-physics nature makes it suitable to splitting methods typical of domain decomposition techniques, where the solution of the global problem is recovered by iteratively solving each subproblem separately.

In the framework of domain decomposition methods, Optimized Schwarz Algorithms are based on Robin-type interface conditions (see [Discacciati et al., SIAM J. Numer. Anal., 45(3):2193--2213, 2007]) and guarantee convergence also in the absence of overlap between subdomains, making them a natural framework to deal with spatial decompositions of the computational domain that are driven by a multi-physics problem such as the one at hand. Following the ideas developed in [Gerardo-Giorda et al., SIAM J. Numer. Anal., 48(6):2091--2116, 2010] for Fluid-Structure Interaction problems, we optimize the performance of the corresponding algorithm, both in terms of an iterative solver and as a preconditioner for the fully coupled problem [Discacciati and Gerardo-Giorda, Submitted, 2016]. A practical strategy to compute optimal Robin coefficients is presented which takes into account both the physical parameters of the problem and the size of the mesh. Numerical results show the effectiveness of our approach.

---

### **Optimized Schwarz methods for heterogeneous problems**

---

*Tommaso Vanzan\*, Martin J. Gander*

Optimized Schwarz methods have increasingly drawn attention over the last decades because of their improvements in terms of robustness and computational cost with respect to the classical Schwarz algorithm [Gander, *Optimized Schwarz methods*, SIAM J. Numer. Anal., 2006]. Optimized transmission conditions have been obtained through analytical or numerical procedures in many different situations which mostly involved the same PDE on each of the subdomains, see [Gander et al., *Optimized Schwarz methods without overlap for the Helmholtz equation*, SIAM J. Sci. Comp., 2002], [Dubois and Gander, *Optimized Schwarz methods for a diffusion problem with discontinuous coefficient*, Numer. Algor., 2015] and references therein. There are however also exceptions with different PDEs in different subdomains, see e.g. [Giorda and Perego, *Optimized Schwarz methods for the bidomain system in electrocardiology*, ESAIM Math. Mod. and Numer. Anal., 2013], [Discacciati and Giorda, *Optimized Schwarz methods for the Stokes-Darcy coupling*, submitted to IMA J. Numer. Anal.]. Due to their convergence properties in the absence of overlap, optimized Schwarz methods are a natural framework to study heterogeneous phenomena where the spatial decomposition is provided by the multi-physics of the problem. In this presentation we will discuss some analytical results for optimized transmission conditions for heterogeneous problems.

---

### **Heterogeneous domain decomposition**

---

*Martin J. Gander, Laurence Halpern, Véronique Martin\**

Heterogeneous domain decomposition methods are domain decomposition methods where different models are solved in different subdomains. Models can be different because problems are heterogeneous, i.e. there are connected components with different physical properties, or because one wants to approximate a homogeneous object with different approximations, depending on their validity and cost. We are interested in the latter case.

Typically we have a complex, expensive model which would give the best possible solution, and we aim to design heterogeneous domain decomposition methods to give

a good approximation to this best possible solution at a lower computational cost. Then, to quantify the quality of the method, it is important to compare the expensive solution on the entire domain to the coupled solution. In this talk, we use multiscale analysis to compare three heterogeneous domain decomposition methods from the literature (see [Gastadi and Quarteroni, Appl. Numer. Math., 6:3-31, 1989/90] and [Gander, Halpern and Martin, Num. Algo., 73:167--195, 2016]) to solve time dependent advection reaction diffusion equations, with advection reaction approximations in parts of the domain. We give sharp error estimates as the viscosity goes to zero. We show numerical results in 1-D and 2-D.

## Coffee Break at Radisson and Culture Centre

Chair: MS-organizer

<b>16:45~18:50</b>	<b>MS03 Domain decomposition methods in biomedical modelling and simulation</b>	<b>Culture Centre</b>
--------------------	---	-----------------------

### **A heterogeneous finite element joint model to support orthopaedic hip surgery planning**

*Ralf Kornhuber, Oliver Sander and Jonathan Youett\**

In this talk we present a dynamic heterogeneous finite element hip joint model and show an application of this toolbox in virtual medicine. The geometric model comprises the proximal femur, pelvis and the three main ligaments. The bones are modelled as geometrically exact St. Venant–Kirchhoff materials. Between the femoral head and the acetabulum dynamic large deformation contact is imposed. The ligaments are represented by quasi- static one-dimensional Cosserat rods which support bending and shearing as well as stretching and torsion. Furthermore, we enforce non-penetration of the ligaments and bones. The heterogeneous coupling of the lower dimensional Cosserat ligaments and the 3D bone continua leads to non-standard transmission conditions at the insertion sites. For the discretisation of the continua we apply a contact-stabilised midpoint rule and the dual mortar finite element method while the manifold-valued Cosserat rods are discretised by geodesic finite elements. The resulting sub-problems are non-convex constrained minimisation problems which we solve using a novel globally convergent inexact filter-trust-region method. The discrete heterogeneous problem is solved by a Dirichlet–Neumann algorithm, leading to decoupled large deformation contact problems for the bones and obstacle problems for the ligaments. As a first application of this model we present a patient- specific range of motion and femoroacetabular impingement analysis of the human hip joint.

### **Coupling eikonal and monodomain equations**

*Martin Weiser\*, Adrian Sali*

Electrical excitation of the myocardium is described by the bidomain, or, slightly simplified, monodomain equations, a system of a reaction-diffusion equation and pointwise ODEs. Due to fast reaction and slow diffusion, the emerging excitation fronts are thin, such that fine grids and small time steps are required for acceptable accuracy. As a much simplified model, eikonal equations describe only the excitation time at which the front pass through a certain point in space. They are stationary elliptic equations with rather smooth solutions, and can be solved on coarse grids very quickly. In many situations, their approximation of monodomain solutions is quite good, but some settings cannot be reasonably described by eikonal approaches. We will investigate a heterogeneous domain decomposition, coupling eikonal to monodomain equations, and solving the expensive monodomain model only where needed. Appropriate coupling conditions are formulated, leading to a time dependent

solution of eikonal equations. Numerical examples illustrate the accuracy gain compared to pure eikonal models and the speedup compared to pure monodomain simulations.

---

### **A parallel solver for electromechanical simulations of ventricular hypertrophy**

*P. Colli Franzone, F. Del Bianco, L. Fassina, L. F. Pavarino, S. Scacchi\*, S. Zampini*

Cardiac hypertrophy is a pathological remodelling caused by a long-term pressure or volume overload inside the ventricles, see e.g. [Taber, *Appl. Mech. Rev.*, 48:487--545, 1995]. The main phenotypic consequence is a progressive intracellular deposition of new sarcomere units in parallel or in series to the pre-existent ones, see [Goktepe et al., *J. Theor. Biol.*, 265:433--442, 2010]. In the present study, we investigate the effects of such growth on the electromechanical response of the ventricular tissue. Our strongly-coupled electromechanical model joins together an electrophysiological model, a cellular active tension generation model and a finite elasticity model. The space discretization is performed by trilinear finite elements, whereas the time discretization by a decoupled semi-implicit method. The parallel solver employed for the solution of non-linear mechanics was developed in [Pavarino et al., *Comput. Meth. Appl. Mech. Eng.*, 295:562--580, 2015] and is based on a Newton-Krylov method, where GMRES is accelerated by a BDDC preconditioner.

---

### **Simulating wear on total knee replacements**

*Ansgar Burchardt\*, Oliver Sander*

With rising life expectancy, an increasing number of people needs implants to replace their natural joints and needs them for a longer time. To avoid implant revisions, with all the risks additional surgery brings, an increased life-time of implants is therefore important. One of the limiting factors for more durable implants is wear.

Design and later admission of new implant models requires in-vitro experiments using mechanical knee simulators with controlled environment conditions and a standardized gait cycle. This is expensive and time consuming: prototypes need to be manufactured, and a run in the simulator takes about three months.

Recreating the physical experiments in a computer simulation avoids these limitations during the design cycle: instead of prototypes, only meshes have to be created and numerical simulations take hours instead of months. We therefore present a software knee simulator based on the Dune framework to simulate wear on total knee replacements following the standardized test procedure described in ISO 14243-1. Our model uses frictionless contact between elastic bodies using a dual mortar formulation. For a single gait cycle, we use a quasi-static time discretization and compute the wear using Archard's law. On a second, larger time scale, we keep track of the local mass loss and modify the implant geometry accordingly.

CAD data and experimental results were available for two different knee implant models. Our simulation results using material constants from the literature are comparable to the experimental results for both the total wear mass loss and its spatial distribution.

---

### **A fictitious domain method for fluid-structure interaction based on the pseudo- $L^2$ -projection between non conforming overlapping meshes**

*Maria Nestola\*, Patrick Zulian, Alena Kopanicakova, Cyrill VonPlanta, Rolf Krause*

The use of variational approaches for the stable information transfer between non-matching meshes along surfaces or overlapping meshes in volumes, represents an essential ingredient for the numerical discretization of coupled multi-physics or multi-scale problems arising from contact or fracture mechanics, fluid-structure interaction (FSI) and geoscience. In particular, in the context of the FSI, several approaches have been developed, which can be classified in boundary-fitted and not boundary-fitted

method. Although the boundary-fitted method, such as the Arbitrary Lagrange Eulerian approach, is known to guarantee accurate results at the interface between the solid structure and the fluid flow, for scenarios that involve large displacements and/or rotations, the fluid grid becomes severely distorted, thus affecting both the numerical stability of the problem and the accuracy of the solution. In particular, it is prohibitively hard to preserve a good mesh quality in the simulation of heart valves where the leaflets deformation and their contact during the valve closure change the topology of the fluid domain. To this aim, non- boundary-fitted approaches, such as the Immersed Boundary and the Fictitious Domain, have been introduced and widely used for simulating the complex dynamics of heart valves. In this work we present a new framework for FSI simulations. In particular, by taking inspiration from the fictitious domain method proposed by Baaijens in [Baaijens, International Journal for Numerical Methods in Fluids, 35(7):743-761, 2001], we employ the finite-element method for discretizing both the equations of the solid structure and the fluid flow. The two codes are coupled by using an pseudo  $-L^2-$  projection approach to transfer the data from the fluid to the solid grid and vice-versa. The way such a transfer operator is constructed may affect the convergence, the accuracy, and the efficiency of the numerical method adopted to solve the FSI problem. In addition, although the serial case already may already lead to a technical difficulties, in the parallel approach, when the associated meshes are distributed arbitrarily onto different processors, the situation becomes even more complicated, since the neighboring elements have to be identified in a global parallel search before the quadrature can be carried out. In this talk we describe in details the approach adopted to solve the FSI problem as well as the main ingredients to get a stable transfer operator between non-matching associated meshes [Zullian et al., SIAM J. on Sci. Comp., 38(3): C307--C333, 2016]. Moreover, some details concerning the solver adopted for the solid problem are provided. Indeed, the elastodynamic equations are solved by adopting a geometric multigrid method where a hierarchy is generated by progressively refining a coarse mesh into finer levels. In this case, the use of the same variation approach adopted to transfer the data between the fluid and the solid grids allows for the construction of prolongation/restriction operator without requiring the additional programming of complex parallel code. For validation and evaluation of the accuracy of the proposed methodology, we present results for an FSI benchmarking configuration which describes the self-induced oscillating deformations of an elastic beam in a flow channel. Moreover, applications to the FSI simulations of a bioprosthetic aortic valve are presented.

Chair: MS-organizer

**16:45~18:50 MS09 Parallel approaches for PDE based mesh generation North Pole**

### **A parallel moving mesh method based on optimal transport**

*Emily Walsh\**

An r-adaptive moving mesh method which is suitable for the numerical solution of partial differential equations (PDEs) in several spatial dimensions will be presented. This method is based on equidistribution of a scalar monitor function combined with optimal transport theory. The mesh is obtained by taking the gradient of a scalar function which satisfies a nonlinear parabolic PDE. It will be shown that whilst this method only requires the solution of one simple scalar time-dependent equation in arbitrary dimension, the coupled system of mesh PDE together with some underlying PDE, is challenging to solve. We consider here a parallel approach using a Schwarz domain decomposition algorithm, which has previously been applied to a class of time dependent MMPDEs based on equidistribution. We will study the convergence properties of this algorithm in one spatial dimension, considering both Neumann and periodic boundary conditions. Numerical examples will be presented in which either the monitor function is prescribed in advance, or is given by the solution of a partial differential equation.

## **Space-time adaptive meshing with the XBraid library**

---

*Jacob B. Schroder\*, Robert D. Falgout, Ben O'Neill, Thomas A. Manteuffel*

The need for parallelism in time is being driven by changes in computer architectures, where future speedups will be available through greater concurrency, but not faster clock speeds, which are stagnant. This leads to a bottleneck for sequential time marching schemes, because they lack parallelism in the time dimension. In this talk, we examine the multigrid reduction in time (MGRIT) algorithm, which is an approach for exploiting parallelism in the time dimension and is designed to wrap existing time integration routines and build on existing codes. Our experiments will use the XBraid library, an open source implementation of MGRIT. However since many current simulation codes rely on adaptivity in both space and time, XBraid must also be extended to accommodate such adaptivity. The goal is an approach which generates adaptive space-time meshes in parallel. We will discuss recent approaches taken in XBraid to support such adaptivity, and explore their effectiveness in the context of several application problems that employ a combination of spatial mesh refinement, spatial mesh motion, and temporal refinement.

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344, LLNL-ABS-704022

## **Optimally-transported meshes for global weather prediction**

---

*Andrew McRae\*, Colin J. Cotter, Christopher J. Budd*

We consider the generation of meshes adapted to a scalar monitor function through equidistribution. Together with an optimal transport condition, this leads to a nonlinear equation of Monge-Ampre type for generating meshes on the plane. As our interest is in numerical methods for global weather prediction, we must formulate the equivalent equation for performing this task on the sphere. Finally, we present robust iterative numerical methods for generating appropriate meshes, using finite element methods to solve the nonlinear equations.

## **Towards a parallel-in-time multigrid solver for fluid-structure interaction problems in real-world applications**

---

*Andreas Henthaler\*, Robert D. Falgout, Jacob B. Schroder, David A. Nordslettern, Oliver Röhrle*

We present a non-intrusive multigrid-reduction-in-time (MGRIT) algorithm<sup>1</sup> that provides an additional layer of parallelism for the efficient solution of a time-dependent and generally nonlinear system. It is based on a true multilevel approach with multigrid techniques (temporal grid hierarchy, V- and F-cycles, relaxation and coarse-grid updates) applied to the temporal domain. To provide a thorough basis for further investigations of multi-physics problems (e.g. fluid-structure interaction phenomena in cardiac biomechanics), we study the application of the MGRIT algorithm to single-physics problems, such as flow over a moving domain. We focus on properties of the space-time solver, demonstrate that traditional time stepping schemes (i.e. direct temporal solvers) and the MGRIT solver (i.e. iterative temporal solver) solve the same system of equations and investigate multigrid scaling as well as weak and strong scaling.

## **Probabilistic domain decomposition for parallel mesh generation**

---

*Alexander Bihlo, Ronald D. Haynes\*, Leah Genge*

We show how probabilistic domain decomposition can be used to obtain a parallel version of the classical Winslow mesh generator. We evaluate the stochastic form of the exact solution of the Winslow mesh generator numerically to provide the approximate solution for the mesh along suitably chosen interfaces that split the entire



computational domain into smaller subdomains. Once the interface solutions are obtained using Monte Carlo methods, the mesh over the sub-domains can be computed completely in parallel using a deterministic solver. We present several numerical examples for mesh generation on the plane and on the sphere. Stochastic domain decomposition for Maxwell's equations

Chair: *Jun Zou*

**16:45~18:50**    **Contributed Talks Session 2: Domain decomposition algorithms and applications**    **Northern Light**

---

**Design of small coarse spaces for two level overlapping Schwarz algorithms for problems with irregular subdomains**

*Clark Dohrmann, Olof Widlund\**

The design of effective and small coarse spaces are central in the development of domain decomposition algorithms in particular for overlapping Schwarz algorithms. Further developing ideas first presented at DD21, we develop and analyze coarse spaces with elements only associated with the subdomain vertices of potentially quite irregular subdomains such as those generated by mesh partitioners. We relate our current work to results on two dimensional problems published in 2012 in SINUM. Both scalar elliptic and problems of linear elasticity will be discussed. We will also describe a related algorithm due to Juan G. Calvo for three-dimensional problems in  $H(\text{curl})$ .

---

**Multigrid methods for  $H(\text{div})$  with nonoverlapping domain decomposition smoothers**

*Duk-Soon Oh\*, Susanne Brenner*

We design and analyze V-cycle multigrid methods for  $H(\text{div})$  problems discretized by the lowest order Raviart-Thomas element. Unlike standard elliptic problems, multigrid methods for the  $H(\text{div})$  problems with traditional smoothers do not work well. We introduce smoothers in the multigrid methods using nonoverlapping domain decomposition preconditioners that are based on substructuring. We prove uniform convergence of the V-cycle method. This is joint work with Susanne Brenner.

---

**A three-level BDDC method for incompressible Navier-Stokes equations**

*Martin Hanek\**

We deal with numerical simulation of incompressible flows using Balancing Domain Decomposition based on Constraints (BDDC). We compare two and three-level BDDC method for non-symmetric problems arising from the linearization of incompressible Navier-Stokes equations. Picard iteration is used for the linearization, and the arising linear systems are solved by BiCGstab method using one step of BDDC as the preconditioner. Numerical results for the benchmark problem of 3-D lid-driven cavity are presented.

