Domain Decomposition Method using Genetic Algorithms for Solving Transonic Aerodynamic Problems

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1 Introduction

Parallel computers and multiprocessor supercomputers are becoming more readily available; therefore highly parallel algorithms for solving practical complex problems are of great interest. Domain decomposition techniques [1], traced to Schwarz alternating procedure, are often used in parallel environments and genetic algorithms (GAs) are reported to be highly parallel [2]. The application of GAs to domain decomposition problems can be expected to have high parallelism. This paper explores GAs for solving 2-D aerodynamic problems through Schwarz's domain splitting with overlapping in order to develop a new kind of domain decomposition approach with high parallelism.

In the present method, GAs are the key techniques through the definition of a fitness function based on solutions on overlapping subdomains. The Schwarz alternating sequences are then guided by genetic algorithms. Binary codings for multiparameters and small population size are used in genetic iterations. The method presented is tested for 2-D transonic flows in a nozzle. The numerical results show that the method presented successfully finds the near optimum solutions within the finite genetic generations.

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2 Description of the Problem

The aerodynamic test problem we treat here is the flows in a nozzle(see Fig. 2.1)

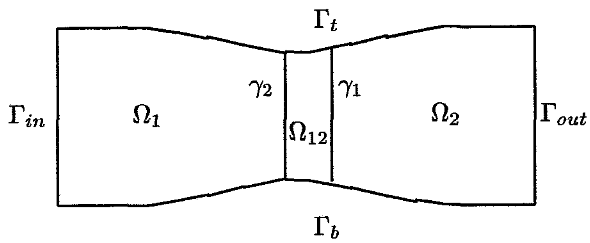

Fig. 2.1

which is modeled by

$$\begin{aligned} a^2 \nabla^2 \Phi - \frac{1}{2} \nabla \Phi \bullet \nabla(|\nabla \Phi|^2) = 0 \text{ in } \Omega \quad (\Omega_1 \bigcup \Omega_2) \\ \partial\Phi/\partial n = 0 \quad \text{on} \quad \Gamma_t \bigcup \Gamma_b \\ \Phi = g_{in} \quad \text{on} \quad \Gamma_{in} \\ \Phi = g_{out} \quad \text{on} \quad \Gamma_{out}. \end{aligned} \tag{2.1}$$

Here Φ is a flow potential, n denotes the unit outward normal vector and a, the speed of sound given by

$$a^2 = \frac{1}{M_\infty^2} + \frac{\gamma - 1}{2}(1 - |\nabla \Phi|^2), \tag{2.2}$$

where γ is the ratio of specific heats and M_∞ represents the reference Mach number at infinity.

As shown in Fig. 2.1, we decompose the computational domain Ω into two subdomains Ω_1 and Ω_2 with overlap Ω_{12}; the interfaces are denoted by γ_1 and γ_2. We shall take values

$$g_1 \quad \text{on} \quad \gamma_1 \quad \text{and} \quad g_2 \quad \text{on} \quad \gamma_2$$

as extra boundary conditions in order to solve (2.1) in each subdomain. Using domain decomposition techniques, the problem can be reduced to minimizing the following function:

$$J(g_1, g_2) = \frac{1}{2} \| \Phi_1 - \Phi_2 \|^2,$$

where Φ_1 and Φ_2 are the solutions in the overlapping subdomain Ω_{12}.

3 Algorithm

In this section, we present an implementation of genetic algorithms for the domain decomposition problem described above.

Genetic algorithms (GAs) [2] are search algorithms which are different from the conventional search procedures encountered in engineering optimization. The core structure of the algorithm is based upon the principle of natural selection and natural genetics. For the function being optimized, the variables are rewritten in a code to form a structured string that the GAs can operate directly on. For the problem described above, binary codings for multiparameters are used, and we only code g_1, which can be given by

$$g_{1i}, i = 1, \cdots n \, (n \text{ parameters });$$

each g_{1i} is coded in l_i bits, thus the length of the strings

$$L = \sum_i^n l_i.$$

Let us consider a population size 25 (i.e. 25 strings). GAs decode each string to return the values of g_{1i}. With the g_{1i} known, we can compute the solutions in the domain Ω_1, namely Φ_1. Like Schwarz's alternative method, we take g_2 based on Φ_1, thus the solution, Φ_2, in the domain Ω_2 can be calculated. Now, we send both values in the overlapping region to the cost function:

$$J(g_1) = \frac{1}{2} \parallel \Phi_1 - \Phi_2 \parallel^2 .$$

Thus, each string has a cost value. A new generation of strings can be produced by performing GA operation [2], such as reproduction, crossover and mutation. By the principle of the survival of the fittest, strings with better cost values will be selected and structures containing the desired characteristics will survive while others die off as successive generations are produced. This process continues until convergence is achieved.

4 Numerical Results and Analysis

The approach presented has been tested for the domain decomposition problem described in the section 2. Fig. 4.1 shows the mesh used to solve the test problem and the overlap Ω_{12} whose interface γ_1 is located just by the throat.

The interface γ_1 has 6 grid points (i.e. 6 parameters). During the GA iteration, the length of the string, 30, is used with 5 bits for each parameter. The probability of crossover $Pc = 0.85$ and the probability of mutation $Pm = 0.015$ are not carefully selected but are fixed for the flow test cases.

