Quasi-optimality of BDDC Methods for MITC Reissner-Mindlin Problems

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1 Introduction

The goal of this paper is to improve a condition number bound proven in [5] for a Balancing Domain Decomposition Method by Constraints (BDDC) for the Reissner-Mindlin plate bending problem discretized with MITC elements. This BDDC preconditioner is based on selecting the plate rotations and deflection degrees of freedom at the subdomain vertices as primal continuity constraints. In [5], we proved that the resulting BDDC algorithm is scalable in the number of subdomains N and independent of the plate thickness t and that the condition number κ of the preconditioned Reissner-Mindlin plate problem is bounded by

$$\kappa \leq C(H/h),$$

with C a constant independent of the plate thickness t, the mesh size h and the subdomain size H. In the present contribution, we prove the improved quasi-optimal result

$$\kappa \leq C(1+\log^3(H/h)).$$

We remark that the MITC discretization of Reissner-Mindlin problems can lead to very ill-conditioned discrete system, with condition number

$$\kappa_{no} \sim Ch^{-2}t^{-2}.$$

Introduced in [11] and analyzed in [17, 21, 22], BDDC methods have evolved from previous domain decomposition work on Balancing Neumann-Neumann methods. BDDC algorithm have been extended in recent years from scalar elliptic problems to almost incompressible elasticity [12, 24], the Stokes system [18], flow in porous

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media [28], and spectral element discretizations [15, 23, 24]. BDDC and overlapping Schwarz methods for Reissner-Mindlin plate problems discretized with Falk-Tu elements have been studied in the recent Ph.D. thesis [16], while multigrid method for plates have been studied in [26]. Among the several finite element works for plates, we mention [2, 3, 7–10, 13, 14, 19, 20, 27].

2 The MITC Reissner-Mindlin Plate Bending Problem

Continuous problem. Let Ω be a polygonal domain in $\mathbb{R}^2$ representing the midsurface of the plate, for simplicity assumed to be clamped on the whole boundary $\partial\Omega$. The Reissner-Mindlin plate bending problem (see [1, 7]) reads

$$\begin{cases} \text{Find } \boldsymbol{\theta}^{ex} \in [H_0^1(\Omega)]^2, u^{ex} \in H_0^1(\Omega) \text{ such that} \\ a(\boldsymbol{\theta}^{ex}, \boldsymbol{\eta}) + \mu k t^{-2}(\boldsymbol{\theta}^{ex} - \nabla u^{ex}, \boldsymbol{\eta} - \nabla v) = (f, v) \quad \forall \boldsymbol{\eta} \in [H_0^1(\Omega)]^2, v \in H_0^1(\Omega) \,, \end{cases} \tag{1}$$

with μ the shear modulus, k is the shear correction factor, t the plate thickness, u^{ex} the deflection, $\boldsymbol{\theta}^{ex}$ the rotation of the normal fibers and f the applied scaled normal load. Moreover, $(\cdot,\cdot)$ stands for the standard scalar product in $L^2(\Omega)$ and $a(\cdot,\cdot)$ is the bilinear form

$$a(\boldsymbol{\theta}^{ex}, \boldsymbol{\eta}) = (\mathbb{C}\varepsilon(\boldsymbol{\theta}^{ex}), \varepsilon(\boldsymbol{\eta})),$$

with $\mathbb{C}$ the positive definite tensor of bending moduli and $\varepsilon(\cdot)$ the symmetric gradient operator. Introducing the scaled shear stresses $\boldsymbol{\gamma}^{ex} = \mu k t^{-2}(\boldsymbol{\theta}^{ex} - \nabla u^{ex})$, problem (1) can be written in terms of the following mixed variational formulation, where for simplicity we have assumed $\mu k = 1$:

$$\begin{cases} \text{Find } \boldsymbol{\theta}^{ex} \in [H_0^1(\Omega)]^2, u^{ex} \in H_0^1(\Omega), \boldsymbol{\gamma}^{ex} \in [L^2(\Omega)]^2 \text{ such that} \\ a(\boldsymbol{\theta}^{ex}, \boldsymbol{\eta}) + (\boldsymbol{\gamma}^{ex}, \boldsymbol{\eta} - \nabla v) = (f, v) \quad \forall \boldsymbol{\eta} \in [H_0^1(\Omega)]^2, v \in H_0^1(\Omega) \\ (\boldsymbol{\theta}^{ex} - \nabla u^{ex}, \boldsymbol{s}) - t^2(\boldsymbol{\gamma}, \boldsymbol{s}) = 0 \quad \forall \boldsymbol{s} \in [L^2(\Omega)]^2 \,. \end{cases} \tag{2}$$