---

**Solving large sparse linear systems with a variable s-step GMRES preconditioned by DD**

*David Imberti, Jocelyne Erhel\**

Krylov methods such as GMRES are efficient iterative methods to solve large sparse linear systems, with only a few key kernel operations: the matrix-vector product, solving a preconditioning system, and building the orthonormal Krylov basis. Domain Decomposition methods allow parallel computations for both the matrix-vector products and preconditioning by using a Schwarz approach combined with deflation (similar to a coarse-grid correction). However, building the orthonormal Krylov basis involves scalar products, which in turn have a communication overhead. In order to

avoid this communication, it is possible to build the basis by a block of vectors at a time, sometimes at the price of a loss of orthogonality. We define a sequence of such blocks with a variable size. We show through some theoretical results and some numerical experiments that increasing the block size as a Fibonacci sequence improves stability and convergence.

### **Non-local transmission operators for non-overlapping DDM with exponential convergence for Helmholtz equation**

---

*M. Lecouvez\*, P. Joly, F. Collino, B. Stupfel*

Scattering problems by electrically large objects are of wide interest in many fields. The numerical computation of such problems remains limited by computer resources, due to the large number of unknowns, especially when inhomogeneous materials are present. Domain decomposition methods (DDM) are particularly attractive for the solution of large finite elements problem. It is decomposed into several coupled sub-problems which can be solved independently, thus reducing considerably the memory storage requirements. Moreover, DDM are intrinsically well suited for numerical implementation on parallel computers. Non-overlapping domain decomposition methods for solving Helmholtz equation are considered here. The key point of these methods is how the sub-domains are coupled to each other, i.e. which transmission conditions are imposed on the interfaces between two sub-domains. The most commonly used transmission condition in electromagnetism is the one proposed by Despres in [B. Despres, SIAM, 197--206, 1993] or higher order ones [see e.g. M. Gander et al., SIAM JSC, 24, 38--60, 2002]. But these transmission conditions are local (i.e. expressed in terms of tangential differential operators) and generally lead to arithmetic convergence. In a general framework, we present a domain decomposition method for which an exponential convergence can be established. This framework introduces a class of non-local transmission operators, verifying some properties of positiveness and isomorphism between Sobolev spaces. A few numerical results highlighting the properties of our DDM will also be presented.

Chair: MS-organizer

**16:45~18:50 MS15-2 Heterogeneous domain decomposition methods North Pole**

### **INTERNODES: an interpolation-based approach for the numerical solution of PDEs on nonconforming discretizations**

---

*Paola Gervasio\*, Simone Deparis, Davide Forti, Alfio Quarteroni*

We present a new approach for numerically solving PDE's on computational domains split into two (or several) subdomains featuring non-conforming interfaces. Non-conformity can be of either grid, polynomial or geometric type, being the latter the situation when the non-matching grids (from the two sides of a curved interface) may either overlap or feature thin holes. Local PDE's can be either of the same type (homogeneous) or not (heterogeneous). In each subdomain, classic Galerkin methods (such as finite elements or spectral elements) are used to approximate the local solution. The two sides of each interface are identified one as master, the other as slave. INTERNODES consists in using two different intergrid operators to transfer information across the interface between the two subdomains. The one from master to slave is used to ensure the continuity of the solution trace, while that from slave to master enforces the continuity of the normal flux. If Lagrange interpolation is used to define the intergrid operators, INTERNODES is as accurate as the mortar method, in spite of the fact that the implementation complexity of the former method is less involved than that of the latter [Deparis et al., Computer and Fluids, 2016]. The case of geometrically non-conforming interfaces can be successfully tackled by using the Radial Basis Functions (RBF) interpolation in setting the intergrid operators [Deparis et al., SIAM J. Sci. Comp., 36:A2745--A2762, 2014]. INTERNODES has been

successfully applied to fluid structure interaction problems [Deparis et al., In Adv. in Comp. Fluid-Structure, 2015].

---

### **Embedded boundary methods and domain decomposition**

---

*Santiago Badia\*, Francesc Verdugo*

Embedded boundary methods are very attractive, because eliminate the need to define body-fitted meshes. In particular, at large scales, the meshing step is a bottleneck of the simulation pipeline, since mesh generators do not usually scale properly. In some other situations, like in additive manufacturing simulations, the geometry evolves in time, and the use of body-fitted meshes is not suitable. On the contrary, algorithms to create adaptive cartesian meshes are highly scalable.

However, using embedded boundary methods, one can destroy the condition number of the linear systems to be solved, since cut elements can have close to zero support. As a result, these techniques require direct linear solvers, since standard preconditioned iterative solvers are not robust and scalable.

In this work, we take as a starting point a balancing domain decomposition by constraints (BDDC) preconditioner. Next, we consider a recent physics-based version of the method that is robust with respect to high variations of the materials. Finally, we show to how to make these preconditioners robust also for embedded boundary methods for coercive PDEs, by a proper modification of the inter-subdomain constraints.

---

### **Reduced representation of the Steklov-Poincaré operator for coupled multi-physics systems**

---

*Matteo Aletti\*, Damiano Lombardi*

Motivated by the study of a complex multi-domain and multi-physics system such as the human eye, we have developed a technique to reduce the computational cost of such simulations. The framework we are dealing with consists of two (or more) compartments in interaction. The first, main, compartment is represented by a generic nonlinear PDE while the other, secondary, compartment is described by a linear, stationary and homogeneous PDE. The idea is to replace, in the Domain Decomposition iterations, the solution of the secondary problem by an approximation of the corresponding Steklov operator. This is obtained by exploiting an offline-online strategy. During the offline phase a set of basis functions of the interface is obtained by solving a Laplace-Beltrami eigenproblem on the interface, the operator is then evaluated on these functions and the corresponding images are used to build the reduced representation that is used during the online phase. The approach has been tested on three-dimensional fluid-fluid and fluid-structure problems. Results, in terms of convergence and speed up, as well as the limitations of the proposed approach will also be discussed.

---

### **Multipatch discontinuous Galerkin isogeometric analysis on non-matching domain decompositions**

---

*Christoph Hofer, Ulrich Langer\*, Ioannis Touloupoulos*

We propose and analyze new multipatch discontinuous Galerkin (dG) Isogeometric Analysis (IgA) techniques for discretizing elliptic diffusion problems. We admit segmentations crimes, i.e., incorrect domain decompositions containing undesirable gaps and / or overlaps between the subdomains (patches). The normal fluxes on the non-matching interfaces, which are coming from the interior of the gap or overlapping regions must be approximated by Taylor expansions using the available values of the solution computed on the faces of the gap and overlapping regions. These approximations are used in order to build up the numerical fluxes of the final dG IgA scheme. This work is a continuation of the work presented in our two previous papers [Hofer et al., *Discontinuous Galerkin isogeometric analysis of elliptic diffusion problems on segmentations with gaps*, SIAM J. Sci. Comp., 2016, accepted] and

[Hofer and Touloupoulos, *Discontinuous Galerkin isogeometric analysis of elliptic problems on segmentations with non-matching interfaces*, Computers and Mathematics with Applications, 2016, accepted], where we have treated decompositions having only gap regions. The resulting large-scale linear systems of IgA equations are solved by means of efficient domain decomposition methods based on dual-primal tearing and interconnecting technologies. These so-called dG IETI-DP solvers, introduced in [Hofer and Langer, *Dual-Primal isogeometric tearing and interconnecting solvers for multipatch dG-IgA equations*, Comput. Meth. Appl. Mech. Engrg, 2016, accepted] for matching interfaces, can easily be parallelized. We also present numerical results for a series of 2d and 3d test problems that impressively confirm our theoretical results. More information can be found in [Hofer et al., Tech. Report 2016-23, RICAM, 2016]

## Wednesday, February 8<sup>th</sup> 2017

	Culture Centre	North Pole	Northern Light	Polar Bear
06:00	Breakfast served			
07:10	Bus departs from Coal Miners' Cabin			
07:30	PL08 Susanne Brenner			Culture Centre
08:15	Stretch your legs			
08:25	MS12: 5-8	MS11: 1-4	MS19: 1-4	MS07: 1-4
10:05	Coffee at Culture Centre and Radisson			
10:35	MS12: 9-11	MS11: 5-7	MS19: 5-7	MS07: 5-7
11:50	Lunch at Radisson			
	Excursion			

Chair: *Hyea Hyun Kim*

**07:30~08:15 PL08 Susanne Brenner Culture Centre**

### An additive Schwarz analysis of multiplicative Schwarz methods

*Susanne Brenner\**

Multiplicative Schwarz methods and additive Schwarz methods are the two main classes of iterative methods since the time of Gauss and Jacobi. These two classes of methods share a common framework, but their convergence analyses have historically followed separate paths. On one hand the additive theory is simple and understandable by all. On the other hand the multiplicative theory is sophisticated and mysterious to many. In this talk we will present a convergence analysis of multiplicative Schwarz methods that is based on the additive theory and discuss its applications to multigrid and domain decomposition methods.

Chair: MS-organizer

**08:25~10:05 MS12-2 Robust solvers for multiphysics problems Culture Centre**

### A parallel-in-time algorithm for variable step size multistep methods

*M. Lecouvez\*, R.D. Falgout, C.S. Woodward*

As the number of cores increases on current and future architectures, the natural sequential approach to time integration is becoming a more serious bottleneck for achieving high scalability. One alternative to overcome this problem is the use of multigrid-in-time algorithms such as MGRIT [Falgout et al., SIAM J. Sci. Comput., 36(6):C635-C661, 2014]. Although first designed for one-step methods, we apply the MGRIT algorithm to multistep *Backward Difference Formula* (BDF) methods for the integration of fully implicit *Differential Algebraic Equations* (DAE) on variable time step grids. To obtain efficient time integration methods, adaptivity in time is indeed a requirement as soon as multiscale events are simulated. Our step function solves the nonlinear problem

$$F(t_n, y_n, \dot{y}_n) = 0, \quad \dot{y}_n = \sum_{j=0}^s \alpha_{n,j} y_{n-j},$$

where  $s$  is the order of the BDF method used, and the coefficients  $\alpha_{n,j}$  depend on the order  $s$  and the previous timestep size. We will present one approach for implementing variable stepsize BDF methods in a parallel-in-time context based on the XBraid software library. Results from power grid applications will be presented to demonstrate the efficiency of our adaptive- and parallel-in-time method.

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC.

---

### **Abstract multigrid method**

*Jinchao Xu\**

In this work, we present a general framework for the design and analysis of Algebraic or Abstract Multi-Grid (AMG) methods. Given a smoother, such as Gauss-Seidel or Jacobi, we provide a general approach to the construction of a quasi-optimal coarse space and we prove that under appropriate assumptions the resulting two-level AMG method for the underlying linear system converges uniformly with respect to the size of the problem, the coefficient variation, and the anisotropy. Our theory applies to most existing multigrid methods, including the standard geometric multigrid method, the classic AMG, energy-minimization AMG, unsmoothed and smoothed aggregation AMG, and spectral AMGe.

---

### **A unified approach to the design and analysis of AMG**

*Jinchao Xu, Hongxuan Zhang\*, Ludmil Zikatanov*

In this work, we present a general framework for the design and analysis of two-level AMG methods. The approach is to find a basis for locally-the-best coarse space, such as the space of constant vectors for standard discretizations of scalar elliptic partial differential equations. The locally defined basis elements are glued together using carefully designed linear extension maps to form a global coarse space. Such coarse spaces, constructed locally, satisfy global approximation property and by estimating the local Poincaré constants, we obtain sharp bounds on the convergence rate of the resulting two-level methods. To illustrate the use of the theoretical framework in practice, we prove the uniform convergence of the classical two level AMG method for finite element discretization of a jump coefficient problem and anisotropic problems on a shape regular mesh.

---

### **Multigrid method for a class of new mixed discretization of linear elasticity**

*Shihua Gong, Shuonan Wu\*, Jinchao Xu*

In this talk, we will first present a family of new mixed finite elements (in terms of the symmetric stress tensor and the displacement variables) for the linear elasticity in any spatial dimension. By using a hybridization process, namely introducing some Lagrangian multipliers to impose interelement continuity for stress, the solution of the discretized indefinite system is reduced to that of obtain a symmetric semi-positive-definite (SSPD) system. We then develop a multigrid method to the resulting SSPD system and prove its uniform convergence with respect to both the mesh size and Poisson ratio. The new discretization (which gives a more accurate approximation for stress than displacement) together with the robust solver provide a new competitive approach for stress analysis in computational structure mechanics. Numerical tests are presented to validate the theoretical results.

**A parareal method for optimality systems in control***Julien Salomon\*, Felix Kwok, Martin J. Gander*

We introduce a new method to solve approximately optimality systems associated with optimal control problems. Our approach is based on parareal algorithm idea, i.e., it consists in using a coarse solver to build an approximation of the Jacobian matrix involved in a Newton Loop. We study the preconditioning properties of our solver and obtain necessary conditions for convergence. Numerical tests show the efficiency of our algorithm.

This work is done in collaboration with Felix Kwok and Martin J. Gander.

**Preconditioners for time-dependent PDE-constrained optimization based on parareal time-domain decomposition***Stefan Ulbrich\**

We consider optimization problems governed by time-dependent parabolic PDEs and discuss the construction of parallel preconditioners based on the parareal time domain decomposition method for the solution of quadratic subproblems which arise within SQP methods. In the case without control constraints, the optimality system of the subproblem is directly reduced to a symmetric PDE system, for which we propose a preconditioner that decouples into a forward and backward PDE solve. In the case of control constraints we apply a semismooth Newton method and apply the preconditioner to the semismooth Newton system. We prove bounds on the condition number of the preconditioned system which shows no or only a weak dependence on the size of regularization parameters for the control. We propose to use the parareal time domain decomposition method for the forward and backward PDE solves within the PDE preconditioner to construct an efficient parallel preconditioner. Numerical results are presented.

**An optimized domain decomposition method for optimal control problems***Bérandère Delourme\*, Laurence Halpern*

The present talk deals with the development of fast and robust numerical methods for the resolution of controllability problems for parabolic systems using a discretized version of the penalized Hilbert Uniqueness Method. This method, based on the simultaneous computation of an evolution problem toward positive time (the direct problem) and a backward in time problem (the adjoint problem), requires a large amount of computer operations. Optimized domain decomposition methods, furnishing a general framework for the parallel computing, permit to design efficient algorithms for solving this problem.

As a preliminary work, we consider the optimal control elliptic problem presented in [Benamou and Déprés, J. Comput. Phys., 136:68–82, 1997], which leads to the resolution of the Helmholtz-like problem  $-\Delta u - i\mu u = f$ , where  $\mu > 0$  is a penalization parameter that is supposed to be large. We apply the Robin-Schwarz domain decomposition algorithm (with parameter  $p$ ) to the previous equation. Optimizing the associated convergence factor leads to solving the min-max problem

$$\inf_{p \in \mathbb{C}} \max_{k \in [k_{\min}, k_{\max}]} \left| \frac{\sqrt{k^2 - i\mu - p}}{\sqrt{k^2 - i\mu + p}} \right|^2 \quad (k_{\min}, k_{\max}) \in \mathbb{R}^2, \quad (1)$$

In practice, the range of frequencies  $[k_{\min}, k_{\max}]$  depend on the characteristic mesh size  $h$  of the corresponding discretized problem, the parameter  $k_{\max}$  being conversely proportional to  $h$ .

General results in [Bennequin, Gander and Halpern, Math. Comp., 78:185—223, 2009] apply, proving that Problem (1) has a unique solution  $p^*$ . We use geometric properties of the problem to find a very simple formula of  $p^*$ . For large  $k_{\max}$ , we

provide asymptotic formulas for  $p^*$  and its associated optimal convergence factor. In particular as suggested by Boyer, Huber and Le Rousseau [Boyer et al., J. Math. Pures Appl., 93:240–276, 2010], we investigate the case where the penalization parameter  $\mu$  is linked to  $k_{\max}$  via  $\mu = k_{\max}^s$ ,  $s \geq 0$ .

### **Parallel D-D type domain decomposition algorithm for optimal control problem governed by parabolic partial differential equation**

---

*Bo Zhang, Jixin Chen\*, Danping Yang*

A parallel domain decomposition algorithm for solving an optimal control problem governed by a parabolic partial differential equation is proposed. This algorithm is based upon non-overlapping domain decomposition. In every iteration, the global problem is reduced to solve simultaneously some implicit sub-problems on many sub-domains by using explicit flux approximations near inner-boundaries at each time-step. Both a priori error bounds of optimal orders and optimal rates of convergence for the iterative schemes are presented. Numerical experiments are also performed to verify the theoretical analysis.

