Preconditioning of Discontinuous Galerkin FEM of Second Order Elliptic Problems

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The popularity of DG approximations in recent years is largely due to the nice features like local mass conservation and flexible choice of the finite element spaces. However, to make the method attractive for the computational practice it is necessary to develop fast and efficient solution methods for the corresponding system and means of reducing the excessive number of degrees of freedom. The latter could be achieved by hybridization, while the former could use preconditioning based on multigrid/multilevel and/or domain decomposition. The talk summarises some recent results in preconditioning of algebraic systems arising in interior penalty DG approximations of second order elliptic problems. These were obtained jointly with my colleagues V. Dobrev, S. Margenov, P. Vassilevski, and L. Zikatanov.

We introduce the two-level iteration that uses the finite element space V of discontinuous functions and an auxiliary, in general smaller, space V_0. First we show the convergence of the two-level method for three particular choices of V_0, (1) continuous FE, (2) Crouzeix-Raviart nonconforming FE, (3) piece-wise constant functions, all defined on the original FE partition of the domain. The last space is quite interesting since it leads to a system with a "graph-Laplacian" matrix, which for general FE partitions does not have approximation property. Further we investigate two possibilities for an algebraic multilevel extension of the two-level method in the case of piece-wise constant functions, one based on the algebraic multigrid method of Falgout, Vassilevski, and Zikatanov, NLAA, 2005, and the second that utilizes the algebraic multilevel iteration of Axelsson and Vassilevski, SINUM, 1990. Under certain assumptions we prove optimal convergence for both multilevel methods.