Discrete problem. We discretize the plate problem by MITC (Mixed Interpolation of Tensorial Components) elements; see e.g. [1, 7, 8] for more details on this family of elements. Let τ_h denote a triangular or quadrilateral conforming finite element mesh on Ω, of characteristic mesh size h. Let $\boldsymbol{\Theta}$, U and $\boldsymbol{\Gamma}$ be the discrete spaces for rotations, deflections and shear stresses, respectively and define $\mathbf{X} = \boldsymbol{\Theta} \times U$. Then the Reissner-Mindlin plate bending problem (2) discretized with MITC elements reads

$$\begin{cases} \text{Find } (\boldsymbol{\theta}, u) \in \mathbf{X}, \ \boldsymbol{\gamma} \in \boldsymbol{\Gamma} \text{ such that} \\ a(\boldsymbol{\theta}, \boldsymbol{\eta}) + (\boldsymbol{\gamma}, \Pi\, \boldsymbol{\eta} - \nabla v) = (f, v) \quad \forall (\boldsymbol{\eta}, v) \in \mathbf{X} \\ (\Pi\, \boldsymbol{\theta} - \nabla u, \boldsymbol{s}) - t^2(\boldsymbol{\gamma}, \boldsymbol{s}) = 0 \quad \forall \boldsymbol{s} \in \boldsymbol{\Gamma} \,, \end{cases} \tag{3}$$

where $\Pi : ([H^1(\Omega)]^2 + \boldsymbol{\Gamma}) \longrightarrow \boldsymbol{\Gamma}$ is the MITC reduction operator. Using the second equation of (3), shear stresses can be eliminated to obtain the following positive definite discrete formulation:

$$\begin{cases} \text{Find } (\boldsymbol{\theta}, u) \in \mathbf{X} \text{ such that} \\ b((\boldsymbol{\theta}, u), (\boldsymbol{\eta}, v)) = (f, v) \quad \forall (\boldsymbol{\eta}, v) \in \mathbf{X}\,, \end{cases} \tag{4}$$

where we have defined $b((\boldsymbol{\theta}, u), (\boldsymbol{\eta}, v)) := a(\boldsymbol{\theta}, \boldsymbol{\eta}) + t^{-2}(\Pi\, \boldsymbol{\theta} - \nabla u, \Pi\, \boldsymbol{\eta} - \nabla v)$. In this paper, we address directly the positive definite problem (4), in the spirit of [4, 5], instead of the mixed formulation (3). For the convergence analysis of the MITC elements, see e.g. [3, 8, 13, 25]. The MITC elements perform optimally with respect to the polynomial degree and regularity of the solution, and their rate of convergence is independent of the thickness parameter t.

3 Iterative Substructuring and BDDC Preconditioning

Subspace decomposition and Schur complement. We decompose the domain Ω into N open, nonoverlapping subdomains Ω_i of characteristic size H forming a shape-regular finite element mesh τ_H. This coarse triangulation τ_H is further refined into a finer triangulation τ_h of characteristic size h; both meshes will typically be composed of triangles or quadrilaterals. In the sequel, we assume that the material tensor $\mathbb{C}$ is constant on the whole domain.

As it is standard in iterative substructuring methods, we first reduce the problem to the interface $\Gamma = \left(\bigcup_{i=1}^{N} \partial\Omega_i\right) \setminus \partial\Omega$, by implicitly eliminating the interior degrees of freedom. In variational form, this process consists in a suitable decomposition of the discrete space $\mathbf{X} = \boldsymbol{\Theta} \times U$. More precisely, let us define $\boldsymbol{W} = \mathbf{X}_{|\Gamma}$, i.e. the space of the traces of functions in $\mathbf{X}$, as well as the local spaces $\mathbf{X}_i = \mathbf{X} \cap [H_0^1(\Omega_i)]^3$. The space $\mathbf{X}$ can be decomposed as $\mathbf{X} = \oplus_{i=1}^{N} \mathbf{X}_i \oplus \overline{\mathscr{H}}(\boldsymbol{W})$. Here $\overline{\mathscr{H}} : \boldsymbol{W} \longrightarrow \mathbf{X}$ is the discrete "plate-harmonic" extension operator defined by solving the problem

$$\begin{cases} \text{Find } \overline{\mathscr{H}}(\boldsymbol{w}_\Gamma) \in \mathbf{X} \;\text{ such that } \overline{\mathscr{H}}(\boldsymbol{w}_\Gamma)|_\Gamma = \boldsymbol{w}_\Gamma \text{ and} \\ b(\overline{\mathscr{H}}(\boldsymbol{w}_\Gamma), \boldsymbol{v}_I) = 0 \qquad \forall\, \boldsymbol{v}_I \in \mathbf{X}_i \quad i = 1, 2, \ldots, N. \end{cases}$$