Chair: MS-organizer

08:25~10:05

**MS19-1 Space-time parallel methods based on waveform relaxation techniques**

**Northern Light**

### **Best approximation problem and application to Schwarz waveform relaxation**

---

*Laurence Halpern\**

Optimized Schwarz waveform relaxation methods are fast and robust (with respect to the size of the subdomains) domain decomposition iterative algorithms, based on the use of Robin or Ventcell transmission conditions. The coefficients minimize the maximum of the convergence factor over a compact set of frequencies. This minimization can be written in a best approximation setting, with  $n = 0$  for Robin, and  $n = 1$  for Ventcell,  $\inf_{p \in P_n} \sup_{k \in K} |\rho(f(k), p)|^2$ . When  $\rho$  is a linear function of  $p$  and  $f(k)$ , and  $K \subset \mathbb{R}$ , this is the well-known Tschebycheff approximation problem, treated by Tschebycheff [Saint Petersburg, 1899] and de la Vallée-Poussin [Gauthier-Villars, 1919]. They have shown that the solution is unique, and alternates  $n + 2$  times exactly. The proof goes by contradiction, supposing less than  $n + 2$  alternation points. Our function  $\rho$  is homographic, that is of the form  $\rho(z, p) = (z - p)/(z + p)$ . For the Laplace equation and  $n = 0$ , the problem can be solved by hand [Gander, SIAM J. Numer. Anal., 44(2):699-731, 2006]. It is somewhat more difficult for  $n = 1$  (Ventcell), and when the time comes into play and more dimensions, it becomes furthermore a best approximation problem in the complex plane, on which very little is known. The study of the problem, and the computation of the parameters require strong theoretical results, see [Bennequin et al., Numer. Math., 134(3):513--567, 2016] and [Bennequin et al., Math. Comp., 78(265):185-232, 2009], that deserve to be exploited and will be explained.

### **Experiments on the space-time FETI method**

---

*Hui Zhang\**

Space-Time domain decomposition methods have been developed in the last decade to improve the parallelism in the time. Many spatial domain decomposition methods have been extended in this sense. In my talk, I will present some experimental results on the space-time FETI method and make comparisons with the spatial FETI method to illustrate the potential of the new method.



## Optimization of overlapping waveform relaxation methods for RC circuits

---

Pratik M. Kumbhar\*, Martin J. Gander

Waveform relaxation (WR) methods are based on partitioning large circuits into sub-circuits which then are solved separately for multiple time steps. The classical WR approach was developed in the late 1980s for circuit solver applications. However it has slow convergence especially when long time windows are used. To overcome this issue, the optimized WR approach was introduced which is based on optimized transmission conditions that transfer information between the sub-systems more efficiently than classical WR. These methods are closely related to domain decomposition techniques. We analyze the effect of overlapping sub-circuits in optimized waveform relaxation techniques for RC circuits. We consider both infinite and finite RC circuits and study the effect of overlap on the convergence factor and on the optimization parameter alpha, which is an important parameter in the convergence factor. We also show how the optimal alpha behaves close to the limiting case of a particular circuit.

## Pipeline implementations of Dirichlet-Neumann and Neumann-Neumann waveform relaxation methods

---

Bankim Chandra Mandal\*

With the increasing availability of multiprocessor supercomputers, one needs to reformulate existing methods in a way that permits multiple parallel operations. In my talk, we reformulate two newly found WR-based methods, namely Dirichlet-Neumann and Neumann-Neumann Waveform Relaxation (DNWR and NNWR) so that successive waveform iterates can be computed in a parallel pipeline fashion after an initial start-up phase. We illustrate the performance of these reformulated algorithms applied to parabolic partial differential equation with numerical results.

Chair: MS-organizer

08:25~10:05 MS07-1 Recent progress on Trefftz methods

Polar Bear

## The Trefftz-UWVF method for Friedrichs systems with relaxation

---

Bruno Després\*, Christophe Buet, Guillaume Morel

We study the extension of the Trefftz-UWVF method to Friedrichs systems with stiff relaxation. A model problem which gathers most of the difficulties is as follows. Our notations cover the works [Kretschmar et al., *A Priori Error Analysis of Space, Time Trefftz Discontinuous Galerkin Methods for Wave Problems*, 2015; Kretschmar et al., *J. Comput. Appl. Math.*, 270:211--222, 2014; Gabard, *J. Comp. Physics*, 225(2):1961--1984, 2007]. The presentation below is restricted to the simplest situation.

Let  $\Omega \subset \mathbb{R}^d$  be a bounded domain with a regular boundary  $\partial\Omega$  and exterior normal denoted as  $\mathbf{n}(x)$  for  $x \in \partial\Omega$ . The simplest Friedrichs system is the transport equations which writes  $\mathbf{a} \cdot \nabla u = -\sigma u + f$  for  $x \in \Omega$ . Here  $\mathbf{a} \subset \mathbb{R}^d$  is a given velocity vector and  $\sigma \geq 0$  is the absorption. The velocity may be function of  $x$ . In this case one may assume  $\mathbf{a} \in C^1(\bar{\Omega})$ . We consider that  $\sigma$  is piecewise constant and it can be large, which introduces stiffness. The boundary condition is compatible with the method of characteristics: it writes  $bu - cu = 0$ ,  $x \in \partial\Omega$  where  $bu = \frac{\mathbf{a}(x) \cdot \mathbf{n}(x)}{\sqrt{2|\mathbf{a}(x) \cdot \mathbf{n}(x)|}} u$  and  $cu =$

$\sqrt{\frac{|\mathbf{a}(x) \cdot \mathbf{n}(x)|}{2}} u$  for  $x \in \partial\Omega$ . The boundary condition vanishes identically on  $\Gamma_+ = \{x \in \partial\Omega \mid \mathbf{a}(x) \cdot \mathbf{n}(x) \geq 0\}$  and is equivalent to  $u(x) = 0 \forall x \in \Gamma_- = \{x \in \partial\Omega \mid \mathbf{a}(x) \cdot \mathbf{n}(x) < 0\}$ .

The special notation for the boundary condition is the algebraic crux of this work which allows to adapt to the transport equation which is a first order differential equation the UWVF techniques [Cessenat et al., *SIAM J. Num. Anal.*, 35:255--299, 1998; Imbert-Gérard et al., *IMA J. Num. Anal.*, 34, 2014] which were initially

developed for second order differential operators with time harmonic dependence. The connection with Trefftz techniques is explicit. The main asset is, after discretization by means of exact solution of the adjoint equation, an a priori error estimate which is very robust with respect to the stiffness of the problem. It makes connection with the recent theory of Asymptotic Preserving numerical schemes [Gosse, SIMAI Springer Series, 2. Springer, 2013].

---

### **Variable coefficients and Trefftz methods**

*Lise-Marie Imbert-Gérard\**

We will discuss the use of approximated basis functions in Trefftz methods for problems with variable coefficients. On the one hand, we propose a method to design basis functions with high order interpolation properties. On the other hand, we will discuss a numerical method coupling a Trefftz formulation and with these basis functions.

---

### **Space–time Trefftz discontinuous Galerkin methods for wave problems**

*Andrea Moiola\**

We present a space–time discontinuous Galerkin (DG) method for linear wave propagation problems. The special feature of the scheme is that it is a Trefftz method, namely that all discrete functions are solution of the partial differential equation to be discretised in each element of the (space–time) mesh. The DG scheme is defined for unstructured meshes whose internal faces need not be aligned to the space–time axes. The Trefftz approach can be used to improve and ease the implementation of explicit schemes based on “tent-pitched” meshes. We show that the scheme is well-posed, quasi-optimal and dissipative, and prove a priori error bounds for general Trefftz discrete spaces. The error is measured in both a skeleton norm and in a mesh-independent volume norm. A concrete discretisation can be obtained using piecewise polynomials that satisfy the wave equation elementwise, for which we show high orders of convergence. The Trefftz- DG scheme is related to domain decomposition methods as, at least in one space dimension, it can be written as an ultra weak variational formulation (UWVF) using a trace-flipping operator and an isometry, which is defined by solutions of local impedance problems. The acoustic wave equation, Maxwell equations and more general hyperbolic systems are considered. This is a joint work with Ilaria Perugia (Vienna).

---

### **An enriched discontinuous Galerkin method for the time-harmonic eddy current problem**

*Raffael Casagrande\**

The solution of the time-harmonic eddy current problem exhibits thin, exponential type boundary layers at the surface of conductors for high frequencies. We present a method for the approximation of these boundary layers for the 2D problem that is based on the Non-symmetric Interior Penalty formulation. The polynomial approximation space is enriched with Trefftz functions that model the exponential decay. This rather simple method resolves the boundary layers on flat/smooth surfaces of the conductor very well, even on very coarse meshes [Casagrande, Winkelmann, Hiptmair, Ostrowski: A Trefftz Method for the time-harmonic Eddy Current Equation, to appear]. However, at corners of the conductor the solution exhibits two different types of singularities which dominate the overall approximation error at high frequencies. We therefore enrich the approximation space additionally with explicit expressions of these singularities and obtain a method that is robust for all frequencies and shows exponential convergence in the polynomial degree  $p$ . Lastly, we discuss the generalization to the 3D eddy current problem.

This talk is based on joint work with Ralf Hiptmair (ETH Zürich).

Chair: MS-organizer

10:35~11:15 MS12-3 Robust solvers for multiphysics problems

Culture Centre

---

**A segregated Uzawa smoother in multigrid for poroelastic problems**

*Fransisco Gasper\**

Coupling of fluid flow and mechanical deformation within a porous media is an important multiphysics problem appearing in many applications, ranging from geomechanics and petrol engineering to biomechanics and food processing, more recently. Numerical simulation is mandatory for real applications and therefore, in this work, we propose a solution strategy for poroelastic problems based on multigrid methods.

In order to develop an efficient multigrid solver, it is necessary to carefully consider each component of the algorithm. In particular, the choice of the smoother often requires special attention. In this work, we propose a simple and efficient segregated smoother based on a Uzawa-type iteration. The Uzawa-like procedure amounts to a simple Richardson iteration for the smoothing of the pressure unknowns. As usual, such iteration involves a relaxation parameter which has to be carefully selected. Local Fourier analysis is applied to determine the optimal value of the aforementioned relaxation parameter as a function of the main problems characteristics. Finally, it is shown that a multigrid method based on such Uzawa smoother results in an efficient solver for poroelastic problems.

---

**Monolithic multigrid method for the coupled Darcy-Stokes problem**

*F.J. Gaspar, P. Luo, C.W. Oosterlee, C. Rodrigo\**

The multigrid solution of coupled porous media and Stokes flow problems is considered. The Darcy equation as the saturated porous medium model is coupled to the Stokes equations by means of appropriate interface conditions. The finite volume method on a staggered grid is considered as the discretization scheme for the Darcy-Stokes problem, giving rise to saddle point linear system. Special treatment is required regarding the discretization at the interface. We focus on an efficient monolithic multigrid solution technique for the coupled problem, in which we do not distinguish the subproblems and the internal interface.

Highly satisfactory multigrid convergence is reported, and, moreover, the algorithm performs well for small values of the hydraulic conductivity and fluid viscosity, that are relevant for applications.

---

**Optimal multigrid methods for  $H(\text{div},\Omega)$ -conforming discontinuous Galerkin discretizations of the Brinkman problem**

*J. Kraus\**

In this talk we consider discontinuous Galerkin  $H(\text{div},\Omega)$ -conforming discretizations of elasticity type and Brinkman equations. We analyze their uniform stability and describe a simple Uzawa iteration for the solution of the Brinkman problem. The major part of the computational work for executing this algorithm is spent on solving in every iteration a nearly incompressible linear elasticity type problem. Based on a special subspace decomposition of  $H(\text{div},\Omega)$ , we analyze variable V-cycle and W-cycle multigrid methods with non-nested bilinear forms.

We prove that these methods are robust and their convergence rates are independent of the material parameters such as Lamé parameters and Poisson ratio and of the mesh size. Numerical results that confirm the theoretical analysis are presented.

This is a joint work with Qingguo Hong (University of Duisburg-Essen, Germany).

**A generalized Suzuki–Trotter type method in optimal control of coupled Schrödinger equations**

---

*Manfred Liebmann\**

A generalized Suzuki–Trotter (GST) method for the solution of an optimal control problem for quantum molecular systems is presented in this work. The control of such systems gives rise to a minimization problem with constraints given by a system of coupled Schrödinger equations. The computational bottleneck of the corresponding minimization methods is the solution of time-dependent Schrödinger equations. To solve the Schrödinger equations we use the GST framework to obtain an explicit polynomial approximation of the matrix exponential function. The GST method almost exclusively uses the action of the Hamiltonian and is therefore efficient and easy to implement for a variety of quantum systems.

Following a first discretize, then optimize approach we derive the correct discrete representation of the gradient and the Hessian. The derivatives can naturally be expressed in the GST framework and can therefore be efficiently computed. By recomputing the solutions of the Schrödinger equations instead of saving the whole time evolution, we are able to significantly reduce the memory requirements of the method at the cost of additional computations. This makes first and second order optimization methods viable for large scale problems. In numerical experiments we compare the performance of different first and second order optimization methods using the GST method for different hardware architectures.

**Preconditioners for edge element systems arising from various Maxwell equations**

---

*Jun Zou\**

We will review in this talk several new preconditioners for solving the discrete real and complex systems arising from the edge element discretization of various Maxwell systems. Two variational and algebraic frameworks are proposed to study the spectral behaviors of the preconditioned systems.

This talk covers the major research results from our joint works with Qiya Hu (Chinese Academy of Sciences), Shi Shu (Xiangtan University), Na Huang (Fujian Normal University), Hua Xiang and Shiyang Zhang (Wuhan University). The research projects were substantially supported by Hong Kong RGC General Research Fund (projects 14322516 and 14306814).

**Preconditioning time-dependent PDE-constrained optimization problems**

---

*John Pearson\**

PDE-constrained optimization problems have a wide range of applications across numerical mathematics and engineering, so it is important to develop fast and feasible methods to solve such problems. We employ preconditioned iterative methods to tackle the large-scale matrix systems that arise from their discretization, and consider a number of issues related to such solvers for time-dependent problems. In particular, we discuss the use of interior point methods to handle additional inequality constraints on the state and control variables, the solution of problems involving fractional differential equation constraints, and the application of deferred correction schemes to reduce the discretization error in time. For each problem, we motivate and derive our proposed approach, and demonstrate the validity of our method using numerical examples.

**Functional iterations with Laplace inversion for DLCs***Shu-Lin Wu\**

We introduce in this talk a functional iteration algorithm for solving the differential linear complementarity problems (DLCs) consisting of two systems, a linear ODE system and a linear complementarity system. The solutions of these two systems are respectively called state variable and constraint variable. For each iteration the algorithm proceeds in a systems decoupling way: by using a rough approximation of the state variable, obtained from the previous iteration, we solve the complementarity problems for all the time points of interest; then we solve the ODE system and update the state variable for preparing for the next iteration, by using the obtained constraint variable as a known source term. The algorithm is highly parallelizable, because in each iteration the computations of both the complementarity and ODE systems at all the time points of interest can start simultaneously. The parallelism for solving the complementarity system is natural and for the ODE system such a parallelism is achieved by using the Laplace inversion technique. For the P-matrix case, we prove that the algorithm converges superlinearly with arbitrarily chosen initial iterate and for the Z-matrix case the algorithm still converges superlinearly if we use the initial value as the initial iterate. We show that this algorithm is superior to the widely used time-stepping method, e.g., the implicit Euler method, with respect to robustness, flexibility and computation time.

**Parareal Dirichlet-Neumann and parareal Neumann-Neumann waveform relaxation for the heat equation***Bo Song\*, Yaolin Jiang*

The Dirichlet-Neumann and Neumann-Neumann waveform relaxation methods are non-overlapping spatial domain decomposition methods to solve evolution problems, while the parareal algorithm is producing directly time parallelism. Based on the combinations of space and time parallel strategies, we present and analyze parareal Dirichlet-Neumann and parareal Neumann-Neumann waveform relaxation for parabolic problems. Between these two algorithms, parareal Neumann-Neumann waveform relaxation is a space-time parallel algorithm, which increases the parallelism both in space and time. We derive for the heat equation the convergence results for both algorithms in one spatial dimension. We also illustrate our theoretical results with numerical experiments.

**Space-time domain decomposition methods and a posteriori error estimates for the subsurface***Sarah Ali Hassan, Caroline Japhet\*, Michel Kern, Martin Vohralik (given by Iain Smears)*

We present a posteriori error estimates and stopping criteria for global-in-time, nonoverlapping domain decomposition methods to model flow and transport problems in a porous medium. The method considered uses the optimized Schwarz waveform relaxation method with Robin transmission conditions whose coefficients can be optimized to improve convergence rates. A mixed formulation of an interface problem on the space-time interface is derived, and different time grids are used to adapt to different time scales in the subdomains [Hoang et al., SIAM J. Numer. Anal., 51(6):3532--3559, 2013]. We estimate separately the error of the space-time DD method, that of the spatial discretization as well as that of the time discretization [Ern and Vohralik, SIAM J. Numer. Anal., 48(1):198--223, 2010]. Consequently, an effective criterion to stop the DD iterations is developed with local time stepping. Our a posteriori estimates are based on the reconstruction techniques for pressures and

fluxes following [Ern and Vohralik, SIAM J. Numer. Anal., 48(1):198--223, 2010] and [Pencheva et al., SIAM J. Numer. Anal., 51:526--554, 2013]. This work is supported by ANDRA and the ANR project DEDALES.

Chair: MS-organizer

10:35~11:15 MS07-2 Recent progress on Trefftz methods

Polar Bear

### **Gaussian beam approximations**

*Olof Runborg\**

Gaussian beam superpositions are asymptotically valid high frequency solutions to linear hyperbolic partial differential equations and the Schrödinger equation. In this talk we discuss the construction of the beams and their accuracy in terms of the frequency. We show some recent error estimates measured in Sobolev and max norms.