As mentioned above, each calculation of $J(g_1)$ requires the solution of the full potential flow equation in each subdomain, which results in a computing cost which is directly proportional to the number of CFD evaluation. Thus a small population

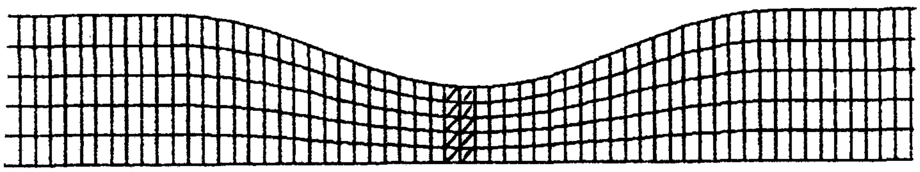

Fig. 4. 1 grid of the nozzle

size is used. The presented method begins by randomly generating a population of 25 strings. We keep in mind that each string represents one possible solution g_1 to the problem. But the values on the interface γ_1 are very sensitive for the transonic case due to the limitation of the sonic throat. If g_1 is given beyond this physical limitation, the CFD solver [3] will fail to converge. This is why traditional Schwarz alternative method may fail. Using GAs the individuals may have the same problem, but with the genetics population, the information before divergence can be still used.

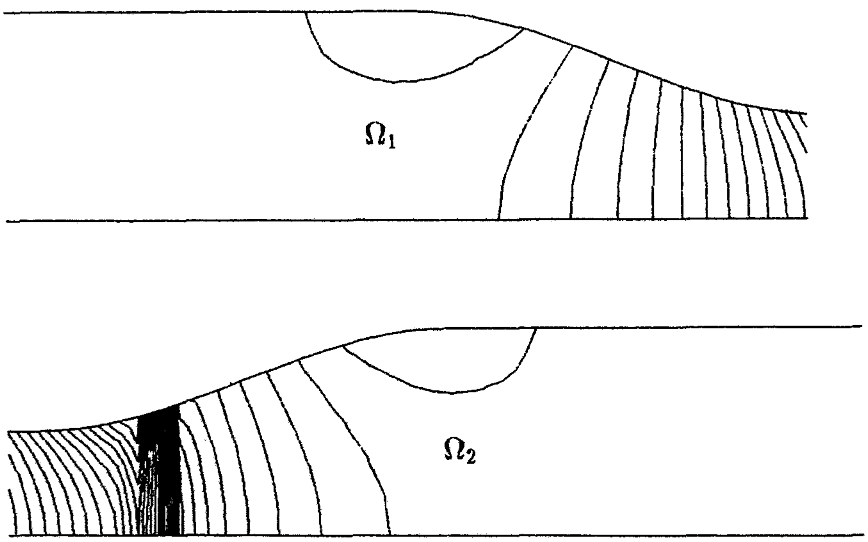

Fig. 4. 2

With the boundary conditions, $g_{in} = -1.8$ and $g_{out} = 1.8$, we have tested different flow cases. Here one typical example for the transonic case with $M_\infty = 0.70$ is presented. The corresponding isomach lines for each subdomain are also presented in Fig. 4.2. Fig. 4.3 shows the computed fitness function $J(g_1)$ for all the examined g_1.

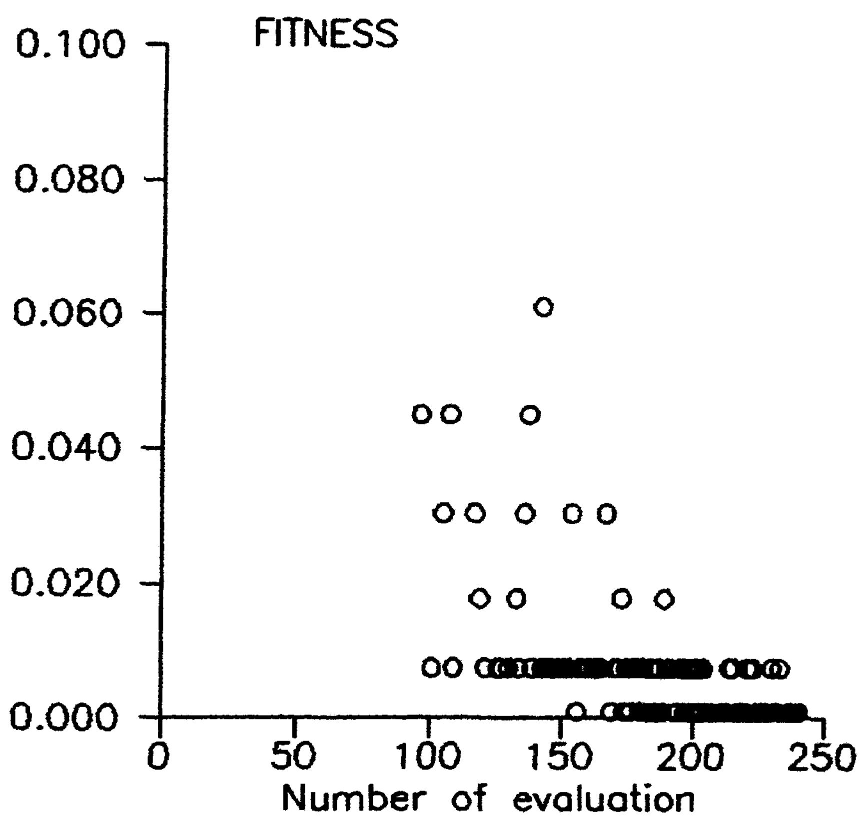

Fig. 4. 3 Isomach lines

It can be noted that the method presented can realize near optimal solutions within the finite genetic generations.

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