New Theoretical Coefficient Robustness Results for FETI-DP

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1 Introduction

In this short note, we present new weighted Poincaré inequalities (WPIs) with weighted averages that allow a robustness analysis of dual-primal finite element tearing and interconnecting (FETI-DP) methods in certain cases where jumps of coefficients are not aligned with the subdomain partition.

Let Ω be a bounded Lipschitz domain in $\mathbb{R}^2$ or $\mathbb{R}^3$. We consider the weak form of the scalar elliptic PDE

$$-\mathrm{div}(\alpha \nabla u) \;=\; f \qquad \text{in } \Omega, \tag{1}$$

with a uniformly positive diffusion coefficient $\alpha \in L^\infty(\Omega)$ that is piecewise constant with respect to a (possibly rather fine) partitioning of Ω. The discretization by continuous and piecewise linear finite elements (FEs) on a mesh $\mathscr{T}(\Omega)$ leads to the sparse (but in general large) linear system

$$\mathbf{K}\mathbf{u} \;=\; \mathbf{f}.$$

We consider FETI-DP solvers (see [2, 4, 5]) for the fast (and parallel) solution of this system, and we follow the structure described in [12, Sect. 6.4]. To this end, we partition the domain Ω into non-overlapping subdomains Ω_i, $i = 1, \ldots, N$ such that the global mesh $\mathscr{T}(\Omega)$ resolves the interface $\bigcup_{i \neq j} \partial\Omega_i \cap \partial\Omega_j$. The interface itself can be divided into subdomain vertices, edges, and faces (for $d = 3$), cf. [12, Sect. 4.2].

Without loss of generality, we assume that α is constant on each element of $\mathscr{T}(\Omega)$. Crucially, we do *not* assume that α is constant on each subdomain. However, we need assumptions on the *kind of jumps*. Let α_i denote the restriction of α to Ω_i and note that it has a well-defined trace in $L^2(\partial\Omega_i)$. For each subdomain edge (face)

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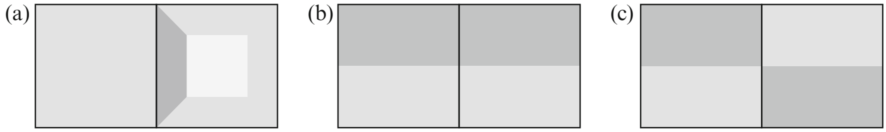

Fig. 1. Different types of coefficient jumps along an edge between two subdomains: (**a**) across (**b**) along (**c**) both across and along

$\mathscr{E}$ on Ω_i, let $V^h(\mathscr{E})$ denote the restriction of the global FE space to $\overline{\mathscr{E}}$ and let us define the weighted average

$$\overline{v}^{\mathscr{E},\alpha_i} := \frac{\int_{\mathscr{E}} \alpha_i v}{\int_{\mathscr{E}} \alpha_i} \qquad \text{for } v \in V^h(\mathscr{E}). \tag{2}$$

Assumption A1. Whenever two Ω_i and Ω_j share an edge (face) $\mathscr{E}$, the weighted averages of any function $v \in V^h(\mathscr{E})$ coincide: $\overline{v}^{\mathscr{E},\alpha_i} = \overline{v}^{\mathscr{E},\alpha_j}$.

A sufficient condition for Assumption A1 is that the coefficient jumps *either across or along*, but not both at the same time. For an illustration see Fig. 1. Our assumptions rules out situations of type (c).

Following [12, Algorithm B], we define the *primal space* $\widehat{W}_\Pi$ spanned by the vertex nodal basis functions at subdomain vertices, the subdomain edge cut-off functions and subdomain face cut-off functions (all of them extended discrete α-harmonically from the interface to the subdomain interiors). The *dual space* W_Δ contains FE functions that are discontinuous across the subdomain interfaces with vanishing α-weighted averages over the subdomain faces, edges, and vertices. We formally perform a change of basis, such that we have a splitting of the degrees of freedom (DOFs) into primal and dual ones, and work in the space $\widetilde{W} = \widehat{W}_\Pi \oplus W_\Delta$.