### **Adaptive refinement for hp-version Trefftz discontinuous Galerkin methods for the homogeneous Helmholtz problem**

*Scott Congreve\*, Paul Houston, Ilaria Perugia*

We consider a Trefftz discontinuous Galerkin finite element (TDG) approximation of the solution to the homogeneous Helmholtz problem  $-\nabla u - k^2 u = 0$  in  $\Omega \subset R^d$ ,  $d = 2, 3$ , for a given wavenumber  $k$ . The TDG method uses (local) solutions to the Helmholtz equation as basis functions on an element rather than polynomial basis functions. See [Hiptmair et al., LNCSE, volume 114, 2016] and the references therein for an extensive survey of Trefftz methods for the Helmholtz equation. Here, we consider plane wave basis functions  $e^{ik\mathbf{d}_l \cdot (\mathbf{x} - \mathbf{x}_K)}$ , where  $\mathbf{d}_l, l = 1, \dots, p_K$ , are distinct unit propagation directions on an element  $K$  with element centre  $\mathbf{x}_K$ .

Existing work [Kapita et al., SIAM J. Sci. Comput., 37(3):A1525--A1553, 2015] as already studied adaptive h-refinement (refinement of mesh elements). We study a basic extension to this work to look at performing p-refinement (increasing the number of plane wave basis functions) and also make use of a modified version of an existing algorithm [Melenk and Wohlmuth, Adv. Comp. Math., 15(1--4):311--331, 2001] to decide on whether to perform h- or p-refinement. For p-refinement we increase the number of polynomial degrees by increasing the *effective polynomial degree*  $q_K$  on an element  $K$ , which is the degree of a polynomial-based approximation that would give the same convergence rate as the TDG approximation, cf. [Moiola et al., Z. Angew. Math. Phys., 62:809--837, 2011].

We also demonstrate how the propagation directions  $\mathbf{d}_l$  used in the basis functions can be modified. Wave propagation problems often have a primary propagation direction and if one of the propagation directions of the plane wave basis functions aligns with this primary direction the error in the numerical solution of the TDG method will be reduced. We show how we can use the eigenvalues and eigenvectors of the Hessian of the numerical solution to the Trefftz discontinuous Galerkin finite element method to approximate this primary propagation direction and, hence, select suitable directions for the plane wave basis functions.

### **Trefftz-UWVF discretization of transport equations with physical absorption**

*Guillaume Morel\*, Bruno Després, Christophe Buet*

Trefftz methods are discontinuous Galerkin type schemes which allow to incorporate some informations about the solution in the basis functions. Among them, the Ultra Weak Variational Formulation (UWVF) has been widely used for time-harmonic wave problems such as the Helmholtz equation. We adapt the UWVF to time dependent transport equations. As a first step, we study the P1 model. We adapt the UWVF's algebra to this kind of equation and show how to construct the basis functions.

Numerical results are also presented to show the accuracy of the method in both one and two dimensions for time dependent problems.

## Thursday, February 9<sup>th</sup> 2017

	Culture Centre	North Pole	Northern Light	Polar Bear
07:00	Breakfast is served			
08:10	Bus departs from Coal Miners' Cabin			
08:30	PL09 Simone Scacchi			
09:15	PL10 Jed Brown			
10:00	Coffee at Culture Centre and Radisson			
10:30	MS01: 1-4	MS04: 1-4	MS05: 1-4	MS18: 1-4
12:10	Lunch at Radisson			
13:40	PL11 Stefano Zampini			
14:25	Moving/Stretching legs			
14:35	MS01: 5-8	MS04: 5	MS05: 5-7	CT03: 1-4
		MS16: 1		
16:15	Coffee at Culture Centre and Radisson			
16:45	MS01: 9-12	MS16: 2-5	MS08: 1-4	
18:50	End of talks			
19:00	Bus departs for Coal Miners' Cabin			
20:00	Conference Banquet 19:45 Bus from Radisson & Coal Miners' Cabin. Attention! Warm clothes			

Chair: *Xiao-Chuan Cai*

**08:30~09:15 PL09 Simone Scacchi**

**Culture Centre**

### **Scalable multilevel preconditioners for cardiac electro-mechanics**

*Simone Scacchi\**

In the last twenty years, computer modeling has become an effective tool to push forward the understanding of the fundamental mechanisms underlying the origin of life-threatening arrhythmias and contractile disorders in the human heart and to provide theoretical support to cardiologists in developing more successful pharmacological and surgical treatments for these pathologies. The spread of the electrical impulse in the cardiac muscle and the subsequent contraction-relaxation process are quantitatively described by the cardiac electro-mechanical coupling (EMC) model, which consists of the following four components: the quasi-static finite elasticity model of the deformation of cardiac tissue, derived from a strain energy function which characterizes the anisotropic mechanical properties of the myocardium; the active tension model, consisting of a system of non-linear ordinary differential



equations (ODEs), describing the intracellular calcium dynamics and cross bridges binding; the electrical current flow model of the cardiac tissue, called Bidomain model, which is a degenerate parabolic system of two non-linear partial differential equations of reaction-diffusion type, describing the evolution in space and time of the intra- and extracellular electric potentials; the membrane model of the cardiac myocyte, i.e. a stiff system of ODEs, describing the flow of the ionic currents through the cellular membrane. This complex non-linear model poses great theoretical and numerical challenges. At the numerical level, the approximation and simulation of the cardiac EMC model is a very demanding and expensive task, because of the very different space and time scales associated with the electrical and mechanical models, as well as their non-linear and multiphysics interactions. In this talk, we present the finite element solver that we have developed to simulate the cardiac electro-mechanical activity on parallel computational platforms. The solver is based on a Multilevel Additive Schwarz preconditioner for the linear system arising from the discretization of the Bidomain model and on a Newton-Krylov-BDDC method for the non-linear system arising from the discretization of finite elasticity. Three-dimensional numerical tests show the effectiveness and scalability of the solver up to  $O(10^5)$  cores on Bluegene and Linux clusters. Finally, we present an application of the electro-mechanical solver to the simulation of cardiac arrhythmias and to the validation of non-invasive mechanical markers of the cardiac performance.

**09:15~10:00 PL10 Jed Brown**

**Culture Centre**

### **On nonlinear adaptivity with heterogeneity**

*Jed Brown\**

Conventional adaptive methods depend on the solution being smooth in much of the domain, such that coarse grids provide adequate resolution everywhere but in some localized areas that require higher resolution. Heterogeneity impedes standard adaptive techniques because it is not possible to have accurate solutions without resolving the heterogeneity. We present an adaptive method that can be viewed as a nonlinear multigrid or domain decomposition scheme in which fine levels resolve the heterogeneity but are rarely visited in most of the domain during the nonlinear solution process. This requires judicious use of overlap similar to the highly parallel segmental refinement methods of [Adams et al., *SIAM J. Sci. Comp.*, 38(4):C426--C440, 2016]. We will compare and contrast our method with other nonlinear multilevel domain decomposition methods such as ASPIN.

### **Coffee Break at Radisson and Culture Centre**

Chair: MS-organizer

**10:30~12:10 MS01-1 Achieving robustness with new coarse spaces and enlarged Krylov subspaces**

**Culture Centre**

### **Optimal coarse spaces and their approximation**

*Martin J. Gander\**

Domain decomposition methods for elliptic problems need a coarse space to be scalable, and there are well established convergence results for these so called two level domain decomposition methods, for both overlapping and non-overlapping subdomains. Coarse spaces can however do much more than just make a domain decomposition method scalable. This is well illustrated by the many recent coarse spaces proposed for high contrast problems, where the coarse space helps the underlying domain decomposition iteration with error components where it is not

efficient. These recent coarse spaces are based on improving inequalities in the condition number estimates of the associated Schwarz and substructuring methods, and are therefore improving the algorithms by improving their convergence estimates. I will explain a different approach for finding good coarse space components, based directly on the functioning of the underlying algorithm, not on the convergence theory. The idea is to identify first an optimal coarse space, like there exist optimal transmission conditions. Optimal here does not mean scalable, but is in the true sense of the word optimal: there is no better choice possible. Like optimal transmission conditions, optimal coarse spaces lead to algorithms that are nilpotent, i.e. they converge in a finite number of steps, and are thus representing direct solvers. Once the optimal coarse space for a domain decomposition method is identified, one can approximate it to obtain an effective two level method. The advantage of this approach is that one can design good coarse spaces for domain decomposition methods that only have partial convergence theory, like for example restricted additive Schwarz or optimized Schwarz methods.

---

### **A new coarse space for Neumann-Neumann method**

*Martin J. Gander, Kevin Santugini-Répiquet, Faycal Chaouqui\**

It is well known that domain decomposition methods need coarse grids for global communication between subdomains in order to be scalable. However, the coarse correction can do more than just make the method scalable: There exist an optimal coarse space in the sense that we have convergence after exactly one coarse step i.e. the method becomes a direct solver. We introduce and analyze here a new coarse space for the Neumann-Neumann method for the so called positive definite Helmholtz equation and give an optimal coarse space for the one and two dimensional case. We then study the limit when the zeroth order term in the equation goes to zero and we obtain the Poisson equation, where the subdomain problems of the Neumann-Neumann method can become ill posed. We show how this problem can be addressed, and also present numerical experiments for our new coarse space.

---

### **Robust coarse spaces for abstract Schwarz preconditioners via generalized eigenproblems.**

*Emmanuel Agullo, Luc Giraud, Louis Poirel\**

The domain decomposition methods (DDM) community has developed many efficient and robust algorithms for solving sparse linear systems in the last decades. While many of these solvers fall in Abstract Schwarz (AS) framework, their robustness has often been demonstrated on a case-by-case basis. Extending the GenEO method, we propose a bound for the condition number of all deflated AS methods provided that the coarse grid consists of the assembly of local components that contain low-energy modes of some local operators. Parallel numerical experiments will be given to illustrate the robustness of the resulting algorithms.

---

### **SHEM: An optimal coarse space for RAS and its multiscale approximation**

*Martin J. Gander, Atle Loneland\**

In the context of domain decomposition methods, coarse spaces are traditionally added to make the underlying method scalable. Coarse spaces can however do much more: They can act on other error components that the subdomain iteration has difficulties with, and thus accelerate the overall solution process. In this talk we identify the optimal coarse space for RAS, where optimal does not refer to scalable, but to best possible and show that this coarse space leads to convergence of the domain decomposition method in one iteration. Also, since the optimal coarse space is very large, we propose an approximation by eigenfunctions that turns out to be also very effective for multiscale problems.

**Optimized Iterative mesh tying**

---

*Hui Zhang\*, Yingxiang Xu*

Non-conforming meshes are often used in numerical PDE applications. In many cases, we have a few mesh patches covering different regions of the domain either with overlap or not. It is convenient to solve the model on each mesh patch independently but we need to recouple them through transmission conditions. Optimized transmission conditions for various models have been studied at the continuous level. In our case, we study the optimized transmission conditions at the (semi-)discrete level to see the impact of the non-conforming meshes.

**Optimized Schwarz method for different geometric interface configurations**

---

*C. Vergara\*, G. Gigante*

We propose convergence analyses of the generalized Schwarz method applied to linear elliptic problems for different kind of interfaces. We start providing a unified analysis in the case of unbounded flat, cylindrical or spherical interfaces in any dimension. Then, we consider the case of circular flat interfaces, arising when e.g. a cylindrical domain is split orthogonally to the axial direction. Finally, we consider the case of geometric heterogeneous coupling, when a three-dimensional problem is coupled with a one-dimensional one. In all the cases, we provide the exact convergence set of the interface symbols related to the operators involved in the transmission conditions. We also provide general procedures to obtain estimates of the optimized interface symbols within the constants. We apply such general results to the diffusion-reaction problem and to the fluid-structure interaction problem arising in hemodynamics. A proper choice of the interface parameters involved in the latter problem allows one to obtain efficient algorithms which do not suffer from the high added mass effect which characterizes hemodynamic applications. We present several numerical results both in ideal and real geometries highlighting the suitability of the proposed interface conditions.

**Optimized Schwarz methods for the optimal control of systems governed by elliptic partial differential equations**

---

*Yingxiang Xu\*, Xin Chen*

In the literature [A domain decomposition method with coupled transmission conditions for the optimal control of systems governed by elliptic partial differential equations, SIAM J. Num. Anal., 33(6):2401-2416, 1996], Benamou proved the convergence of a nonoverlapping Schwarz domain decomposition (DD) method with various Robin type transmission conditions. However, to achieve the possibly best performance of the DD method, the free parameters involved in these transmission conditions should be further optimized, which is the central idea of the well-known Optimized Schwarz Methods (OSMs) that have been shown their efficiency in many applications. In this talk, we show the results on the OSMs for the optimal control of systems governed by elliptic partial differential equations without constraint. We obtain the optimized transmission parameters contained in various type of transmission conditions, including those considered by Benamou, and the variants of the second order and the two-sided Robin types applied in OSMs for second order elliptic PDEs, as well as the corresponding asymptotic convergence rate estimates. We find that the weight parameter enters both the predicted transmission parameters and the asymptotic convergence rate estimates. We find as well that the weight parameter plays an important role on the performance of the OSMs: the smaller the weight parameter is, the faster the OSMs converge. While when the weight parameter goes to infinity, we arrive at the results for solving a biharmonic problem using mixed form. We use numerical experiments to validate the theoretical

findings.

### **A double source transfer domain decomposition method for Helmholtz equations in unbounded domain**

---

*Xueshuang Xiang\**

We propose and study a double source transfer domain decomposition method for solving the truncated perfectly matched layer approximation in bounded domain of Helmholtz problems. The method is based on the decomposition of the domain into non-overlapping layers and instead of transferring source along one direction in STDDM, double STDDM transfers the source in each layer both along two directions. Double STDDM is used as an iterative algorithm and in each iteration, it first transfers source from down to up and produces Up wave (the wave propagates from down to up), and then transfers source from up to down and produces Down wave (the wave propagates from up to down). Besides producing Up wave and Down wave in each iteration, it also produces the residual source used in next iteration. The output of double STDDM is the sum of Up waves and Down waves produced in each iteration. By using the similar techniques in STDDM, the convergence of double STDDM is proved for the case of constant wave number. Numerical examples are included to show the efficient behaviour of double STDDM both for constant and heterogenous wave number: the iteration number is independent of wave number, mesh size and number of layers.

Chair: MS-organizer

**10:30~12:10 MS05-1 Parallel solvers for isogeometric analysis**

**Northern Light**

### **Symbol-based multigrid methods for isogeometric analysis**

---

*M. Donatelli\*, C. Garoni, C. Manni, S. Serra-Capizzano, H. Speleers*

We consider the stiffness matrices arising from the Galerkin B-spline Isogeometric Analysis approximation of classical elliptic problems. By exploiting their specific spectral properties, compactly described by a symbol, we design an efficient multigrid method for the fast solution of the related linear systems. Despite the theoretical optimality, the convergence rate of the two-grid methods with classical stationary smoothers worsens exponentially when the spline degree increases. With the aid of the symbol we provide a theoretical interpretation of this exponential worsening. Moreover, by a proper factorization of the symbol we design an ad hoc multigrid method with preconditioned Krylov method “smoother” at the nest level, in the spirit of the multi-iterative strategy, that allows us to obtain a good convergence rate independent both of the matrix size and of the spline degree. A numerical experimentation confirms the effectiveness of our proposal and the numerical optimality with a uniformly high convergence rate, also for the V-cycle multigrid method and large spline degrees.

### **Single and multi-patch iterative solvers for isogeometric analysis**

---

*Giancarlo Sangalli, Mattia Tani\**

We address the problem of numerically solving linear systems that arise in Isogeometric Analysis (IGA). It is known that many standard methods, when applied to IGA systems, have a computational cost which significantly increase with the degree  $p$  of the splines employed as basis functions. This fact contributes in making high degree splines prohibitive for real world applications.

The purpose of this talk is twofold. First, we describe an effective preconditioning strategy for single-patch problems. In the preconditioning step the original differential problem is replaced with an analogous one with constant coefficients and trivial domain geometry. This new problem can be efficiently solved by fully exploiting the

tensor structure of the basis functions, and robustness with respect to  $p$  is achieved. Second, we extend the above approach to domains with multi-patch geometry, where we do not have a global tensor structure. We overcome this difficulty by taking a Domain Decomposition approach. In this way, the local tensor structure can be exploited in the subdomains, making the overall strategy robust and efficient.

---

### **New robust and efficient multigrid methods for isogeometric analysis**

---

*C. Hofreither\**

We present an efficient and fully robust multigrid method for Isogeometric Analysis using splines of maximum continuity. The method is based on a theoretical result concerning the stable splitting of spline spaces into a regular interior part and one or several smaller spaces which capture boundary effects. The construction of the multigrid smoother is based on an additive subspace correction approach, constructing a suitable Kronecker product smoother for each of the subspaces. Only in a very small space associated to the corners of the domain, we use an exact solver. The resulting multigrid method is fully robust in the spline degree and can be realized in quasi-optimal time, independently of the space dimension.

We then present recent extensions to this method. In particular, by choosing the first coarse space below the finest space to be a space of piecewise linear splines using the fine knot vector, it seems possible to maintain robustness while obtaining improved convergence rates as well as significantly faster implementation. The new method achieves a significant speedup over the original robust multigrid method in several numerical examples.