Defining the Schur complement bilinear form $s(\boldsymbol{w}_\Gamma, \boldsymbol{v}_\Gamma) = b(\overline{\mathscr{H}}(\boldsymbol{w}_\Gamma), \overline{\mathscr{H}}(\boldsymbol{v}_\Gamma))$, the Schur complement system reads $s(\boldsymbol{u}_\Gamma, \boldsymbol{v}_\Gamma) = < \tilde{\boldsymbol{f}}, \boldsymbol{v}_\Gamma > \;\; \forall\, \boldsymbol{v}_\Gamma \in \boldsymbol{W}$, for a suitable right-hand side $\tilde{\boldsymbol{f}}$.

The BDDC Reissner-Mindlin plate preconditioner. BDDC preconditioners, introduced in [11] and analyzed in [21], can be regarded as an evolution of Balancing Neumann-Neumann preconditioners for the Schur complement system. In this section, we briefly recall the BDDC preconditioner of [5].

Define $\Gamma_i := \partial\Omega_i$, and $\Gamma_{ij} = \partial\Omega_i \cap \partial\Omega_j$, $i, j \in \{1, 2, \ldots, N\}$, the common edge between two adjacent subdomains Ω_i and Ω_j. The local spaces $\overline{\boldsymbol{W}}_i$ are the spaces of discrete functions defined by $\overline{\boldsymbol{W}}_i = \boldsymbol{W}_{|\Gamma_i}$, $i = 1, 2, \ldots, N$. Let $\overline{\mathscr{H}}_i : \overline{\boldsymbol{W}}_i \longrightarrow \mathbf{X}|_{\Omega_i}$, $i = 1, 2, \ldots, N$, represent the restriction of the operator $\overline{\mathscr{H}}$ to the subdomain Ω_i

$$\begin{cases} \text{Find } \overline{\mathscr{H}}_i(\boldsymbol{w}_i) \in \mathbf{X}|_{\Omega_i} \text{ such that } \overline{\mathscr{H}}(\boldsymbol{w}_i)|_{\Gamma_i} = \boldsymbol{w}_i \text{ and} \\ b_i(\overline{\mathscr{H}}_i(\boldsymbol{w}_i), \boldsymbol{v}_i) = 0 \qquad \forall\, \boldsymbol{v}_i \in \mathbf{X}_i, \end{cases}$$

where the $b_i(\cdot,\cdot)$ are given by restricting the integrals in $b(\cdot,\cdot)$ to the domain Ω_i, $i = 1,2,\dots,N$. The local bilinear forms are $s_i(\boldsymbol{w}_i,\boldsymbol{v}_i) = b_i(\overline{\mathscr{H}_i\boldsymbol{w}_i}, \overline{\mathscr{H}_i\boldsymbol{v}_i}), \forall \boldsymbol{w}_i,\boldsymbol{v}_i \in \overline{\boldsymbol{W}}_i$. Let R_i^T, $i = 1,2,\dots,N$ be the prolongation operators which extend any function of $\overline{\boldsymbol{W}}_i$ to the function of $\boldsymbol{W}$ which is zero at all the nodes not on Γ_i. Note that for $\boldsymbol{w},\boldsymbol{v} \in \boldsymbol{W}$, $\sum_{i=1}^N s_i(R_i\boldsymbol{w}, R_i\boldsymbol{v}) = s(\boldsymbol{w},\boldsymbol{v})$. For $x \in \Gamma$, we also define the weight $N_x = \#\{j \in \mathbb{N} \mid x \in \partial\Omega_j\}$ and the weighted counting operators $\delta_i : \overline{\boldsymbol{W}}_i \longrightarrow \overline{\boldsymbol{W}}_i$ (and their inverses $\delta_i^\dagger$) by

$$\delta_i \boldsymbol{v}_i\,(x) = N_x \boldsymbol{v}_i(x), \qquad \delta_i^\dagger \boldsymbol{v}_i\,(x) = N_x^{-1}\boldsymbol{v}_i(x), \quad \forall x \text{ node of } \Gamma_i \cap \Gamma.$$