Let $B : \widetilde{W} \to U$ be the usual jump operator. The FETI-DP system

$$F\lambda \;=\; B\widehat{K}^{-1}\widehat{f} \tag{3}$$

is solved by preconditinioned conjugate gradients, where $F := B\widehat{K}^{-1}B^\top$ and where $\widehat{K}$, $\widehat{f}$ denote the stiffness matrix and load vector partially assembled at the primal DOFs, respectively. The overall solution is then given by

$$u \;=\; \widehat{K}^{-1}(\widehat{f} - B^\top\lambda).$$

Next, we define a FETI-DP preconditioner that is slightly modified to allow for certain coefficient jumps (cf. [3, 7]). Let $i = 1,\ldots,N$ be fixed and let $\mathscr{T}(\Omega_i)$ denote the mesh restricted to subdomain Ω_i. For each mesh node x^h on $\overline{\Omega}_i$, we set

$$\widehat{\alpha}_i(x^h) := \max_{T \in \mathscr{T}(\Omega_i): x^h \in \overline{T}} \alpha_{|T}\,. \tag{4}$$

Furthermore, if $\mathscr{N}_{x^h}$ denotes the index set of subdomains sharing the mesh node x^h, we define the weighted counting function

$$\delta_i^\dagger(x^h) := \begin{cases} \dfrac{\widehat{\alpha}_i(x^h)}{\sum_{j\in\mathcal{N}_{x^h}} \widehat{\alpha}_j(x^h)}, & \text{if } x^h \text{ lies on } \overline{\Omega}_i\,, \\ 0, & \text{otherwise.} \end{cases}$$

Using these counting functions we define the scaled jump operator B_D according to [12, Sect. 6.4.1] (for details see also [9] where the same scaled jump operator was used to define a one-level FETI preconditioner). The FETI-DP preconditioner is finally given by

$$M^{-1} := B_D S B_D^\top\,, \qquad (5)$$

where $S = \operatorname{diag}(S_i)_{i=1}^N$ is the block-diagonal Schur complement of the block stiffness matrix $K = \operatorname{diag}(K_i)_{i=1}^N$, eliminating the interior DOFs in each subdomain. Alternatively, one may replace B and B_D in (3), (5) by the respective operators which only act on the dual DOFs, which reduces the number of redundancies in λ.

2 Weighted Poincaré Inequalities with Weighted Averages

Let D be a bounded Lipschitz polytope and let $\{Y_\ell\}_{\ell=1}^n$ be a subdivision of D into open Lipschitz polytopes such that

$$\alpha_{|Y_\ell} = \alpha_\ell = \text{const.} \qquad (6)$$

Furthermore, let $\mathscr{X} \subset \partial D$ be a manifold of dimension $0 \le d_{\mathscr{X}} \le d-1$ (usually a vertex, an open subdomain edge or an open face, or a union of these). We define

$$\mathscr{X}_\ell := \overline{Y}_\ell \cap \mathscr{X}\,.$$

Some of these sets may be empty or have lower dimension than $\mathscr{X}$. However, with the index set $I_{\mathscr{X}} := \{\ell : \operatorname{meas}_{d_{\mathscr{X}}}(\mathscr{X}_\ell) > 0\}$ we can write

$$\overline{\mathscr{X}} = \bigcup_{k\in I_{\mathscr{X}}} \overline{\mathscr{X}}_k\,.$$

In general, for different indices k, $\ell \in I_{\mathscr{X}}$, the manifolds $\mathscr{X}_k$ and $\mathscr{X}_\ell$ may have a non-trivial intersection or even coincide. For simplicity, we assume that

$$k \neq \ell \in I_{\mathscr{X}} \quad \Longrightarrow \quad \operatorname{meas}_{d_{\mathscr{X}}}(\mathscr{X}_k \cap \mathscr{X}_\ell) = 0.$$

The general case needs more formalism and will be treated in an upcoming paper [10]. Finally, we can define a meaningful trace $\alpha_{\text{tr}} \in L^\infty(\mathscr{X})$ of α by

$$\alpha_{\text{tr}}(x) = \alpha_k \qquad \text{for } x \in \mathscr{X}_k\,.$$

Let $\{V^h(D)\}_h$ be a family of H^1-conforming FE spaces associated with a quasi-uniform family of triangulations of D. For $v \in V^h(D)$, we define the weighted (semi)norms and the weighted average on $\mathscr{X}$ by

$$\|v\|^2_{L^2(D),\alpha} := \int_D \alpha\, v^2\,, \quad |v|^2_{H^1(D),\alpha} := \int_D \alpha\, |\nabla v|^2 \quad \text{and} \quad \overline{v}^{\mathscr{X},\alpha_{\mathrm{tr}}} := \frac{\int_{\mathscr{X}} \alpha_{\mathrm{tr}}\, v}{\int_{\mathscr{X}} \alpha_{\mathrm{tr}}}\,.$$