---

### **Multigrid solvers for isogeometric discretizations of the Stokes problem**

---

*S. Takacs\**

We will discuss how to set up fast multigrid solvers for linear systems arising from the discretization of a Stokes flow problem. The aim of the talk is to show that the results proposed recently for the Poisson equation can be extended to more advanced problems, like saddle point problems. The Stokes problem can be seen as a non-trivial example for a saddle point problem.

Following the spirit of isogeometric analysis, we will set up a discretization with B-splines. Having a saddle point problem, it is important to have a discretization which satisfies the Ladyzhenskaya-Babuska-Brezzi condition (inf-sup condition). We will recall known results on the stability of certain classes of isogeometric discretizations, particularly Taylor Hood like discretizations and macro-element like discretizations. These results will be connected with the approximation error and inverse estimates, which we have developed in a recent paper.

In the talk, we will see how a multigrid solver can be constructed based on that knowledge. In the talk, we will present numerical experiments comparing the different approaches (approximation error, convergence speed of the multigrid solver) and comment on the convergence theory.

Chair: MS-organizer

**10:30~12:10 MS18-1 Fast solvers for space-time discretizations**

**Polar Bear**

---

### **Parallel space–time multigrid for the time–periodic Navier–Stokes equations**

---

*Pietro Benedusi\*, Daniel Hupp, Peter Arbenz, Rolf Krause*

We present a parallel and efficient space-time multilevel solution method for the nonlinear systems arising from the discretization of Navier–Stokes (N–S) equations with finite differences. In particular we study the incompressible, unsteady N–S equations with periodic boundary condition in time. A sequential time integration limits the parallelism of the solver to the spatial variables and can therefore be an

obstacle to parallel scalability. Time periodicity allows for a space-time discretization, which adds time as an additional direction for parallelism and thus can improve parallel scalability. To achieve fast convergence, we used a space-time multigrid algorithm with a SCGS smoothing procedure (symmetrical coupled Gauss–Seidel, a.k.a. box smoothing). This technique, proposed by S. P. Vanka [Vanka, J. Comp. Phys, 65:138--165, 1986], for the steady viscous incompressible Navier–Stokes equations is extended to the unsteady case and its properties are studied using local Fourier analysis. We used numerical experiments to analyze the scalability and the convergence of the solver, focusing on the case of a pulsatile flow.

---

### **Space–time isogeometric analysis of parabolic evolution problems**

*Ulrich Langer, Stephen E. Moore, Martin Neumüller\**

We present and analyze a new stable spacetime Isogeometric Analysis (IgA) method for the numerical solution of parabolic evolution equations. The discrete bilinear form is elliptic on the IgA space with respect to a discrete energy norm. This property together with a corresponding boundedness property, consistency and approximation results for the IgA spaces yields an a-priori discretization error estimate with respect to the discrete norm. The theoretical results are confirmed by several numerical experiments with low- and high-order IgA spaces. Moreover for low order IgA spaces the related linear system can be solved in parallel with respect to space and time by applying standard solver packages.

---

### **Preconditioned space–time boundary element methods for the heat equation**

*Stefan Dohr\*, Olaf Steinbach*

Regarding time-dependent initial boundary value problems, there are different numerical approaches to compute an approximate solution. In addition to finite element methods and time-stepping schemes one can use boundary element methods to solve time-dependent problems. As for stationary problems, one can use the fundamental solution of the partial differential equation and the given boundary and initial conditions to derive boundary integral equations and apply some discretization method to compute an approximate solution of those equations. In this talk, we describe the boundary element method for the discretization of the time-dependent heat equation. In contrast to standard time-stepping schemes we consider an arbitrary decomposition of the space-time cylinder into boundary elements. Besides adaptive refinement strategies, this approach allows us to parallelize the computation of the global solution of the whole space-time system. In addition to the analysis of the boundary integral operators and the derivation of boundary element methods for the Dirichlet initial boundary value problem, we state convergence properties and error estimates of the approximations. Those estimates are based on the approximation properties of boundary element spaces in anisotropic Sobolov-spaces, in particular in  $H^{\frac{1}{2}-\frac{1}{4}}(\Sigma)$  and  $H^{-\frac{1}{2}-\frac{1}{4}}(\Sigma)$ . The systems of linear equations, which arise from the discretization of the boundary integral equations, are being solved with GMRES. For an efficient computation of the solution we need preconditioners. Based on the mapping properties of the single layer- and hypersingular boundary integral operator we construct and analyse a preconditioner for the discretization of the first boundary integral equation. Finally we present numerical examples for the spatial one-dimensional heat equation to confirm the theoretical results.

---

### **Space-time finite and boundary element methods for the heat equation**

*Olaf Steinbach\**

Space-time finite and boundary element methods rely on the variational formulation of the time-dependent partial differential equation in the space-time domain by means of a Galerkin-Petrov formulation. We discuss related stability estimates for space-time finite and boundary element formulations, and for their coupling. For an efficient and

parallel solution of the resulting linear systems we comment on suitable solution strategies and available preconditioning strategies.

Chair: *David Keyes*

**13:40~14:25 PL11 Stefano Zampini Culture Centre**

---

**Recent advances on adaptive multilevel BDDC methods for div- and curl-conforming**

*Stefano Zampini\**

In this talk we will discuss some recent advances on Balancing Domain Decomposition methods for div- and curl-conforming finite element spaces, possibly using non-conforming meshes arising from adaptive mesh refinement techniques, with a specific focus on the implementation of the method in the Portable and Extensible Toolkit for Scientific computing (PETSc). Experimental results will be provided to prove the robustness of the method for higher-order spaces, as well as for more exotic discretization spaces obtained by upscaling techniques that preserve the de Rham sequence.

Chair: MS-organizer

**14:35~16:15 MS01-2 Achieving robustness with new coarse spaces and enlarged Krylov subspaces Culture Centre**

---

**Adaptive BDDC/FETI-DP algorithms**

*Eric T. Chung, Junxian Wang, Hyea Hyun Kim\**

Adaptive BDDC and FETI-DP algorithms are presented for second order elliptic problems in both two and three dimensions. The adaptive constraints are found by solving generalized eigenvalue problems on each equivalence class, edges in two dimensions and edges and faces in three dimensions. Enhanced with these constraints, the condition number of the both algorithms can be bounded by the user-defined tolerance value. Some implementation issues and computational efficiency of these adaptive algorithms will be discussed.

---

**FETI-DP and BDDC for composite materials - adaptive coarse spaces in 3D**

*Axel Klawonn\*, Martin Kühn, Oliver Rheinbach*

Coarse spaces for FETI-DP and BDDC, based on the adaptive computation of small local eigenvalue problems, are considered. A special emphasis is put on the three dimensional case and on materials with heterogeneous material parameters where coefficient jumps are not aligned with the interface of the domain decomposition. Our new approach is based on solving local eigenvalue problems on faces, enriched by a selected, small number of additional local eigenvalue problems on edges. The additional edge eigenvalue problems make the method provably robust with a condition number bound which depends only on the tolerance of the local eigenvalue problems and some properties of the domain decomposition. The introduction of relevant edge eigenvalue problems for FETI-DP and BDDC methods yields a condition number estimate which is independent of discontinuities of the material parameters. Numerical results are presented for linear elasticity and composite materials supporting our theoretical findings. The problems considered include those with randomly distributed coefficients and almost incompressible material components.

## **Robust domain decomposition methods in industrial context applied to large business cases**

*Augustin Parret-Fréaud\*, Christian Rey, Frédéric Feyel, Christophe Bovet, Pierre Gosselet, Basile Marchand, Nicole Spillane*

The route to virtual testing during the conception phases of new products, relying on the development of new materials of increasingly complex microstructure, often leads to large scale finite element problems which need robust and efficient HPC solvers. In an industrial context, new multi-preconditioned FETI algorithms are especially promising in order to go beyond some weakness of historical FETI methods (heterogeneities, jagged subdomains). Thanks to a recent test campaign on the new 38 528 cores cluster of French CEA/CCRT (cobalt, ranked 63th at the last TOP500 list), we have been able to evaluate the performances our solvers on more than 30 000 cores (up to 10000 subdomains and 2 billion degrees of freedom). We will discuss the strength and weakness and show applications on large scale industrial 3D problems (both linear and nonlinear), such as woven composites or polycrystalline turbine blade (which exhibits various types of heterogeneities).

## **Robustness assessment and new clustering techniques for the adaptive multi-preconditioned FETI method**

*Christophe Bovet\*, Pierre Gosselet, Augustin Parret-Fréaud, Nicole Spillane*

The resolution of FE problems of industrial complexity requires scalable and reliable parallel solvers. Classical DD methods often exhibit poor convergence when applied to this kind of problems involving, for instance quasi-incompressible materials, large heterogeneities or bad shaped finite elements.

The multipreconditioned FETI algorithm (MPFETI, previously called Simultaneous FETI) was recently proposed to ensure robustness in those cases. Thanks to an enlarged search space, MPFETI solves critical problems in significantly fewer iterations than FETI. Each iteration involves however a larger computational effort. Adaptive variants of MPFETI use a selection process of generated search directions, it aims to achieve a balance between robustness and computational efficiency. In this contribution we assess these solvers on realistic industrial problems and hardware, and we present a new clustering techniques allowing AMPFETI to be applied efficiently on larger-scale problems.

Chair: MS-organizer

14:35~16:15	<b>MS04-2 Optimized transmission conditions in domain decomposition methods</b>	<b>North Pole</b>
-------------	---	-------------------

## **Optimized double sweep Schwarz method by complete radiation boundary conditions for the Helmholtz equation**

*Seungil Kim\*, Hui Zhang*

We introduce a sweeping preconditioning algorithm in the nonoverlapping domain decomposition framework for solving the Helmholtz equation of high-frequency regime [Kim and Zhang, *Comp. Math. Appl.*, 72(6):1573--1589, 2016]. First, it will be shown that Neumann data of propagating wave fields in each subdomain can be used for incoming data sources in neighbouring subdomains and be imposed in the radiation condition based on the Dirichlet-to-Neumann map (DtN). Next, the DtN map is approximated by the complete radiation boundary conditions (CRBCs), local and high-order absorbing boundary condition. We prove the convergence of the domain decomposition algorithm with the increasing order of CRBCs. The sweeping DDM can be used to accelerate GMRES iterations, which is illustrated in numerical examples.



---

**Recent progress in domain decomposition methods for the Helmholtz equation**

---

*Christiaan C. Stolk\**

Recent research on domain decomposition method for the Helmholtz equation has led to methods that converge in few iterations, while employing a large number of (layer-like) subdomains. However, domain decomposition methods are not the only way to obtain efficient and scalable solvers. The number of degrees of freedom used for a problem should be made as small as possible by choosing an appropriate discretization, for example an optimized finite difference discretization. Multigrid methods form another way to improve the efficiency of Helmholtz solvers. We will show how, for finite difference schemes, the mentioned techniques can be combined to yield a highly efficient solver. It is challenging to extend such results to finite element methods. In this talk I will discuss progress in this area.

---

**Recursive and additive sweeping preconditioners for the helmholtz equation**

---

*Fei Liu\*, Lexing Ying*

We present the recursive and additive sweeping preconditioners for the numerical solution of the Helmholtz equation based on the perfectly matched layer (PML). The recursive sweeping preconditioner applies to the 3D Helmholtz equation and it is based on the earlier work of the sweeping preconditioner with the moving PMLs. The key idea is to apply the sweeping preconditioner recursively to the quasi 2D auxiliary problems introduced in the 3D sweeping preconditioner. Compared to the non-recursive 3D sweeping preconditioner, the setup cost of this new approach drops from  $O(N^{4/3})$  to  $O(N)$ , the application cost per iteration drops from  $O(N \log N)$  to  $O(N)$ , and the iteration number only increases mildly when combined with the standard GMRES solver. The additive sweeping preconditioner divides the domain of interest into thin layers and proposes a new transmission condition between the sub-domains where the emphasis is on the boundary values of the intermediate waves. This approach can be viewed as an effective approximation of an additive decomposition of the solution operator. When combined with the standard GMRES solver, the iteration number is essentially independent of the frequency.

Chair: MS-organizer

14:35~16:15 MS05-2 Parallel solvers for isogeometric analysis

Northern Light

---

**Isogeometric FETI-DP methods for incompressible Stokes equations**

---

*A. Cortes, L. Dalcin, A. Sarmiento Rodriguez, S. Zampini\**

In the last decade, FETI-DP methods have proven to be robust preconditioners for linear systems arising from various finite elements discretizations and from IsoGeometric Analysis. In addition, recent research on FETI-DP have broadened the range of applicability of these methods to a large class of saddle point systems.

In this talk we will present experimental results combining state-of-the-art open-source software packages for IsoGeometric Analysis (PetIGA) and the numerical solution of PDEs (PETSc), and provide numerical evidence for the robustness of the FETI-DP method for div-conforming discretizations of the incompressible Stokes equations.

---

**Exact and inexact IETI-DP solvers for continuous and discontinuous Galerkin IgA equations**

---

*Christoph Hofer\*, Ulrich Langer*

In this talk, we construct and investigate fast solvers for large-scale linear systems of algebraic equations arising from isogeometric analysis (IgA) of diffusion problems with heterogeneous diffusion coefficient on multipatch domains. In particular, we investigate the adaption of the Dual-Primal Finite Element Tearing and

Interconnecting (FETI-DP) method to IgA, called Dual-Primal Isogeometric Tearing and Interconnecting (IETI-DP) method, [Hofer and Langer, Springer ECCOMAS series “Computational Methods in Applied Sciences”, to appear, 2016]. We consider the cases where we have matching and non-matching meshes on the interfaces. In the latter case we use a discontinuous Galerkin (dG) method to couple the different patches. This requires a special extension of the IETI-DP method to the dG-IgA formulation, see [Hofer and Langer, Computer Methods in Applied Mechanics and Engineering, in press, 2016]. We use ideas from the finite element case in order to formulate the corresponding IETI-DP method, called dG-IETI-DP. We design the dG-IETI-DP method in such a way that it can be seen as a IETI-DP method on an extended discrete interface space. Furthermore, we extend the dG-IETI-DP method to the case of non-matching interfaces due to incorrect segmentation producing gaps and overlaps in the domain decomposition, [Hofer et al., RICAM-Report 2016-44, 2016]. We investigate inexact versions utilizing multigrid methods, cf. [Hofreither and Takacs, NFN Report 2016-45, 2016], for the formulation with an energy minimizing primal subspace. These methods are highly suited for parallelization. We investigate the scaling behaviour up to 1024 cores. We present numerical results for complicated two and three dimensional domains. We observe that the condition number  $\kappa$  behaves like  $O(1 + \log(\frac{H}{h})^2)$ , and is robust with respect to jumping diffusion coefficients and changing mesh-sizes across patch interfaces. We also study the dependence of  $\kappa$  on the underlying polynomial degree  $p$  of the NURBS used.

---

### Isogeometric BDDC and FETI-DP preconditioners for linear elasticity

---

*L. F. Pavarino\*, S. Scacchi, O. B. Widlund, S. Zampini*

We extend our previous works on isogeometric BDDC deluxe preconditioners for scalar elliptic problems to the system of compressible linear elasticity. By using a relatively rich primal space based on fat subdomain vertices and fat edge/face averages, we obtain scalable and quasi-optimal BDDC deluxe preconditioners, which also show good convergence rates with respect to the polynomial degree  $p$  and regularity  $k$  of the isogeometric basis functions. We also study how the performance of these preconditioners is affected by domain deformation and the presence of discontinuous elastic coefficients in composite materials. Several parallel results in 3D using PETSc PCBDDC preconditioner illustrate our findings. If instead we use a mixed formulation of almost incompressible elasticity and Stokes systems, we can alternatively solve the resulting isogeometric saddle point problem with a FETI-DP type preconditioner as in previous finite element work by Li and Tu. The resulting solver is scalable and quasi-optimal, as confirmed by the results of several serial numerical tests in the plane.

Chair: *Laurence Halpern*

**14:35~16:15    Contributed Talks Session 3: FETI domain decomposition    Polar Bear**

---

### Acceleration techniques for multilevel FETI methods

---

*Lubomir Riha\*, Alexandros Markopoulos, Ondrej Meca, Tomas Brzobohaty, Tomas Kozubek*

This contribution presents the acceleration technique for the Hybrid Total FETI method (HTFETI). HTFETI is a suitable technique for cases where classical Total FETI (TFETI) method fails due to large number of subdomains and resulting large coarse problem. The penalty is that the resulting linear system that needs to be solved requires more iterations for the same domain decomposition when compared to TFETI method. This negative effect can be partially reduced using the transformation of the basis.

In our implementation the transformation of the basis is realized by additional Lagrange multipliers which are calculated directly in each iteration. This is an

alternative to the approach which modifies the subdomain stiffness matrices which can generate higher filling of the Cholesky factors which increases both the memory requirements and the processing time. To keep the algorithm efficient, the set of additional LM is chosen to be of minimal yet sufficient size. For instance, in case of a three dimensional linear elasticity problem each interface between two neighboring subdomains is connected by 6 additional LM. These extra LM enforce three zero displacements and three zero rotations based on average values calculated from all nodes on the interface. This process is repeated for all interfaces among all subdomains inside a cluster. We will present the method, implementation and large scalability tests for both strong and weak scalability.