Let $C_i : \overline{\boldsymbol{W}}_i \to \mathbb{R}^{3cc_i}$ be local constraint operators that read function values at the corners of the subdomain Ω_i, with cc_i the number of corners of the subdomain. Then we define the local constrained spaces

$$\boldsymbol{W}_i = \{\boldsymbol{w}_i \in \overline{\boldsymbol{W}}_i \mid C_i\boldsymbol{w}_i = \boldsymbol{0}\},$$

and a global coarse space $\boldsymbol{W}_0 \subset \boldsymbol{W}$ associated with the function values at the subdomain vertices. Given the number m of such subdomain vertices, let $w_c \in \mathbb{R}^{3m}$ be a vector representing the respective nodal values. Then the space $\boldsymbol{W}_0$ is defined by

$$\boldsymbol{W}_0 = \{\sum_{i=1}^N R_i^T \delta_i^\dagger \boldsymbol{w}_{0,i} \mid C_i\boldsymbol{w}_{0,i} = R_i^C w_c, w_c \in \mathbb{R}^{3m}, s_i(\boldsymbol{w}_{0,i},\boldsymbol{w}_{0,i}) \to \min\},$$

with R_i^C the operator extracting the vertex values for the subdomain Ω_i from the global vector w_c of all the subdomain vertex values. Any element $\boldsymbol{w} \in \boldsymbol{W}$ can be uniquely decomposed as $\boldsymbol{w} = \boldsymbol{w}_0 + \sum_{i=1}^N \boldsymbol{w}_i$, with $\boldsymbol{w}_0 \in \boldsymbol{W}_0$, $\boldsymbol{w}_i \in \boldsymbol{W}_i$ for $i = 1,\dots,N$. We use inexact bilinear forms defined by

$$\begin{aligned}\tilde{s}_i(\boldsymbol{w}_i,\boldsymbol{v}_i) &= s_i(\delta_i\boldsymbol{w}_i,\delta_i\boldsymbol{v}_i) \qquad \forall \boldsymbol{w}_i,\boldsymbol{v}_i \in \boldsymbol{W}_i,\ i = 1,2,\dots,N,\\ \tilde{s}_0(\boldsymbol{w}_0,\boldsymbol{v}_0) &= \sum_{i=1}^N s_i(\boldsymbol{w}_{0,i},\boldsymbol{v}_{0,i}) \qquad \forall \boldsymbol{w}_0,\boldsymbol{v}_0 \in \boldsymbol{W}_0.\end{aligned}$$

Finally, we define the coarse operator $P_0 : \boldsymbol{W} \longrightarrow \boldsymbol{W}_0$ by

$$\tilde{s}_0(P_0\boldsymbol{u},\boldsymbol{v}_0) = s(\boldsymbol{u},\boldsymbol{v}_0) \ \ \forall \boldsymbol{v}_0 \in \boldsymbol{W}_0,$$

and the local operators $P_i = R_i^T \tilde{P}_i : \boldsymbol{W} \longrightarrow R_i^T \boldsymbol{W}_i$ by

$$\tilde{s}_i(\tilde{P}_i\boldsymbol{u},\boldsymbol{v}_i) = s(\boldsymbol{u}, R_i^T\boldsymbol{v}_i) \ \ \forall \boldsymbol{v}_i \in \boldsymbol{W}_i.$$

Then, our BDDC method is defined by the preconditioned operator

$$P = \sum_{i=0}^N P_i\,. \tag{5}$$

The matrix form of P and the associated preconditioner can be found in [5].

4 A Quasi-optimal BDDC Convergence Bound

We start by recalling the following assumption from [5], using the same notations.