We are interested in the following WPI with weighted average:

$$\|u - \overline{u}^{\mathscr{X},\alpha_{\mathrm{tr}}}\|^2_{L^2(D),\alpha} \;\leq\; C_{P,\alpha}(D,\mathscr{X};h)\,\mathrm{diam}(D)^2\, |u|^2_{H^1(D),\alpha} \qquad \forall u \in V^h(D). \qquad (7)$$

In particular, we are interested under which assumptions the parameter $C_{P,\alpha}(D,\mathscr{X};h)$ is independent of the values $\{\alpha_\ell\}$.

Sufficient conditions for robustness. We need two crucial assumptions for (7) to be independent of the values $\{\alpha_\ell\}$. The first assumption is a quasi-monotonicity assumption on α. It has been introduced in [1] and generalized in [4, 8]. The second assumption states that $\mathscr{X}$ "sees" the largest coefficient.

Definition 1. *Let $0 \leq m < d$ and let $\ell^* := \operatorname{argmax}_{1\leq \ell \leq s} \alpha_\ell$ denote the index of the largest coefficient.*[4]

(a) We call the region $P_{\ell_1,\ell_s} := (\overline{Y}_{\ell_1} \cup \ldots \cup \overline{Y}_{\ell_s})^\circ$, $1 \leq \ell_1, \ldots, \ell_s \leq n$ a type-m quasi-monotone path *from Y_{ℓ_1} to Y_{ℓ_s} (with respect to α), if*
  (i) the regions Y_{ℓ_i} and $Y_{\ell_{i+1}}$ share a common m-dimensional manifold, and
  (ii) $\alpha_{\ell_1} \leq \alpha_{\ell_2} \leq \ldots \leq \alpha_{\ell_s}$.

(b) We say that α is type-m quasi-monotone on D, *if for all $k = 1,\ldots,n$ there exists a quasi-monotone type-m path from Y_k to Y_{ℓ^*}.*

Assumption A2. α is type-m quasi-monotone on D for some $0 \leq m < d$.

Assumption A3. $\mathrm{meas}_{d_{\mathscr{X}}}(\mathscr{X} \cap \overline{Y}_{\ell^*}) > 0$.

In order to formulate our main theorem, we first need some definitions of generalized Poincaré constants/parameters.

Definition 2. *(i) For any bounded Lipschitz domain $Y \subset \mathbb{R}^d$ let $C_P(Y)$ be the smallest constant such that*

$$\|v - \overline{v}^Y\|^2_{L^2(Y)} \;\leq\; C_P(Y)\,\mathrm{diam}(Y)^2\, |v|^2_{H^1(Y)} \qquad \forall v \in H^1(Y).$$

(ii) Let Z be the finite union of bounded Lipschitz polytopes such that $\overline{Z}$ is connected, and let $\{\mathscr{T}^h(Z)\}_h$ be a quasi-uniform family of triangulations of Z with the associated continuous piecewise linear FE spaces $\{V^h(Z)\}_h$. Let X, $W \subset \overline{Z}$ be manifolds/subdomains of (possibly different) dimension $\in \{0,\ldots,d\}$. Let $C_P(Z,X,W;h)$ be the best parameter such that

$$\|v - \overline{v}^X\|^2_{L^2(W)} \;\leq\; C_P(Z,X,W;h)\,\frac{|W|}{|Z|}\,\mathrm{diam}(Z)^2\, |u|^2_{H^1(Z)} \qquad \forall v \in V^h(Z).$$

$|W|$ and $|Z|$ denote the measures of W and Z (in the respective dimension).

[4] We can assume without loss of generality that ℓ^* is unique. By definition, type-m quasi-monotonicity implies that otherwise all maximal subregions can be combined into a single subregion.

If Z is connected and if the dimensions of X and W are $\geq d-1$, we can define a constant $C_P(Z,X,W)$ independent of the discretization parameter h such that the inequality in Definition 2(ii) holds for all functions in $H^1(Z)$.