### **An efficient and reliable stopping criterion for the solution of symmetric saddle point problems with FETI**

---

*Ange B. Toulougoussou\*, Pierre Gosselet, Francois-Xavier Roux*

Symmetric saddle point systems arise in a wide range of applications in science and engineering. For systems of large size, it is convenient to use iterative methods that take advantage of parallel high performance computing and save memory space. We have recently combined FETI and BDD into a single substructuring method with natural coarse spaces for solving the linear systems arising from the discretization of Stokes or almost incompressible elasticity with mixed finite element methods such as Hood-Taylor or Mini. We derive a stopping criterion for the preconditioned conjugate gradient based on information that is easily computable by our method. Numerical results using this new stopping condition prove that our method is robust and scalable with respect to mesh size and the number of subdomains.

### **Scalable TFETI based domain decomposition for contact problems with variationally consistent discretization of non-penetration**

---

*Zdeněk Dostál\*, Oldřich Vlach*

The theoretical results on {numerical} scalability of algorithms for the solution of contact problems presented so far relied essentially on the strong linear independence of the rows of constraint matrices that describe the "gluing" of subdomains and non-penetration conditions. The constraint matrices with the desired properties assumed matching grids and node-to-node linearized non-penetration conditions, possibly obtained by an effective orthogonalization of the equality constraints. However, such approach has poor approximation properties and causes problems when the contact conditions are imposed on large curved interfaces or non-matching grids -- the latter are inevitably present in the solution of transient contact problems.

Similarly to the description of the conditions of equilibrium, most of the drawbacks mentioned above can be overcome when we impose the non-penetration conditions in average. The basic tool are so called mortars, which enforce the non-penetration by the discretization of weak non-penetration conditions, in particular the biorthogonal mortars introduced by Wohlmuth. The latter are supported by a nice approximation theory of variationally consistent discretization by Wohlmuth and can be effectively implemented.

Until recently, it was not clear whether they can be plugged into the FETI-based domain decomposition algorithms in a way which preserves their scalability. The reasons were that the constraint matrices resulting from the mortar discretization do not have a nice block-diagonal structure like those arising from the node-to-node schemes and that it was not clear how to orthogonalize effectively the rows corresponding to the inequality constraints against those enforcing the "gluing" of the subdomains, especially those associated with the "wire baskets".

In our lecture, we shall briefly review the description of non-penetration conditions by means of biorthogonal mortars, prove that the constraint matrices arising from the discretization by some biorthogonal bases are well conditioned under natural restrictions, and show that the procedure complies well with the TFETI domain

decomposition method. The theoretical results on scalability are illustrated by numerical experiments.

---

### **Spectral properties of the FETI constraint matrices based on graph theory**

---

*Václav Hapla\**,

This presentation deals with spectral properties of the primal and dual equality constraint matrices within the FETI method (often denoted by B and G, respectively). Properties of various types of the B matrix (with fully redundant, non-redundant and orthonormal rows) and its scaling will be compared and explained in a new way using results from the spectral graph theory. Related practical experiments using the PERMON library, co-developed by the author, will be shown.

Chair: MS-organizer

<b>16:45~18:50</b>	<b>MS01-3 Achieving robustness with new coarse spaces and enlarged Krylov subspaces</b>	<b>Culture Centre</b>
--------------------	---	-----------------------

---

### **New coarse space components for additive Schwarz**

---

*Martin J. Gander, Bo Song\**

The Additive Schwarz method (AS) does not converge in general when used as a stationary iterative method, it can only be used as a preconditioner for a Krylov method. In the two level variant of AS, a coarse grid correction is added to make the method scalable. We propose new coarse space components which allow the two level method to become convergent when used as a stationary iterative method, and show that a suitable choice makes the method even nilpotent, i.e. it converges in two iterations, independently of the overlap and the number of subdomains. One of the coarse spaces we obtain is optimal, which means it is of smallest dimension possible for the method to become nilpotent. We also consider a harmonic enrichment with sine functions to approximate the optimal coarse space, which reduces the number of basis functions needed for good performance. We derive the convergence factor of our new method in the two subdomain case in two spatial dimensions. We also compare our coarse spaces with GenEO recently proposed by Spillane, Dolean, Hauret, Nataf, Pechstein, and Scheichl. We finally illustrate our theoretical results with numerical experiments.

---

### **Asynchronous optimized Schwarz, theory and experiments**

---

*Frédéric Magoulés, Daniel B. Szyld\*, Cedric Venet*

Asynchronous methods refer to parallel iterative procedures where each process performs its task without waiting for other processes to be completed, i.e., with whatever information it has locally available and with no synchronizations with other processes. In this talk, an asynchronous version of the optimized Schwarz method is presented for the solution of differential equations on a parallel computational environment. In a one-way subdivision of the computational domain, with overlap, the method is shown to converge when the optimal artificial interface conditions are used. Convergence is also proved under very mild conditions on the size of the subdomains, when approximate (non-optimal) interface conditions are utilized. Numerical results are presented on large three-dimensional problems illustrating the efficiency of the proposed asynchronous parallel implementation of the method. The main application shown is the calculation of the gravitational potential in the area around the Chicxulub crater, in Yucatan, where an asteroid is believed to have landed 66 million years ago contributing to the extinction of the dinosaurs.

---

### **Adaptive enlarged Krylov conjugate gradient**

---

*Laura Grigori, Olivier Tissot\**

Solving sparse linear systems of equations is often the performance bottleneck in scientific codes due to the ratio between communications and flops. In this talk, we present enlarged Krylov Conjugate Gradient method for reducing communications when solving symmetric positive definite linear systems. Then we propose a method to reduce the number of search directions during the iterations which is both effective and stable. We present numerical results of a parallel implementation of the method. We also compare enlarged Krylov Conjugate Gradient method with other block Conjugate Gradient methods.

### **Adaptive multipreconditioning for symmetric and non-symmetric problems**

---

*Christophe Bovet, Pierre Gosselet, Nicole Spillane\**

Multipreconditioning is a technique that allows to use multiple preconditioners within a Krylov subspace method. For domain decomposition, this means that each contribution to the preconditioner (generated by one subdomain) is used to enlarge the search space instead of the sum of all of these directions. As an illustration, within multipreconditioned Additive Schwarz the search space at a given iteration (with residual  $r$ ) is spanned by  $\{R_i^T A_i^{-1} R_i r\}_{i=1, \dots, N}$  (N-dimensional) instead of  $(\sum_{i=1}^N R_i^T A_i^{-1} R_i) r$  (unidimensional).

Quite naturally, this significantly enlarged search space leads to robust solvers that converge in a small number of iterations. In order for the overall cost of computation to be competitive, adaptivity is needed: only some iterations of the Krylov subspace methods are multipreconditioned. In this talk I will discuss how to choose the adaptivity process and what results can be guaranteed depending on the Domain Decomposition method and the nature of the linear system (symmetric or nonsymmetric).

Chair: MS-organizer

**15:50~18:50 MS16 Nonsmooth and nonlinear problems**

**North Pole**

### **An efficient numerical treatment of a fractured, elastic medium with contact constraints**

---

*Jakub W. Both\*, Oliver Sander*

In the last decades, the prediction of mechanical behavior of fractured media under given load has been of major interest within different fields of engineering as, e.g., hydraulic fracturing, geothermal energy and material science. In our study, we simulate static deformation of a linearly elastic medium with a non-growing fracture and frictionless contact in the fracture. Such a medium can be loaded under both tension and pressure without unphysical self-penetration. Despite all simplifications, solutions are nonsmooth due to a non-Lipschitz geometry. Hence, standard finite element discretizations have non-optimal approximation properties. We discretize the resulting variational inequality using a modified XFEM (eXtended Finite Element Method) approach, motivated by optimal approximation properties for pure fracture opening. The modifications allow to construct an XFEM basis where all inequality constraints become bound constraints on individual variables. We solve the resulting algebraic variational inequality using a simplified Truncated Nonsmooth Newton Multigrid (TNNMG) method, where the application of a geometric coarse grid correction within the multigrid part is exchanged with a simple solver. We show numerical examples to analyze both the discretization error and the efficiency of the solver.

**Coffee Break at Radisson and Culture Centre**

### **On the globally convergent solution of large deformation contact problems**

---

*Ralf Kornhuber, Oliver Sander, Jonathan Youett\**

We present a globally convergent method for the solution of frictionless large deformation contact problems involving hyperelastic materials. For the discretisation we apply the dual mortar method which is known to be more stable than node-to-segment approaches. The resulting non-convex constrained minimisation problems are solved using a filter-trust-region scheme. This method combines techniques from non-linear optimisation to achieve global convergence towards first order optimal points. To speed up the method inexact linearisations of the non-penetration constraint are used whenever the current iterate is far away from a critical point. By controlling the introduced error global convergence of the method is conserved. A combination of the truncated nonsmooth Newton method and the monotone multigrid method is used for the fast solution of the sub-problems.

### **Non-overlapping domain decomposition methods for the minimization of the dual total variation**

---

*Andreas Langer\**

Recently, domain decomposition methods for the minimization of the nonsmooth and non-additive total variation were introduced. In this case the crucial difficulty is the correct treatment of the interfaces of the domain decomposition patches. In order to tackle this difficulty we consider its pre-dual formulation. In this sense we introduce non-overlapping domain decomposition methods for the pre-dual total variation minimization problem. Both parallel and sequential approaches are proposed. For these methods we establish convergence to a minimizer of the original problem. We present several numerical experiments, which show the successful application of the sequential and parallel algorithm for image denoising.

### **Nonlinearly preconditioned inexact Newton methods for nonsmooth optimization**

---

*Carsten Gräser\**

The ASPIN/MSPIN method introduced by Cai and Keyes tries to improve the convergence of Newton's method for smooth nonlinear PDEs by applying an inexact Newton method to a nonlinearly preconditioned system using a nonlinear additive/multiplicative Schwarz preconditioner. This technique can also be used successfully for nonsmooth nonlinear system where Newton's method cannot be applied directly. In contrast to the smooth case the nonlinear preconditioner is essential to make Newton-type methods applicable. In the talk we will show that state of the art nonsmooth solvers like the primal-dual active set method, the TNNMG method are in fact variants of ASPIN and MSPIN.

### **Nonsmooth multigrid for small-strain plasticity problems**

---

*Oliver Sander\**

The numerical solution of small-strain plasticity problems is widely considered to be a solved problem. Somewhat surprisingly, we nevertheless demonstrate that considerable speedup is still possible, by direct application of a nonsmooth multigrid method of TNNMG type. The algorithm is globally convergent, and exhibits convergence rates comparable to a multigrid solver for linear elasticity. It works both for smooth and nonsmooth yield surfaces (multi-surface plasticity), and even for certain gradient-plasticity models.

Chair: MS-organizer

16:45~18:50 MS08 DD-based control and control-based DD

Northern Light

---

## Control and coupling strategies for nonlinear domain-decomposition methods

---

*Rolf Krause\*, Patrick Zulian, Alena Kopanicakova, Christian Groß*

In this talk, we investigate convergence control-strategies for non-linear domain decomposition methods. We furthermore present and discuss coupling strategies for multi-physics problems, which are based on discrete  $L^2$ -projections. In general, showing global convergence for non-linear domain decomposition methods is a difficult task. Although local convergence results are available, for the general non-linear case the question of global convergence is still open.

We present convergence strategies, which are based in the successive minimization of a given target (or energy) functional. Using ideas from Trust-Region methods, we are able to derive non-linear convergence control strategies for the additive case (APTS) and the multiplicative case (MPTS), which guarantee convergence to a critical point. We furthermore show in which way variational transfer between non-matching meshes and different models (i.e. fluid and structure) can be used in order to transfer in a stable and reliable way information such as displacement and stresses for multi-physics simulations. Combining the above ideas, robust non-linear domain decomposition methods for coupled multiphysics problems can be derived. We will present examples from elasticity, fluid-structure interaction, and phase-field models for crack propagation

---

## Interface control domain decomposition (ICDD) method to couple Navier-Stokes and Darcy equations

---

*Paola Gervasio\*, Marco Discacciati, Alfio Quarteroni*

In this talk we consider the Navier-Stokes/Darcy problem modeling the filtration of incompressible fluids through porous media. To realize this coupling we use the so-called Interface Control Domain Decomposition (ICDD) method proposed in [Discacciati, Gervasio and Quarteroni, *SIAM J. Control Optim.*, 51(5):3434--3458, 2013; *Int. J. Numer. Meth. Fluids.*, 76(8):471--496, 2014]. The global computational domain is split into two overlapping subdomains in which we solve Navier-Stokes and Darcy equations, respectively, and the overlap corresponds to the transition region between the two regimes. ICDD introduces new auxiliary control variables on the subdomain internal boundaries (named interfaces) that play the role of the unknown traces of the Stokes velocity and Darcy pressure. Such controls are determined by minimizing a suitable cost functional that measures the jump of the quantities of interest at the interfaces of the decomposition. As a matter of fact we solve an optimal control problem in which both controls and observation are defined on the interfaces and whose constraints are the PDE's on the overlapping subdomains ([Discacciati et al., *SIAM J. Num. Anal.* 54(2):1039--1068, 2016]). In this talk we discuss both theoretical and computational aspects of the ICDD method applied to Stokes-Darcy coupling, and we show some numerical results with the aim of comparing our approach with classic coupling techniques based on non-overlapping decompositions and the Beavers-Joseph-Saffman interface conditions.

---

## Optimization-based approach for problems with non-conforming interface discretizations

---

*Paul Kuberry\*, Pavel Bochev, Kara Peterson*

Discretization of problems involving interfaces may lead to several different types of non-conformities in the interface representation. These can include non-matching or non-coincident interface grids and/or different element types and polynomial degrees on each side of the interface. We present a nontraditional, optimization-based domain decomposition approach for such problems. The idea is to cast the interface problem as a constrained optimization problem with virtual Neumann controls. In this formulation the objective is to minimize jumps in the fields subject to the governing equations on each subdomain, while the Neumann boundary condition on the interface serves as control. We discuss the steps necessary to develop an optimization formulation for

non-conforming interfaces so that the flux is conserved over the interface, the objective remains well-defined, and the method is optimally accurate.

Using extension operators on a virtual interface and flux operators on a parameterized control space, we are able to construct an objective that enforces displacement continuity, even in the presence of gaps and overlaps between the interface representations. We will share and discuss numerical results exploring convergence rates, patch tests, and conservation properties of our method

### **Robust preconditioners for optimality systems - an infinite-dimensional perspective**

---

*Jarle Sogn\*, Walter Zulehner*

In this talk we consider optimization problems in function space with objective functionals of tracking type and elliptic partial differential equations (PDEs) as constraints, like inverse problems for elliptic PDEs or optimal control problems with elliptic state equations. Such problems typically involve an additional regularization/cost term depending on a regularization/cost parameter.

The discretized optimality systems of such problems are typically ill-conditioned due to high-dimensional approximation spaces and/or small regularization/cost parameters. Preconditioners for these discretized optimality systems based on the concept of Schur complements have been frequently proposed in literature leading to robust convergence properties of associated preconditioned Krylov subspace methods.

We will show how to exploit this strategy already for the formulation of the optimality systems in function space, which naturally leads to alternative formulations of the elliptic PDE-constraint. Besides the usual weak form also the strong and the very weak form come into play. We discuss possible implications of this approach for preconditioning the discretized optimality systems.



## Friday, February 10<sup>th</sup> 2017

	Culture Centre	North Pole	Northern Light	Polar Bear
07:00	Breakfast is served			
08:40	Bus departs from Coal Miners' Cabin Attention Coal Miners! Check out before boarding the bus, take your luggage with you			
09:00	PL12 Laura Grigori			Culture Centre
09:45	Conference closing remarks			
11:00	Check out from hotels			
	Bus to airport from Radisson: 11:20 (Norwegian) 12:00 (SAS)			

Chair: *Ralf Kornhuber*

**9:00~9:45 PL12 Laura Grigori**

**Culture Centre**

### **Communication avoiding iterative solvers and preconditioners**

*Laura Grigori\**

The cost of moving data in an algorithm can surpass by several orders of magnitude the cost of performing arithmetics, and this gap has been steadily and exponentially growing over time. This talk will review work performed in the recent years on a new class of algorithms for numerical linear algebra that drastically reduce the communication cost with respect to classic algorithms, or even provably minimize it in several cases. We focus in particular on enlarged Krylov subspace methods and preconditioners based on low rank corrections for solving large sparse linear systems of equations on massively parallel computers. We also discuss several associated computational kernels as computing a low rank approximation of a sparse matrix. The efficiency of the proposed methods is tested on matrices arising from linear elasticity problems as well as convection diffusion problems with highly heterogeneous coefficients.