Assumption 1 *Given any Γ_i, $i=1,2,\ldots,N$, let $\mathscr{E}_i$ represent the set of the edges of Γ_i. Then, we assume that there exist two positive constants k_*, k^* and a boundary seminorm $|\cdot|_{\tau(\Gamma_i)}$ on $\overline{\boldsymbol{W}}_i$, $i=1,2,\ldots,N$, such that*

$$|\boldsymbol{w}_i|^2_{\tau(\Gamma_i)} \leq k^* s_i(\boldsymbol{w}_i,\boldsymbol{w}_i) \quad \forall \boldsymbol{w}_i \in \overline{\boldsymbol{W}}_i, \tag{6}$$

$$|\boldsymbol{w}_i|^2_{\tau(\Gamma_i)} \geq k_* s_i(\boldsymbol{w}_i,\boldsymbol{w}_i) \quad \forall \boldsymbol{w}_i \in \boldsymbol{W}_i, \tag{7}$$

$$|\boldsymbol{w}_i|^2_{\tau(\Gamma_i)} = \sum_{e\in\mathscr{E}_i} |\boldsymbol{w}_i|^2_{\tau(e)} \quad \forall \boldsymbol{w}_i \in \overline{\boldsymbol{W}}_i, \tag{8}$$

where $|\cdot|_{\tau(e)}$ is a given seminorm on the edge e.

We notice that we cannot adopt the obvious choice $|\boldsymbol{w}_i|_{\tau(\Gamma_i)} = s_i(\boldsymbol{w}_i,\boldsymbol{w}_i)$, since it can be shown that it does not satisfy (8), not even with a bound including a uniform constant. We have the following main result.

Theorem 2. *If Assumption 1 holds, then the condition number κ of the Reissner-Mindlin BDDC preconditioned operator P in (5) satisfies the bound*

$$\kappa(P) \leq C\left(1+\log^3(H/h)\right),$$

with the constant C depending only on the material constants and mesh regularity, and not on the plate thickness t.

Here we can only outline the main steps of the proof; full details can be found in [6]. The proof proceeds by showing that Assumption 1 holds for the MITC plate bending problem (4) and by establishing the respective upper and lower bounds for the constants k_*, k^* in (6), (7). These bounds in turn will prove Theorem 2 since $\kappa(P) \leq C(1+5k_*^{-1}k^*)$, see [5, 21] for a proof.
Upper bound (6). The upper bound is established exactly as in [5, Sect. 5.2]. **Lower bound** (7). To prove the lower bound, we note that the local spaces $\overline{\boldsymbol{W}}_i$, $i=1,2,\ldots,N$, are composed of rotation and deflection parts, which we denote by $\overline{\boldsymbol{W}}_i = \overline{\boldsymbol{\Theta}}_i \times \overline{U}_i$. Accordingly, we denote the rotation and deflection parts of the constrained space by $\boldsymbol{W}_i = \boldsymbol{\Theta}_i \times U_i$, where the functions of $\boldsymbol{\Theta}_i$ and U_i vanish at the subdomain corner nodes. We work with the following seminorm defined in [5]: $|\boldsymbol{w}_i|^2_{\tau(\Gamma_i)} = \sum_{e\in\mathscr{E}_i} |\boldsymbol{w}_i|^2_{\tau(e)} \quad \forall \boldsymbol{w}_i = (\boldsymbol{\theta}_i, u_i) \in \overline{\boldsymbol{W}}_i$, where for all edges $e \in \mathscr{E}_i$

$$|\boldsymbol{w}_i|^2_{\tau(e)} = |\boldsymbol{\theta}_i|^2_{\gamma(e)} + ht^{-2}||\Pi\, \boldsymbol{\theta}_i \cdot \boldsymbol{\tau} - u_i'||^2_{L^2(e)},$$

$$|\boldsymbol{\theta}_i|_{\gamma(e)} := \inf_{\boldsymbol{\psi}\in[H^1(\Omega_i)]^2,\, \boldsymbol{\psi}|_e = \boldsymbol{\theta}_i|_e} ||\boldsymbol{\varepsilon}(\boldsymbol{\psi})||_{L^2(\Omega_i)},$$

$\boldsymbol{\tau}$ is the tangent unit vector at the boundary and the apex indicates the derivative, in the direction of $\boldsymbol{\tau}$, for functions defined on the (one dimensional) boundary. We now improve the lower bound proved in [5] by introducing a splitting of the plate rotation variable. Consider $\boldsymbol{w}_i = (\boldsymbol{\theta}_i, u_i) \in \boldsymbol{W}_i$ and define the splitting $\boldsymbol{\theta}_i^{(2)} \in \boldsymbol{\Theta}_i^{(2)} := \mathrm{span}\{B_l^i \boldsymbol{\tau}\}_{l \in \Gamma_i}$, by

