Domain Decomposition Methods in Science and Engineering

Thirteenth International Conference on Domain Decomposition Methods

Lyon, France

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Preface

This volume captures 53 of 100 the presentations of the Thirteenth International Conference on Domain Decomposition Methods, which was hosted by the University of Lyon in the Champfleuri Conference Center in the Province of Rhone-Alps, France, October 9-12, 2000. Approximately 117 mathematicians, engineers, physical scientists, and computer scientists from 22 countries came to this nearly annual gathering.

Since three parallel sessions were employed at the conference in order to accommodate as many presenters as possible, attendees and non-attendees alike may turn to this volume to keep up with the diversity of subject matter that the topical umbrella of "domain decomposition" inspires throughout the community. The interest of so many authors in meeting the editorial demands of this proceedings volume demonstrates that the common thread of domain decomposition continues to justify a regular meeting. "Divide and conquer" may be the most basic of algorithmic paradigms, but theoreticians and practitioners alike are still seeking - and finding - incrementally more effective forms, and value the interdisciplinary forum provided by this proceedings series.

Besides inspiring elegant theory, domain decomposition methodology satisfies the architectural imperatives of high-performance computers better than methods operating only on the finest scale of the discretization and over the global data set. These imperatives include: concurrency on the scale of the number of available processors, spatial data locality, temporal data locality, reasonably small communication-to-computation ratios, and reasonably infrequent process synchronization (measured by the number of useful floating-point operations performed between synchronizations). Spatial data locality refers to the proximity of the addresses of successively used elements, and temporal data locality refers to the proximity in time of successive references to a given element. Spatial and temporal locality are both enhanced when a large computation based on nearest-neighbor updates is processed in contiguous blocks. On cache-based computers, subdomain blocks may be tuned for workingset sizes that reside in cache. On message-passing or cache-coherent nonuniform memory access (cc-NUMA) parallel computers, the concentration of gridpoint-oriented computations - proportional to subdomain volume - between external stencil edge-oriented communications - proportional to subdomain surface area, combined with a synchronization frequency of at most once per volume computation, gives domain decomposition excellent parallel scalability on a per iteration basis, over a range of problem size and concurrency. In view of these important architectural advantages for domain decomposition methods, it is fortunate, indeed, that mathematicians studied the convergence behavior aspects of the subject in advance of the wide availability of these cost-effective architectures, and showed how to endow domain decomposition iterative methods with algorithmic scalability, as well.

Domain decomposition has proved to be an ideal paradigm not only for execution on advanced architecture computers, but also for the development of reusable, portable software. Since the most complex operation in a Schwarz-type domain decomposition iterative method - the application of the preconditioner - is logically equivalent in each subdomain to a conventional preconditioner applied to the global domain, software developed for the global problem can readily be adapted to the local problem, instantly presenting lots of "legacy" scientific code to be harvested for parallel implementations. Furthermore, since the majority of data sharing between subdomains in domain decomposition codes occurs in two archetypal communication operations - ghost point updates in overlapping zones between neighboring subdomains, and global reduction operations, as in forming an inner product - domain decomposition methods map readily onto optimized, standardized message-passing environments, such as MPI.

Finally, it should be noted that domain decomposition is often a natural paradigm for the modeling community. Physical systems are often decomposed into two or more contiguous subdomains based on phenomenological considerations, such as the importance or neglibility of viscosity or reactivity, or any other feature, and the subdomains are discretized accordingly, as independent tasks. This physically-based domain decomposition may be mirrored in the software engineering of the corresponding code, and leads to threads of execution that operate on contiguous subdomain blocks, which can either be further subdivided or aggregated to fit the granularity of an available parallel computer, and have the correct topological and mathematical characteristics for scalability.

Organizing the contents of an interdisciplinary proceedings is an interesting job, and our decisions will inevitably surprise a few authors, though we hope without causing offense. It is often difficult to assign a paper to just one of the categories of theory, algorithms, and applications. Readers are encouraged not to take the primary divisions very seriously, but to trace all the connections.