## List of Participants

First Name	Last Name	Talks	Events	Affiliation	Email
Alexander	Heinlein	Y (1)	MS06	UNIVERSITY OF COLOGNE	<a href="mailto:alexander.heinlein@uni-koeln.de">alexander.heinlein@uni-koeln.de</a>
Alexandre	Halbach	Y (1)	CT/MS13	UNIVERSITY OF LIEGE	<a href="mailto:alexandre.halbach@ulg.ac.be">alexandre.halbach@ulg.ac.be</a>
Alexandros	Markopoulos	Y	--	TECHNICAL UNIVERSITY OF OSTRAVA	<a href="mailto:alexandros.markopoulos@vsb.cz">alexandros.markopoulos@vsb.cz</a>
Alfio	Quarteroni	Y (1)	INV SCI	EPFL LAUSANNE, SWITZERLAND	<a href="mailto:alfio.quarteroni@epfl.ch">alfio.quarteroni@epfl.ch</a>
Amini	Charles	Y	--	INSTITUTE OF ENVIRONMENT HYGIENE	<a href="mailto:aminicharles1234@gmail.com">aminicharles1234@gmail.com</a>
Andrea	Moiola	Y (1)	MS07	READING UNIVERSITY, UK	<a href="mailto:a.moiola@reading.ac.uk">a.moiola@reading.ac.uk</a>
Andreas	Langer	Y (2)	MS13 MS16	UNIVERSITY OF STUTTGART, GERMANY	<a href="mailto:langer@mathematik.uni-stuttgart.de">langer@mathematik.uni-stuttgart.de</a>
Andreas	Hessenthaler	SF (1)	MS09		<a href="mailto:hessenthaler@mechbau.uni-stuttgart.de">hessenthaler@mechbau.uni-stuttgart.de</a>
Andrew	McRae	Y (1)	MS09	BATH UNIVERSITY	<a href="mailto:a.t.t.mcrae@bath.ac.uk">a.t.t.mcrae@bath.ac.uk</a>
Andrew	Knyazev	Y	--	MITSUBISHI LABS	<a href="mailto:knyazev@merl.com">knyazev@merl.com</a>
Ange B.	Toulougoussou	Y (1)	CT	INRIA (RESEARCHER)	<a href="mailto:ange.toulougoussou@inria.fr">ange.toulougoussou@inria.fr</a>
Ansgar Burchardt	Burchardt	Y (1)	MS03	DRESDEN	<a href="mailto:ansgar.burchardt@tu-dresden.de">ansgar.burchardt@tu-dresden.de</a>
Antoine	Rousseau	Y (1)	MS04	INRIA, FRANCE	<a href="mailto:antoine.rousseau@inria.fr">antoine.rousseau@inria.fr</a>
Assyr	Abdulle	Y (1)	MS08	EPFL, SWITZERLAND	<a href="mailto:assy.abdulle@epfl.ch">assy.abdulle@epfl.ch</a>
Atle	Loneland	Y (1)	MS01	SPAREBANKEN VEST	<a href="mailto:atle.loneland@gmail.com">atle.loneland@gmail.com</a>
Augustin	Parret-Fréaud	Y (1)	MS01	SAFRAN TECH	<a href="mailto:augustin.parret-freaud@safrangroup.com">augustin.parret-freaud@safrangroup.com</a>
Axel	Klawonn	Y (2)	MS01 MS13 SCI	KOELN UNIV, GERMANY	<a href="mailto:axel.klawonn@uni-koeln.de">axel.klawonn@uni-koeln.de</a>
Bankim	C Mandal	Y (1)	MS19	MICHIGAN STATE UNIVERSITY	<a href="mailto:bmandal@math.msu.edu">bmandal@math.msu.edu</a>
Berangere	Delourme	Y (1)	MS11	LAGA, UNIVERSITÉ PARIS 13	<a href="mailto:delourme@math.univ-paris13.fr">delourme@math.univ-paris13.fr</a>
Bin	Wu	Y	--	WESTERN NORWAY U. APPL. SCI.	<a href="mailto:bwu@hib.no">bwu@hib.no</a>
Blanca	Ayuso de Dios	Y (1)	MS02	TECHNICAL UNIVERSITY OF HAMBURG	<a href="mailto:blanca.ayuso@tuhh.de">blanca.ayuso@tuhh.de</a>
Bo	Song	Y (2)	MS01 MS19	UNIVERSITY OF GENEVA	<a href="mailto:bo.song@unige.ch">bo.song@unige.ch</a>
Boyan	S. Lazarov	Y (1)	MS06	TECHNICAL UNIVERSITY OF DENMARK	<a href="mailto:bsl@mek.dtu.dk">bsl@mek.dtu.dk</a>
Bruno	Després	Y (1)	MS07	UNIVERSITÉ PARIS 6 (UPMC)	<a href="mailto:despres@ann.jussieu.fr">despres@ann.jussieu.fr</a>
Carmen	Rodrigo Cardiel	Y (1)	MS12	UNIVERSITY OF ZARAGOZA	<a href="mailto:carmenr@unizar.es">carmenr@unizar.es</a>
Caroline	Japhet	Y (1)	MS19	UNIVERSITY PARIS 13	<a href="mailto:japhet@math.univ-paris13.fr">japhet@math.univ-paris13.fr</a>
Carsten	Graser	Y (1)	MS16	FREIE UNIVERSITÄT BERLIN, GERMANY	<a href="mailto:graeser@mi.fu-berlin.de">graeser@mi.fu-berlin.de</a>
Chang-Ock	Lee	Y	--	KAIST	<a href="mailto:colee@kaist.edu">colee@kaist.edu</a>
Christiaan	C. Stolk		MS04	UNIV. OF AMSTERDAM, THE NETHERLANDS	<a href="mailto:c.c.stolk@uva.nl">c.c.stolk@uva.nl</a>
Christian	Vergara	Y (1)	MS04	POLITECNICO DI MILANO, ITALY	<a href="mailto:christian.vergara@polimi.it">christian.vergara@polimi.it</a> <a href="mailto:christian.vergara@unibg.it">christian.vergara@unibg.it</a>
Christoph	Hofer	SF (1)	MS05	JOHANNES KEPLER UNIVERSITY LINZ, PHD	<a href="mailto:christoph.hofer@ricam.oeaw.ac.at">christoph.hofer@ricam.oeaw.ac.at</a>
Christophe	Bovet	Y (1)	MS01	ONERA – THE FRENCH AEROSPACE LAB	<a href="mailto:christophe.bovet@onera.fr">christophe.bovet@onera.fr</a>
Clemens	Hofreither	Y (1)	MS05	JOHANNES KEPLER UNIVERSITY LINZ	<a href="mailto:chofreither@numa.uni-linz.ac.at">chofreither@numa.uni-linz.ac.at</a>
Daisuke	Tagami	Y (1)	CT	KYUSHU UNIVERSITY	<a href="mailto:tagami@imi.kyushu-u.ac.jp">tagami@imi.kyushu-u.ac.jp</a>
Daniel	Szyld	Y (1)	MS01	TEMPLE UNIVERSITY, PHILADELPHIA, USA	<a href="mailto:szyld@temple.edu">szyld@temple.edu</a>
David	Keyes	Y (1)	CT SCI	KAUST	<a href="mailto:david.keyes@kaust.edu.sa">david.keyes@kaust.edu.sa</a>

David	Horak	Y (1)	MS14	OSTRAVA, CZECH REPUBLIC	<a href="mailto:david.horak@vsb.cz">david.horak@vsb.cz</a>
Duk-Soon	Oh	Y (1)	CT	RUTGERS UNIVERSITY, USA	<a href="mailto:duksoon@math.rutgers.edu">duksoon@math.rutgers.edu</a>
Eirik	Keilegavlen	Y (1)	MS06	UNIVERSITY OF BERGEN	<a href="mailto:eirik.keilegavlen@uib.no">eirik.keilegavlen@uib.no</a>
Emanuele	Ragnoli	Y (1)	CT	IBM RESEARCH – IRELAND	<a href="mailto:eragnoli@ie.ibm.com">eragnoli@ie.ibm.com</a>
Emily	Walsh	Y (1)	MS09	UWE BRISTOL	<a href="mailto:emily3.walsh@uwe.ac.uk">emily3.walsh@uwe.ac.uk</a>
Eric T.	Chung	Y (1)	MS06	THE CHINESE UNIVERSITY OF HONG KONG	<a href="mailto:eric.t.chung@gmail.com">eric.t.chung@gmail.com</a>
Erik	Eikeland	Y	--	WESTERN NORWAY U. APPL. SCI.	<a href="mailto:eeik@hib.no">eeik@hib.no</a>
Eun-Hee	Park	Y (1)	MS02	KANGWON NATIONAL UNIVERSITY	<a href="mailto:eh.park@kangwon.ac.kr">eh.park@kangwon.ac.kr</a>
Eun-Jae	Park	Y (1)	MS02	YONSEI UNIVERSITY , S KOREA	<a href="mailto:eunjae.park@gmail.com">eunjae.park@gmail.com</a>
Faycal	Chaouqui	Y (1)	MS01	INRIA BORDEAUX - SUD- OUEST	<a href="mailto:faycal.chaouqui@unige.ch">faycal.chaouqui@unige.ch</a>
Fei	Li	SF (1)	MS04	STANFORD UNIVERSITY, PHD STUDENT	<a href="mailto:liufei@stanford.edu">liufei@stanford.edu</a>
Fei	Wang	Y	--	XI'AN JIAOTONG UNIVERSITY	<a href="mailto:feiwang.xjtu@xjtu.edu.cn">feiwang.xjtu@xjtu.edu.cn</a>
Felix	Kwok	Y (1)	MS17	HONG KONG BAPTIST UNIVERSITY.	<a href="mailto:felix_kwok@hkbu.edu.hk">felix_kwok@hkbu.edu.hk</a>
Fran	Rosamond	Y	--	UNIVERSITY OF BERGEN	<a href="mailto:frances.rosamond4@gmail.com">frances.rosamond4@gmail.com</a>
Francesco	Botti/Bassi	Y	--	UNIVERSITÀ DEGLI STUDI DI BERGAMO	<a href="mailto:francesco.bassi@unibg.it">francesco.bassi@unibg.it</a>
Fransisco	Gasper	Y (1)	MS12	UNIVERSITY OF ZARAGOZA, SPAIN	<a href="mailto:fjgaspar@unizar.es">fjgaspar@unizar.es</a> <a href="mailto:f.j.gaspar@cwi.nl">f.j.gaspar@cwi.nl</a>
Gabriele	Ciaramella	Y (1)	MS15	UNIVERSITY OF GENEVA, SWITZERLAND, POSTDOC	<a href="mailto:gabriele.ciaramella@unige.ch">gabriele.ciaramella@unige.ch</a>
Giovanni	Samaey	Y (1)	MS17	KU LEUVEN	<a href="mailto:giovanni.samaey@cs.kuleuven.be">giovanni.samaey@cs.kuleuven.be</a>
Guanglian	Li	Y (1)	MS06	UNIVERSITY OF BONN, POSTDOC	<a href="mailto:li@ins.uni-bonn.de">li@ins.uni-bonn.de</a>
Guillaume	Morel	Y (1)	MS07	CEA, LJLL (PHD)	<a href="mailto:guillaume.morel3@sfr.fr">guillaume.morel3@sfr.fr</a>
Hee Jun	Yang	Y	--	KYUNG HEE UNIVERSITY	<a href="mailto:yhj109@khu.ac.kr">yhj109@khu.ac.kr</a>
Hermann	Schneider	Y	--	COMPUTER SIMULATION TECHNOLOGY AG	<a href="mailto:hermann.schneider.1000@live.de">hermann.schneider.1000@live.de</a>
Hongxuan	Zhang	Y (1)	MS12	PENNSYLVANIA STATE UNIVERSITY	<a href="mailto:huz114@psu.edu">huz114@psu.edu</a>
Hui	Zhang	Y (2)	MS04, MS19	ZHEJIANG OCEAN UNIVERSITY	<a href="mailto:huiz@zjou.edu.cn">huiz@zjou.edu.cn</a>
Hyea	Hyun Kim	Y	MS01 SCI	KYUNG HEE UNIVERSITY	<a href="mailto:hhkim@khu.ac.kr">hhkim@khu.ac.kr</a> <a href="mailto:hyeahyun@gmail.com">hyeahyun@gmail.com</a>
Iain	Smears	Y (1)	MS02	UNIVERSITY OF OXFORD, UK	<a href="mailto:smears@maths.ox.ac.uk">smears@maths.ox.ac.uk</a> <a href="mailto:iain.smears@inria.fr">iain.smears@inria.fr</a>
Ivan	Graham	Y (2)	INV, MS10	BATH UNIVERSITY, UK	<a href="mailto:i.g.graham@bath.ac.uk">i.g.graham@bath.ac.uk</a>
Jacob	B. Schroder	Y (1)	MS09	LAWRENCE LIVERMORE NATIONAL LABORATORY	<a href="mailto:schroder2@llnl.gov">schroder2@llnl.gov</a>
Jacopo	Corno	Y (1)	MS10	TU DARMSTADT	<a href="mailto:corno@gsc.tu-darmstadt.de">corno@gsc.tu-darmstadt.de</a>
Jakub	Sistek	Y (1)	MS14	INSTITUTE OF MATHEMATICS OF THE ACADEMY OF SCIENCES OF THE CZECH REPUB	<a href="mailto:sistek@math.cas.cz">sistek@math.cas.cz</a>
Jakub	Both	SF (1)	MS16	UNIVERSITY OF BERGEN, NORWAY	<a href="mailto:jakub.both@uib.no">jakub.both@uib.no</a>
James	Adler	Y (1)	MS12	TUFTS UNIVERSITY, USA	<a href="mailto:james.adler@tufts.edu">james.adler@tufts.edu</a>
Jan	Nordbotten	Y (1)	INV	UNIVERSITY OF BERGEN, NORWAY	<a href="mailto:jan.nordbotten@gmail.com">jan.nordbotten@gmail.com</a>
Jarle	Sogn	Y (1)	MS08	JOHANNES KEPLER UNIVERSITY LINZ, AUSTRIA	<a href="mailto:jarle.sogn@gmail.com">jarle.sogn@gmail.com</a>
Jed	Brown	Y (1)	INV	UNIVERSITY OF COLORADO, USA	<a href="mailto:jed@jedbrown.org">jed@jedbrown.org</a>
Jinchao	Xu	Y (1)	MS12 SCI	PENN STATE, USA	<a href="mailto:xu@math.psu.edu">xu@math.psu.edu</a>
Jinru	Chen	Y	MS12	NANJING NORMAL UNIV., CHINA	<a href="mailto:jrchen@njnu.edu.cn">jrchen@njnu.edu.cn</a>
Jixin	Chen	SF (1)	MS11	UNIVERSITE LIBRE DE BRUXELLES, EAST CHINA NORMAL UNIVERSITY	<a href="mailto:cjxhmj22344457@163.com">cjxhmj22344457@163.com</a>
Jocelyne	Erhel	Y (1)	CT	INRIA, FRANCE	<a href="mailto:jocelyne.erhel@inria.fr">jocelyne.erhel@inria.fr</a>
Johannes	Krauss	Y (1)	MS12	UNIVERSITY OF DUISBURG-ESSEN	<a href="mailto:johannes.krauss@oeaw.ac.at">johannes.krauss@oeaw.ac.at</a> <a href="mailto:johannes.krauss@uni-due.de">johannes.krauss@uni-due.de</a>