$$\int_e \boldsymbol{\theta}_i^{(2)} \cdot \boldsymbol{\tau} = \int_e \boldsymbol{\theta}_i \cdot \boldsymbol{\tau} - u_i' \quad \forall e \in \mathscr{E}_i,$$

and let $\boldsymbol{\theta}_i^{(1)} = \boldsymbol{\theta}_i - \boldsymbol{\theta}_i^{(2)}$ so that $\boldsymbol{\theta}_i = \boldsymbol{\theta}_i^{(1)} + \boldsymbol{\theta}_i^{(2)}$. By construction, it holds

$$\int_e u_i' - \boldsymbol{\theta}_i^{(1)} \cdot \boldsymbol{\tau} = 0 \quad \forall e \in \mathscr{E}_i.$$

We introduce also the related splitting of $\boldsymbol{w}_i$

$$\boldsymbol{w}_i = \boldsymbol{w}_i^{(1)} + \boldsymbol{w}_i^{(2)}, \qquad \boldsymbol{w}_i^{(1)} = (u_i, \boldsymbol{\theta}_i^{(1)}), \qquad \boldsymbol{w}_i^{(2)} = (0, \boldsymbol{\theta}_i^{(2)}).$$

An improved lower bound can be obtained by estimating the split terms in the following two lemmas; see [6] for complete proofs.

Lemma 1. *There exists a constant $C > 0$ independent of h such that for all edges e of all subdomains Ω_i*

$$|\boldsymbol{w}_i|_{\tau(e)} = |(u_i, \boldsymbol{\theta}_i)|_{\tau(e)} \geq C\big(|(u_i, \boldsymbol{\theta}_i^{(1)})|_{\tau(e)} + |(0, \boldsymbol{\theta}_i^{(2)})|_{\tau(e)}\big).$$

This lemma follows from the inequality $||(0, \boldsymbol{\theta}_i^{(2)})||_{\tau(e)} \leq C||\boldsymbol{w}_i||_{\tau(e)}$, that is derived in [6] from the definition of $\boldsymbol{\theta}_i^{(2)}$, a scaling argument and an inverse inequality. A similar argument applied to the extension of $\boldsymbol{\theta}_i^2$ by zero inside Ω_i leads to the following lemma.

Lemma 2. *There exists a constant $C > 0$ independent of h such that*

$$s_i(\boldsymbol{w}_i^{(2)}, \boldsymbol{w}_i^{(2)}) \leq C\, |\boldsymbol{w}_i^{(2)}|^2_{\tau(\Gamma_i)}.$$

The main step in the proof of Theorem 2 is the bound of the following proposition, obtained by considering an auxiliary rotated Stokes problem with boundary data $\boldsymbol{\theta}_i^{(1)}$ and several technical estimates, see [6, Proposition 5.5].

Proposition 1. *There exists a constant $C > 0$ independent of h such that*

$$s_i(\boldsymbol{w}_i^{(1)}, \boldsymbol{w}_i^{(1)}) \leq C\,(1 + \log^3(H/h))\, |\boldsymbol{w}_i^{(1)}|^2_{\tau(\Gamma_i)}.$$

The upper bound then follows by combining the three previous results. Indeed, first recalling the splitting $\boldsymbol{w}_i = \boldsymbol{w}_i^{(1)} + \boldsymbol{w}_i^{(2)}$ and using a triangle inequality, then applying Lemma 2 and Proposition 1, finally using Lemma 1 yields

$$s_i(\boldsymbol{w}_i, \boldsymbol{w}_i) \leq 2\Big(s_i(\boldsymbol{w}_i^{(1)}, \boldsymbol{w}_i^{(1)}) + s_i(\boldsymbol{w}_i^{(2)}, \boldsymbol{w}_i^{(2)})\Big)$$

$$\leq C\Big((1 + \log^3(H/h))\, |\boldsymbol{w}_i^{(1)}|^2_{\tau(\Gamma_i)} + |\boldsymbol{w}_i^{(2)}|^2_{\tau(\Gamma_i)}\Big) \leq C(1 + \log^3(H/h))|\boldsymbol{w}_i|^2_{\tau(\Gamma_i)}.$$

Bound (7) is therefore proved with $k_*^{-1} = C\,(1 + \log^3(H/h))$, with the constant C depending only on the material constants and mesh regularity.

We remark that an extensive set of numerical tests, also including jump in the coefficients, which are in complete accordance with Theorem 2, can be found in [5].

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