Theorem 1. *Let Assumptions A2 and A3 be satisfied. Then the parameter* $C_{P,\alpha}(D,\mathscr{X};h)$ *in formula* (7) *is independent of the values* $\{\alpha_\ell\}_{\ell=1}^n$ *and*

$$C_{P,\alpha}(D,\mathscr{X};h) \;\leq\; 2\left[C^{*,1}(h)+C^{*,2}(h)\right] \tag{8}$$

with

$$C^{*,1}(h) := \sum_{\ell=1}^{n} \frac{|Y_\ell|\,\mathrm{diam}(P_{\ell,\ell^*})^2}{|P_{\ell,\ell^*}|\,\mathrm{diam}(D)^2}\, C_P(P_{\ell,\ell^*},\mathscr{X}_{\ell^*},Y_\ell;h),$$

$$C^{*,2}(h) := \frac{|D|}{|\mathscr{X}_{\ell^*}|}\sum_{k\in I_{\mathscr{X}}} \frac{|\mathscr{X}_k|\,\mathrm{diam}(P_{k,\ell^*})^2}{|P_{k,\ell^*}|\,\mathrm{diam}(D)^2}\, C_P(P_{k,\ell^*},\mathscr{X}_{\ell^*},\mathscr{X}_k;h).$$

Proof. Without loss of generality, we may assume that $\overline{u}^{\mathscr{X},\alpha_{\mathrm{tr}}}=0$. For each index $\ell=1,\dots,n$,

$$\tfrac{1}{2}\,\|u\|^2_{L^2(Y_\ell)} \;\leq\; \|u-\overline{u}^{\mathscr{X}_{\ell^*}}\|^2_{L^2(Y_\ell)} + |Y_\ell|\,\big(\overline{u}^{\mathscr{X}_{\ell^*}}\big)^2 .$$

Due to Assumption A2, there is a quasi-monotone path from Y_ℓ to Y_{ℓ^*}. With $c_{\ell,\ell^*} := C_P(P_{\ell,\ell^*},\mathscr{X}_{\ell^*},Y_\ell;h)$, summation over $\ell=1,\dots,n$ yields

$$\tfrac{1}{2}\,\|u\|^2_{L^2(D),\alpha} \;\leq\; \sum_{\ell=1}^{n} c_{\ell,\ell^*}\frac{|Y_\ell|}{|P_{\ell,\ell^*}|}\,\mathrm{diam}(P_{\ell,\ell^*})^2\, \underbrace{\alpha_\ell|u|^2_{H^1(P_{\ell,\ell^*})}}_{\leq\, |u|^2_{H^1(D),\alpha}} \;+\; \underbrace{\sum_{\ell=1}^{n}\alpha_\ell\,|Y_\ell|}_{\leq\,\alpha_{\ell^*}|D|}\,\big(\overline{u}^{\mathscr{X}_{\ell^*}}\big)^2,$$

where we have used Definition 2(ii) and the quasi-monotonicity of P_{ℓ,ℓ^*}. The first sum is bounded by $C^{*,1}(h)\,\mathrm{diam}(D)^2\,|u|^2_{H^1(D),\alpha}$. To bound the remaining term, we use Cauchy's inequality and the definition of α_{tr}:

$$\alpha_{\ell^*}|D|\,\big(\overline{u}^{\mathscr{X}_{\ell^*}}\big)^2 \;\leq\; \frac{|D|}{|\mathscr{X}_{\ell^*}|}\,\alpha_{\ell^*}\,\|u\|^2_{L^2(\mathscr{X}_{\ell^*})} \;\leq\; \frac{|D|}{|\mathscr{X}_{\ell^*}|}\,\|u\|^2_{L^2(\mathscr{X}),\alpha_{\mathrm{tr}}}.$$

A variational argument yields

$$\|u\|^2_{L^2(\mathscr{X}),\alpha_{\mathrm{tr}}} \;\leq\; \|u-\underbrace{\overline{u}^{\mathscr{X},\alpha_{\mathrm{tr}}}}_{=0}\|^2_{L^2(\mathscr{X}),\alpha_{\mathrm{tr}}} \;=\; \inf_{c\in\mathbb{R}}\|u-c\|^2_{L^2(\mathscr{X}),\alpha_{\mathrm{tr}}}$$

$$\leq\; \|u-\overline{u}^{\mathscr{X}_{\ell^*}}\|^2_{L^2(\mathscr{X}),\alpha_{\mathrm{tr}}} \;=\; \sum_{k\in I_{\mathscr{X}}}\alpha_k\,\|u-\overline{u}^{\mathscr{X}_{\ell^*}}\|^2_{L^2(\mathscr{X}_k)}.$$