These proceedings will be of interest to mathematicians, computer scientists, and computational scientists, so we project its contents onto some relevant classification schemes below.

American Mathematical Society (AMS) 1991 subject classifications include:

Optimal control
Numerical simulation, modeling
Iterative methods for linear systems
Multigrid methods, domain decomposition for IVPs
Finite elements, Rayleigh-Ritz and Galerkin methods, finite elements
Spectral, collocation and related methods
Multigrid methods, domain decomposition for BVPs
Integral equations
Parallel computation
Mathematical software

Association for Computing Machinery (ACM) 1998 subject classifications include:

Programming environments, reusable libraries
Analysis and complexity of numerical algorithms
Numerical linear algebra, optimization, differential equations
Mathematical software, parallel implementations, portability
Applications in physical sciences and engineering

Applications for which domain decomposition methods have been specialized in this proceedings include:

Stokes, Euler, Navier-Stokes, multiphase flow, reacting flow
Porous media, atmospheric transport
Phase change, free surface phenomena
Semiconductor device physics
Linear and nonlinear elasticity
Acoustics, electromagnetics

The Neumann-Neumann method - a substructuring preconditioner typically employing

Additive Schwarz on the resulting interface problem - remains a topic of theoretical development and diverse applications [Giraud *et al.*, Alart *et al.*, Pavarino & Widlund], as odes the related Finite Element Tearing and Interconnection (FETI) method [Brenner, Dostal *et al.*]. Primal-dual formulations of FETI were heavily featured in the twelfth symposium in Chiba; primal-dual formulations emerge in further contexts in this proceedings [Klawonn & Widlund, Hoppe *et al.*].

Mortar methods, a nonoverlapping form of domain decomposition permitting flexibility in the form of nonmatching grids, were also a very active area in the Chiba symposium and continue to draw attention [Bjørstad *et al.*, Braess & Dahmen, Oswald & Wohlmuth, Shyy *et al.*, Tai *et al.*]. Another active area in nonoverlapping domain decomposition that is closely tied to the discretization is the optimal parametrization of Robin interface conditions [Bounaim, Gander, Gander *et al.*, Faille *et al.*, Dolean *et al.*, Rapin & Lube, Knopp *et al.*]. Related interface developments are presented under the rubric of optimal control and virtual control [Gervasio *et al.*, Pironneau *et al.*].

Overlapping domain decomposition methods continue to be refined, as well. This volume features two papers that shore up the highly effective Restricted Additive Schwarz (RAS) method. One [Cai *et al.*] shows how RAS, with its asymmetrical communication-saving restriction and extension operators can be rendered symmetric in an appropriate subspace and produces new theoretical bounds that mirror its observed superiority with respect to standard Additive Schwarz. The other [Frommer *et al.*] adopts a purely algebraic approach of oblique projections to produce the same ranking of additive Schwarz variants over the class of M-matrices, and also considers a restricted multiplicative Schwarz.

Two papers on the Aitken-Schwarz method introduced in Chiba [Baranger *et al.*, Garbey *et al.*] extend this overlapping technique, whose analysis depends upon Fourier decomposition of interface modes to nonlinear problems and less regular meshes. Meanwhile, nonlinear Additive Schwarz preconditioning [Cai *et al.*] has been applied to problems with shocks and has been shown to greatly improve the domain of convergence of Newton's method.

A novel purely algebraic method known as "multigraph", providing an algorithmic "spectrum" between exact Gaussian elimination and blocked iteration is presented in [Bank & Smith]. At an opposite extreme, waveform relaxation, a method that avoids forming discrete algebraic problems at common intermediate timesteps is advocated in [Daoud & Gander].

The implications for domain decomposition of several discretization techniques, apart from the customary conforming finite element and finite difference techniques on a single partitioned grid, are taken up by various authors. We mention especially fictitious domain methods [Feng & Karakashian, Lasser & Toselli], spectral methods [Azaiez *et al.*], and the increasingly theoretically supported discretization technique of finite volumes [Cautres *et al.*]. Apart from these methods rooted in differential equation formulations, there is a paper on domain decomposition for integral equation-based boundary element methods [Boubendir & Bendali].