John	Shadid	Y (1)	MS12	SANDIA NATIONAL LABORATORIES, USA	<a href="mailto:jnshadi@sandia.gov">jnshadi@sandia.gov</a>
John	Pearson	Y (1)	MS11	UNIVERSITY OF KENT	<a href="mailto:j.w.pearson@kent.ac.uk">j.w.pearson@kent.ac.uk</a>
Jon Eivind	Vatne	Y	--	WESTERN NORWAY U. APPL. SCI.	<a href="mailto:jon.eivind.vatne@bkkfiber.no">jon.eivind.vatne@bkkfiber.no</a>
Jonathan	Youett	Y (2)	MS03, MS16	FREI UNIVERSITÄT BERLIN, GERMANY	<a href="mailto:youett@math.fu-berlin.de">youett@math.fu-berlin.de</a>
Jongho	Park	Y (SF)	Poster	KAIST	<a href="mailto:jongho.park@kaist.ac.kr">jongho.park@kaist.ac.kr</a>
Jose	C. Gary	SF	Poster	TEMPLE UNIVERSITY, USA	<a href="mailto:tue62222@temple.edu">tue62222@temple.edu</a>
Jose Pablo	Lucero Lorca	Y (SF)	Poster	UNIVERSITÄT HEIDELBERG	<a href="mailto:pablo.lucero@iwr.uni-heidelberg.de">pablo.lucero@iwr.uni-heidelberg.de</a>
Juan	Galvis	Y (3)	INV, MS06, MS13	UNIVERSIDAD NACIONAL DE COLOMBIA	<a href="mailto:jcgalvisa@unal.edu.co">jcgalvisa@unal.edu.co</a>
Julien	Salomon	Y (1)	MS11	UNIVERSITE PARIS-DAUPHINE	<a href="mailto:julien.salomon@dauphine.fr">julien.salomon@dauphine.fr</a>
Jun	Zou	Y (1)	MS11 SCI	CHINESE UNIVERSITY OF HONG KONG	<a href="mailto:zou@math.cuhk.edu.hk">zou@math.cuhk.edu.hk</a>
Kaibo	Hu	Y (1)	MS12	PEKING UNIVERSITY, PHD	<a href="mailto:kaibo@pku.edu.cn">kaibo@pku.edu.cn</a>
Ke	Chen	Y	--	UNIVERSITY OF LIVERPOOL	<a href="mailto:k.chen@liverpool.ac.uk">k.chen@liverpool.ac.uk</a>
Kundan	Kumar	Y (1)	MS13	UNIVERSITY OF BERGEN, NORWAY	<a href="mailto:kundan.kumar@uib.no">kundan.kumar@uib.no</a>
Laura	Grigori	Y (1)	INV	INRIA, FRANCE	<a href="mailto:laura.grigori@inria.fr">laura.grigori@inria.fr</a>
Laurence	Halpern	Y (1)	MS19 SCI	UNIV. PARIS 13	<a href="mailto:halpern@math.univ-paris13.fr">halpern@math.univ-paris13.fr</a>
Leonardo	A. Poveda	SF	CT	UNIVERSIDADE DE SÃO PAULO	<a href="mailto:lpovedac@ime.usp.br">lpovedac@ime.usp.br</a>
Leszek	Marcinkowski	Y (1)	MS06	UNIVERSITY OF WARSAW	<a href="mailto:lmarcin@mimuw.edu.pl">lmarcin@mimuw.edu.pl</a>
Li-yeng	Sung	Y	--		<a href="mailto:sung@math.lsu.edu">sung@math.lsu.edu</a>
Lina	Zhao	SF (1)	Poster	YONSEI UNIVERSITY	<a href="mailto:zlngreat5@gmail.com">zlngreat5@gmail.com</a>
Lise-Marie	Imbert-Gérard	Y (1)	MS07	COURANT INSTITUTE, NY	<a href="mailto:imbertgerard@cims.nyu.edu">imbertgerard@cims.nyu.edu</a>
Lorenzo Alessio	Botti	Y (1)	MS02	UNIVERSITÀ DEGLI STUDI DI BERGAMO	<a href="mailto:bottilorenzo@gmail.com">bottilorenzo@gmail.com</a>
Lori	Badea	Y (1)	MS13	ROMANIAN ACADEMY	<a href="mailto:lori.badea@imar.ro">lori.badea@imar.ro</a>
Louis	Poirel	Y (1)	MS01	INRIA BORDEAUX - SUD-OUEST	<a href="mailto:louis.poirel@inria.fr">louis.poirel@inria.fr</a>
Lubomir	Riha	Y (1)	CT		<a href="mailto:lubomir.riha@vsb.cz">lubomir.riha@vsb.cz</a>
Luca F.	Pavarino	Y (1)	MS05, MS13	UNIVERSITY OF PAVIA	<a href="mailto:luca.pavarino@unimi.it">luca.pavarino@unimi.it</a>
Luca	Gerardo-Giorda	Y (1)	MS15 2 talks	BASQUE CENTER FOR APPLIED MATHEMATICS, SPAIN	<a href="mailto:lgerardo@bcamath.org">lgerardo@bcamath.org</a>
Ludmil	Zikatanov	Y (1)	MS02	PENN STATE UNIVERSITY	<a href="mailto:ludmil@psu.edu">ludmil@psu.edu</a>
Lulu	Liu	Y (1)	MS13	KAST	<a href="mailto:lulu.liu@usi.ch">lulu.liu@usi.ch</a> <a href="mailto:lulu.liu@kaust.edu.sa">lulu.liu@kaust.edu.sa</a>
Manfred	Liebmann	Y (1)	MS11	TU MUENCHEN	<a href="mailto:manfred.liebmann@uni-graz.at">manfred.liebmann@uni-graz.at</a>
Marc	Olm	SF (1)	MS17	UNIVERSITAT POLITÈCNICA DE CATALUNYA	<a href="mailto:olm.marc@gmail.com">olm.marc@gmail.com</a>
Marcella	Bonazzoli	SF (1)	MS10	UNIVERSITY OF NICE, PHD STUDENT	<a href="mailto:marcella.bonazzoli@unice.fr">marcella.bonazzoli@unice.fr</a>
Marco	Donatelli	Y (1)	MS05	UNIVERSITA' DELL'INSUBRIA - SEDE DI COMO	<a href="mailto:marco.donatelli@uninsubria.it">marco.donatelli@uninsubria.it</a>
Marcus	Sarkis	Y (1)	MS06	WORCESTER POLYTECHNIC INSTITUTE	<a href="mailto:msarkis@wpi.edu">msarkis@wpi.edu</a>
Maria Nestola	Nestola	Y (1)	MS03	USI LUGANO	<a href="mailto:maria.giuseppina.chiara.nestola@usi.ch">maria.giuseppina.chiara.nestola@usi.ch</a>
Marie	Rognes	Y (1)	INV	SIMULA	<a href="mailto:meg@simula.no">meg@simula.no</a>
Mario	Bebendorf	Y (1)	MS06		<a href="mailto:mario.bebendorf@uni-bayreuth.de">mario.bebendorf@uni-bayreuth.de</a>
Martin	Neumueller	Y (2)	MS17, MS18	JOHANNES KEPLER UNIV, LINZ, GERMANY	<a href="mailto:neumueller@numa.uni-linz.ac.at">neumueller@numa.uni-linz.ac.at</a>
Martin	Lanser	Y (1)	--	UNIVERSITÄT ZU KÖLN	<a href="mailto:mlanser@uni-koeln.de">mlanser@uni-koeln.de</a>
Martin	J. Gander	Y (2)	MS01 MS10 SCI	UNIVERSITY OF GENEVA, SWITZERLAND	<a href="mailto:martin.gander@unige.ch">martin.gander@unige.ch</a>
Martin	Hanek	SF (1)	CT	CZECH TECHNICAL UNIVERSITY IN PRAGUE, PHD	<a href="mailto:martin.hanek@fs.cvut.cz">martin.hanek@fs.cvut.cz</a>

Martin	Weiser	Y (1)	MS03	ZIB BERLIN	<a href="mailto:weiser@zib.de">weiser@zib.de</a>
Matteo	Aletti	Y (1)	MS15	INRIA, PHD STUDENT	<a href="mailto:matteo.aletti@inria.fr">matteo.aletti@inria.fr</a>
Matthias	Bolten	Y (1)	MS17	KASSEL UNIVERSITY, GERMANY	<a href="mailto:matthias.bolten@mathematik.uni-kassel.de">matthias.bolten@mathematik.uni-kassel.de</a>
Mathieu	Benoit Lecouvez	Y (2)	MS12 CT	LLNL	<a href="mailto:lecouvez1@llnl.gov">lecouvez1@llnl.gov</a>
Mattia	Tani	Y (1)	MS05	UNIVERSITÀ DI PAVIA	<a href="mailto:mattia.tani@unipv.it">mattia.tani@unipv.it</a>
Mi-Young	Kim	Y	--	INHA UNIVERSITY	<a href="mailto:mikim@inha.ac.kr">mikim@inha.ac.kr</a>
Michael	Minion	Y (1)	MS17	LBNL	<a href="mailto:mlminion@lbl.gov">mlminion@lbl.gov</a>
Mike	Fellows	Y	--	UNIVERSITY OF BERGEN	<a href="mailto:michael.fellows@uib.no">michael.fellows@uib.no</a>
Nicolas	Marsic	Y (1)	CT	UNIVERSITÄT DARMSTADT	<a href="mailto:marsic@temf.tu-darmstadt.de">marsic@temf.tu-darmstadt.de</a>
Nicole	Spillane	Y (1)	MS01	CMAP, CNRS, ECOLE POLYTECHNIQUE	<a href="mailto:nicole.spillane@cmap.polytechnique.fr">nicole.spillane@cmap.polytechnique.fr</a>
Olaf	Steinbach	Y (1)	MS18	TU GRAZ	<a href="mailto:o.steinbach@tugraz.at">o.steinbach@tugraz.at</a>
Olaf	Schenk	Y	--	UNIVERSITÀ DELLA SVIZZERA ITALIANA	<a href="mailto:olaf.schenk@usi.ch">olaf.schenk@usi.ch</a>
Oliver	Rheinbach	Y (1)	MS06	TECHNISCHE UNIV. BERGAKADEMIE FREIBERG	<a href="mailto:oliver.rheinbach@math.tu-freiberg.de">oliver.rheinbach@math.tu-freiberg.de</a>
Oliver	Sander	Y (1)	MS16	UNIVERSITY OF DRESDEN, GERMANY	<a href="mailto:oliver.sander@tu-dresden.de">oliver.sander@tu-dresden.de</a>
Olivier	Tissot	Y (1)	MS01	INRIA, PHD STUDENT	<a href="mailto:olivier.tissot@inria.fr">olivier.tissot@inria.fr</a>
Olof	Widlund	Y (1)	CT SCI	COURANT	<a href="mailto:widlund@cims.nyu.edu">widlund@cims.nyu.edu</a>
Olof	Runborg	Y (1)	MS07	KTH, SWEDEN	<a href="mailto:olofr@kth.se">olofr@kth.se</a>
Paola	F. Antonietti	Y (1)	MS02	POLITECNICO DI MILANO, ITALY	<a href="mailto:paola.antonietti@polimi.it">paola.antonietti@polimi.it</a>
Paola	Gervasio	Y (2)	MS08, MS15	UNIVERSITY OF BRESCIA, ITALY	<a href="mailto:gervasio@ing.unibs.it">gervasio@ing.unibs.it</a>
Paul	Kuberry	Y (1)	MS08	SANDIA NATIONAL LABORATORIES	<a href="mailto:pakuber@sandia.gov">pakuber@sandia.gov</a>
Pierre	Jolivet	Y (1)	MS14	TOULOUSE, FRANCE	<a href="mailto:jolivet@ann.jussieu.fr">jolivet@ann.jussieu.fr</a> <a href="mailto:pierre.jolivet@enseiht.fr">pierre.jolivet@enseiht.fr</a>
Pierre-Henry	Cocquet	Y (1)	MS10	UNIVERSITY OF REUNION	<a href="mailto:pierre-henri.cocquet@unige.ch">pierre-henri.cocquet@unige.ch</a>
Pietro	Benedusi	Y	--	UNIVERSITÀ DELLA SVIZZERA ITALIANA, PHD	<a href="mailto:pietro.benedusi@usi.ch">pietro.benedusi@usi.ch</a>
Piotr	Krzyzanowski	Y (1)	MS06	WARSAW UNIVERSITY	<a href="mailto:p.krzyzanowski@mimuw.edu.pl">p.krzyzanowski@mimuw.edu.pl</a>
Pratik	Kumbhar	Y (1)	MS19	UNIV. OF GENEVA, PHD STUDENT	<a href="mailto:pratik.kumbhar@unige.ch">pratik.kumbhar@unige.ch</a>
Radim	Sojka	SF	Poster		<a href="mailto:radim.sojka@vsb.cz">radim.sojka@vsb.cz</a>
Raffael	Casagrande	Y (1)	MS07	ETH ZURICH, SWITZERLAND (PHD)	<a href="mailto:raffael.casagrande@sam.math.ethz.ch">raffael.casagrande@sam.math.ethz.ch</a>
Ralf	Kornhuber	Y	SCI	FREIE UNIVERSITÄT BERLIN, GERMANY	<a href="mailto:kornhuber@math.fu-berlin.de">kornhuber@math.fu-berlin.de</a>
Rolf	Krause	Y (1)	MS08	USI, SWITZERLAND	<a href="mailto:rolf.krause@usi.ch">rolf.krause@usi.ch</a>
Ron	Haynes	Y (1)	INV	MEMORIAL UNIV OF NEWFOUNDLAND, CA	<a href="mailto:rhaynes@mun.ca.rhaynes74@gmail.com">rhaynes@mun.ca.rhaynes74@gmail.com</a>
Santiago	Badia	Y (2)	MS14, MS15	CIMNE	<a href="mailto:sbadia@cimne.upc.edu">sbadia@cimne.upc.edu</a>
Scott	Congreve	Y (1)	MS07	WIEN UNIV.	<a href="mailto:scott.congreve@univie.ac.at">scott.congreve@univie.ac.at</a>
Sebastian	Götschel	Y (1)	MS17	ZUSE INSTITUTE BERLIN, POSTDOC	<a href="mailto:goetschel@zib.de">goetschel@zib.de</a>
Sebastian	Reyes Riffo	Y	--	UNIVERSITÉ PARIS-DAUPHINE (PHD STUDENT)	<a href="mailto:reyesriffo@ceremade.dauphine.fr">reyesriffo@ceremade.dauphine.fr</a>
Seungil	Kim	Y (1)	MS04	KYUNG HEE UNIVERSITY	<a href="mailto:sikim@khu.ac.kr">sikim@khu.ac.kr</a>
Shu-Lin	Wu	Y (1)	MS19	SICHUAN UNIV. OF SCIENCE AND ENGINEERING	<a href="mailto:wushulin84@hotmail.com">wushulin84@hotmail.com</a>
Shuonan	Wu	Y (1)	MS12	PENN STATE, USA	<a href="mailto:snwu@multigrid.org">snwu@multigrid.org</a> <a href="mailto:sxw58@psu.edu">sxw58@psu.edu</a>
Simone	Scacchi	Y (2)	INV, MS03	UNIVERSITY OF MILAN, ITALY	<a href="mailto:simone.scacchi@unimi.it">simone.scacchi@unimi.it</a>
Stefan	Dohr	SF (1)	MS18	GRAZ UNIVERSITY OF TECHNOLOGY	<a href="mailto:stefan.dohr@gmail.com">stefan.dohr@gmail.com</a>

Stefan	Takacs	Y (1)	MS05	RICAM, AUSTRIAN ACADEMY OF SCIENCES	<a href="mailto:stefan.takacs@ricam.oeaw.ac.at">stefan.takacs@ricam.oeaw.ac.at</a>
Stefan	Ulbrich	Y (1)	MS11	TECHNISCHE UNIVERSITÄT DARMSTADT, GE	<a href="mailto:ulbrich@mathematik.tu-darmstadt.de">ulbrich@mathematik.tu-darmstadt.de</a>
Stefano	Zampini	Y (2)	INV, MS05		<a href="mailto:s.zampini@cineca.it">s.zampini@cineca.it</a>
Susanne	Brenner	Y (1)	INV SCI	LOUISIANA STATE UNIVERSITY, USA	<a href="mailto:brenner@math.lsu.edu">brenner@math.lsu.edu</a>
Tomas	Kozubek	Y	--		<a href="mailto:tomas.kozubek@vsb.cz">tomas.kozubek@vsb.cz</a>
Tommaso	Vanzan	Y (1)	MS15	UNIVERSITY OF GENEVA	<a href="mailto:tommaso.vanzan@unige.ch">tommaso.vanzan@unige.ch</a>
Tonatiuh	Sanchez-Vizuet	Y	--	COURANT INSTITUE, NY	<a href="mailto:tonatiuh@cims.nyu.edu">tonatiuh@cims.nyu.edu</a>
Ulrich	Langer	Y (1)	MS15 SCI	JOHANNES KEPLER UNIVERSITY, LINZ	<a href="mailto:ulanger@numa.uni-linz.ac.at">ulanger@numa.uni-linz.ac.at</a>
Ulrich	Ruede	Y (1)	MS14	ERLANGEN, GERMANY	<a href="mailto:ulrich.ruede@fau.de">ulrich.ruede@fau.de</a>
Václav	Hapla	Y (1)	CT	TECHNICAL UNIVERSITY OF OSTRAVA	<a href="mailto:vaclav.hapla@vsb.cz">vaclav.hapla@vsb.cz</a>
Vanessa	Mattesi	Y (1)	MS10	UNIVERSITY OF LIÈGE, POSTDOC	<a href="mailto:vanessa.mattesi@ulg.ac.be">vanessa.mattesi@ulg.ac.be</a>
Veronique	Martin	Y (1)	MS15	UNIVERSITÉ D'AMIENS, FRANCE	<a href="mailto:veronique.martin@u-picardie.fr">veronique.martin@u-picardie.fr</a>
Victorita	Dolean	Y (1)	MS10	UNIVERSITY OF STRATHCLYDE, UK	<a href="mailto:victorita.dolean@strath.ac.uk">victorita.dolean@strath.ac.uk</a>
Wietse	Boon	Y (1)	CT/MS06	UIB, PHD STUDENT	<a href="mailto:wietse.boon@uib.no">wietse.boon@uib.no</a>
Xavier	Claeys	Y	--	INRIA ALPINES/ UPMC LJLL	<a href="mailto:xavier.claeys@inria.fr">xavier.claeys@inria.fr</a>
Xiao-Chuan	Cai	Y (1)	MS14 SCI	UNIV. OF COLORADO, BOLDER	<a href="mailto:cai@colorado.edu">cai@colorado.edu</a>
Xiaozhe	Hu	Y (1)	INV	TUFTS UNIVERSITY, USA	<a href="mailto:xiaozhe.hu@tufts.edu">xiaozhe.hu@tufts.edu</a>
Xiaozhou	Li	Y	--	UNIVERSITÀ DELLA SVIZZERA ITALIANA	<a href="mailto:xiaozhou.li@usi.ch">xiaozhou.li@usi.ch</a>
Xueshuang	Xiang	Y (1)	MS04	QIAN XUESEN LABORATORY OF TECHNOLOGY	<a href="mailto:xiangxueshuang@qxslab.cn">xiangxueshuang@qxslab.cn</a>
Yingxiang	Xu	Y (1)	MS04	NORTHEAST NORMAL UNIVERSITY, CHINA	<a href="mailto:yxxu@nenu.edu.cn">yxxu@nenu.edu.cn</a>
Zdeněk	Dostál	Y (1)	CT	TECHNICAL UNIVERSITY OF OSTRAVA	<a href="mailto:zdenek.dostal@vsb.cz">zdenek.dostal@vsb.cz</a>