Now, we have

$$\alpha_k\,\|u-\overline{u}^{\mathscr{X}_{\ell^*}}\|^2_{L^2(\mathscr{X}_k)} \;\leq\; C_P(P_{k,\ell^*},\mathscr{X}_{\ell^*},\mathscr{X}_k;h)\,\frac{|\mathscr{X}_k|}{|P_{k,\ell^*}|}\,\mathrm{diam}(P_{k,\ell^*})^2\,\alpha_k\,|u|^2_{H^1(P_{k,\ell^*})}.$$

Using the quasi-monotonicity of α on P_{k,ℓ^*} finally leads to (8).

Necessity of the conditions. As discussed in [8, Sect. 3.1], Assumption A2 is necessary to ensure that $C_{P,\alpha}(D,\mathscr{X};h)$ is independent of the values $\{\alpha_\ell\}$.

To see that A3 is necessary as well, assume that $\text{meas}_{d_{\mathscr{X}}}(\mathscr{X}\cap\overline{Y}_{\ell^*})=0$. We choose a function u which is one on Y_{ℓ^*}. Since the average functional $v\mapsto\overline{v}^{\mathscr{X},\alpha_{\text{tr}}}$ is independent of α_{ℓ^*}, we can prescribe values of u on $\mathscr{X}$ such that $\overline{u}^{\mathscr{X},\alpha_{\text{tr}}}=0$ and continuously extend u into $D\subset\overline{Y}_{\ell^*}$. The whole construction of u is independent of α_{ℓ^*}, Since $\nabla u=0$ on Y_{ℓ^*}, the seminorm $|u|_{H^1(D),\alpha}$ is independent of α_{ℓ^*} as well. However, $\|u\|^2_{L^2(D),\alpha}\geq\alpha_{\ell^*}|Y_{\ell^*}|$. Therefore, if $\alpha\leq\alpha_k$ on $D\backslash Y_{\ell^*}$, then $C_{P,\alpha}(D,\mathscr{X};h)=\mathscr{O}\big(\frac{\alpha_{\ell^*}}{\alpha_k}\big)$ for $\alpha_{\ell^*}/\alpha_k\to\infty$. This means that Assumptions A2 and A3 in some sense *characterize* the robustness of the WPI with weighted average.

3 Robustness Proof of FETI-DP

To analyze the robustness of FETI-DP, we need the following assumption.

Assumption A4. For each subdomain Ω_i and for each subdomain edge (face) $\mathscr{E}$ of Ω_i, there is a Lipschitz domain $D_{i,\mathscr{E}}\subset\Omega_i$, such that $\mathscr{E}\subset\partial D_{i,\mathscr{E}}$ and Assumptions A2 and A3 are satisfied for $D=D_{i,\mathscr{E}}$ and $\mathscr{X}=\mathscr{E}$. The union of all the regions $D_{i,\mathscr{E}}$ covers a boundary layer Ω_{i,η_i} of width $\eta_i\geq h$ of Ω_i (see e.g. [6, Definition 2.6]).

Theorem 2. *Let Assumptions A1 and A4 hold. Then the condition number $\kappa(M^{-1}F)$ for the FETI-DP method is independent of the values of the coefficient α, in particular of any non-resolved jumps.*

Due to space limitations we only give a sketch of the proof. A detailed proof will be given in [10], together with a more detailed statement of Theorem 2 that makes precise the dependence of $\kappa(M^{-1}F)$ on geometric parameters, such as the ratios $\text{diam}(\Omega_i)/h$ and $\text{diam}(\Omega_i)/\eta_i$.

Let $\mathscr{H}_i$ denote the discrete α-harmonic extension from $\partial\Omega_i$ to Ω_i and let

$$|w|_S^2:=\sum_{i=1}^N|\mathscr{H}_iw|^2_{H^1(\Omega_i),\alpha}\,.$$

Then, following [12, Sect. 6.4.3], a bound of the kind

$$|P_Dw|_S^2\;\leq\;\omega\,|w|_S^2\qquad\forall w\in\widetilde{W},\tag{9}$$

where $P_D:=B_D^\top B$, implies that $\kappa(M^{-1}F)\leq\omega$.