These highlighted contributions only begin to call attention to technical points of interest in the current proceedings. We also note, sadly, a point of personal interest to all applied mathematicians, whether working in domain decomposition or not: this proceedings contains two of the last contributions of Jacques-Louis Lions.

For the convenience of readers coming recently into the subject of domain decomposition methods, a bibliography of previous proceedings is provided below, along with some major recent review articles and related special interest volumes. This list will inevitably be found embarrassingly incomplete. (No attempt has been made to supplement this list with the larger and closely related literature of multigrid and general iterative methods, except for the books by Hackbusch and Saad, which have significant domain decomposition components.)

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We also mention the homepage for domain decomposition on the World Wide Web, `www.ddm.org`, maintained by Professor Martin Gander of McGill University. This site features links to conference, bibliographic, and personal information pertaining to domain decomposition, internationally.

The technical direction of the Thirteenth International Conference on Domain Decomposition Methods in Scientific and Engineering Computing was provided by a scientific committee consisting of: Petter E. Bjørstad, Tony F. Chan, Peter J. Deuflhard, Roland Glowinski, Ronald Hoppe, Hideo Kawarada, David E. Keyes, Yuri A. Kuznetsov, Jacques Périaux, Olivier Pironneau, Alfio Quarteroni, Zhong-Ci Shi, Olof B. Widlund, and Jinchao Xu.

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CIMNE, UPC Spain, is publishing the proceedings of the International Conference on Domain Decomposition Methods for the second time. The editors are very grateful to Eugenio Oñate for his patience with our process.

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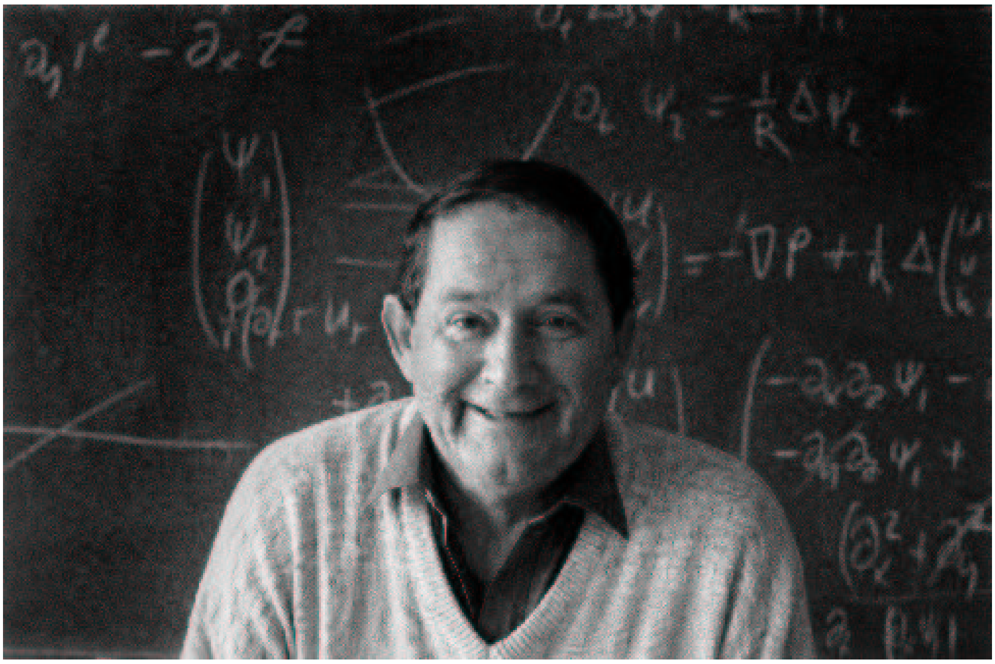

This conference has been dedicated to the memory of Wiktor Eckhaus who was a great applied Mathematician, and a good man. His contribution to the matching asymptotic theory in the 70's was in nature a domain decomposition approach to the construction of uniform asymptotic expansion for singular perturbed problems.