As in the proof of [9, Lemma 5.6; formula (5.24)], we can introduce a set of cut-off functions associated with each subdomain edge (face) $\mathscr{E}$ whose support is contained in $D_{i,\mathscr{E}}$. It then follows that, for any $w\in\widehat{W}_\Pi\oplus W_\Delta$,

$$|P_Dw|_S^2\;\leq\;C\sum_{i=1}^N\Big[|\mathscr{H}_iw_i|^2_{H^1(\Omega_i),\alpha}+\sum_{\mathscr{E}}\frac{1}{\text{diam}(\Omega_i)^2}\,\|\mathscr{H}_iw_i-\overline{w_i}^{\mathscr{E}}\|^2_{L^2(D_{i,\mathscr{E}}),\alpha}\Big],$$

where C depends on $\text{diam}(\Omega_i)/h$ and $\text{diam}(\Omega_i)/\eta_i$, but it is independent of the values $\{\alpha_\ell\}$. By Theorem 1, we can bound each of the weighted L^2 norms by the weighted H^1 seminorm of $\mathscr{H}_iw_i$, and thus obtain (9).

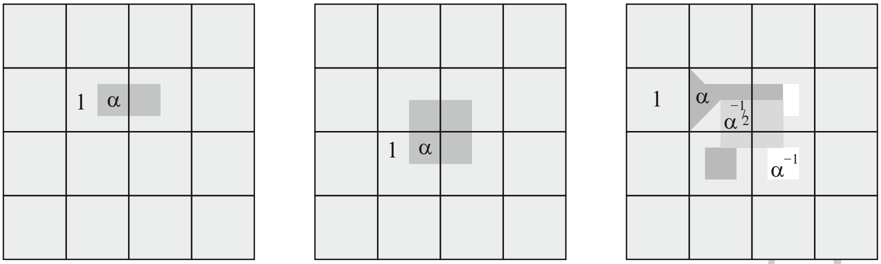

Fig. 2. Edge-island (*left*), cross-point island (*middle*), complicated coefficient (*right*)

α	condition	#iterations
1	1.58	10
10^1	1.57	10
10^3	1.56	10
10^5	1.56	10
10^7	1.56	10
10^{-1}	1.70	10
10^{-3}	1.74	10
10^{-5}	1.74	10
10^{-7}	1.74	11

α	condition	#iterations
1	1.58	10
10^1	1.59	10
10^3	1.59	10
10^5	1.59	10
10^7	1.59	10
10^{-1}	1.57	10
10^{-3}	1.57	10
10^{-5}	1.57	10
10^{-7}	1.57	10

α	condition	#iterations
1	1.58	10
10^1	1.61	11
10^2	1.62	11
10^3	1.62	11
10^4	1.62	11
10^{-1}	1.62	11
10^{-2}	1.60	11
10^{-3}	1.59	11
10^{-4}	1.59	11

Table 1. Edge-island (left), crosspoint-island (middle), complicated coefficient (right), $H/h = 32$.

4 Numerical Results

We provide results for the three examples shown in Fig. 2. Note that in the last example, the coefficient is not quasi-monotone on one of the subdomains, but satisfies Assumptions A1 and A4. In our implementation we used PARDISO [11]. The estimated condition numbers and the number of PCG iterations are displayed in Table 1. They clearly confirm Theorem 2.

5 Conclusion

We analyse a FETI-DP method for the scalar elliptic PDE (1) with possible jumps in the diffusion coefficient alpha. We show that provided weighted edge/face averages are used, the condition number of the preconditioned system is independent of coefficient jumps. The essential assumptions are A1 and A4, i.e., the coefficient does not jump both across and along any interfaces between two subdomains and the coefficient is quasi-monotone in the vicinity of any edge/face within each subdomain. The key theoretical tool that is of interest in itself is a novel weighted Poincaré inequality for functions with suitably chosen vanishing weighted face/edge averages. We are able to show that under Assumption A4, the Poincare constant of each neighborhood $D_{i,\mathscr{E}}$ can be bounded independent of jumps.

As in our previous work [8], the Poincaré constants (and thus also the condition number) will also depend on the "geometry" of the coefficient variation. In particular, for piecewise constant coefficients it will in general depend on the geometry of the subregions where the coefficient is constant. We did not give details of this dependence here, but this will be done in an upcoming paper [10] (using [8]). Cases where the coefficient jumps both along and across subdomain interfaces appear to be substantially harder to be treated and are also the subject of our future investigations.

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