A FETI-DP algorithm for incompressible Stokes equations with continuous pressures

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1 Introduction

The FETI-DP algorithm was first extended to solving incompressible Stokes equations by Li [3], where a Dirichlet preconditioner was considered and the subdomain average pressure degrees of freedoms were selected as a primal constraint, in addition to the coarse level primal velocity constraints. The resulting coarse problem is a saddle point problem. The condition number bound is independent of the number of subdomains and grows only polylogarithmically with the size of the individual subdomain problems.

Recently, Kim, Lee, and Park [2] introduced a different FETI-DP formulation for this problem, where no pressure variables are selected as coarse level primal variables and the resulting coarse level problem is symmetric positive definite. Only the lumped preconditioner is considered in their paper.

Both approaches mentioned above are valid only for discretizations with a discontinuous pressure. Discontinuous pressures have also been used in domain decomposition algorithms for similar type saddle-point problems; see for example [1, 5, 7].

In this paper, we propose a FETI-DP algorithm for incompressible Stokes using either a lumped or a Dirichlet preconditioner with continuous pressure discretization; see also [4, 8] for more details. Our coarse problem includes no pressure variables and is symmetric positive definite.

2 Discretization and domain decomposition

The weak solution of the incompressible Stokes problem, on a bounded, two-dimensional polygonal domain Ω with a zero Dirichlet boundary condition, is given by: find $\mathbf{u}^* \in \left(H_0^1(\Omega)\right)^2 = \{\mathbf{v} \in (H^1(\Omega))^2 \mid \mathbf{v} = \mathbf{0} \text{ on } \partial\Omega\}$ and $p^* \in L^2(\Omega)$, such that

$$\begin{cases} a(\mathbf{u}^*, \mathbf{v}) + b(\mathbf{v}, p^*) = (\mathbf{f}, \mathbf{v}), \ \forall \mathbf{v} \in \left(H_0^1(\Omega)\right)^2, \\ b(\mathbf{u}^*, q) \qquad\qquad = 0, \qquad \forall q \in L^2(\Omega), \end{cases} \tag{1}$$

where

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$$a(\mathbf{u}^*,\mathbf{v}) = \int_\Omega \nabla\mathbf{u}^*\cdot\nabla\mathbf{v}, \quad b(\mathbf{u}^*,q) = -\int_\Omega (\nabla\cdot\mathbf{u}^*)q, \quad (\mathbf{f},\mathbf{v}) = \int_\Omega \mathbf{f}\cdot\mathbf{v}.$$

We note that the solution of (1) is not unique, with the pressure p^* determined only up to an additive constant.

A Q_2-Q_1 Taylor-Hood mixed finite element is used in this paper to solve (1). The domain Ω is partitioned into shape-regular rectangular elements of characteristic size h. The pressure finite element space, $Q \subset L^2(\Omega)$, is taken as the space of continuous piecewise bilinear functions while the velocity finite element space, $\mathbf{W} \in \left(H_0^1(\Omega)\right)^2$, is formed by the continuous piecewise biquadratic functions.

The finite element solution $(\mathbf{u}, p) \in \mathbf{W}\bigoplus Q$ of (1) satisfies

$$\begin{bmatrix} A & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ p \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix}, \tag{2}$$

where A, B, and $\mathbf{f}$ represent, respectively, the restrictions of $a(\cdot,\cdot)$, $b(\cdot,\cdot)$ and $(\mathbf{f},\cdot)$ to the finite-dimensional spaces $\mathbf{W}$ and Q. We use the same notation in this paper to represent both a finite element function and the vector of its nodal values. The solution of (2) always exists and is uniquely determined when the pressure is considered in the quotient space $Q/Ker(B^T)$, where $Ker(B^T)$ represents the kernel of B^T and is the space of constant pressures in Q. In this paper, when $q \in Q/Ker(B^T)$, q always has a zero average.

The Taylor-Hood mixed finite element space $\mathbf{W}\times Q$ is inf-sup stable in the sense that there exists a positive constant β, independent of h, such that, in matrix/vector form,

$$\sup_{\mathbf{w}\in\mathbf{W}} \frac{\langle q, B\mathbf{w}\rangle^2}{\langle \mathbf{w}, A\mathbf{w}\rangle} \geq \beta^2 \langle q, Zq\rangle, \qquad \forall q \in Q/Ker(B^T). \tag{3}$$

Here, as elsewhere in this paper, $\langle\cdot,\cdot\rangle$ represents the inner product of two vectors. The matrix Z represents the mass matrix defined on the pressure finite element space Q, i.e., for any $q \in Q$, $\|q\|_{L^2}^2 = \langle q, Zq\rangle$. It is easy to see, cf. [6, Lemma B.31], that Z is spectrally equivalent to $h^2 I$ for two-dimensional problems, where I represents the identity matrix of the same dimension, i.e., there exist positive constants c and C, such that

$$ch^2 I \leq Z \leq Ch^2 I. \tag{4}$$

Here, as in other places of this paper, c and C represent generic positive constants which are independent of the mesh size h and the subdomain diameter H (discussed below).

The domain Ω is decomposed into N non-overlapping polygonal subdomains Ω_i, $i = 1,2,...,N$. Each subdomain is the union of a bounded number of elements, with the diameter of the subdomain on the order of H. The nodes on the interface of neighboring subdomains match across the subdomain boundaries $\Gamma = (\cup\partial\Omega_i)\backslash\partial\Omega$. Γ is composed of subdomain edges, which are regarded as open subsets of Γ, and of the subdomain vertices, which are end points of edges.

The velocity and pressure finite element spaces $\mathbf{W}$ and Q are decomposed into $\mathbf{W} = \mathbf{W}_I \bigoplus \mathbf{W}_\Gamma$, $Q = Q_I \bigoplus Q_\Gamma$, where $\mathbf{W}_I$ and Q_I are direct sums of independent

subdomain interior velocity spaces $\mathbf{W}_I^{(i)}$, and interior pressure spaces $Q_I^{(i)}$, respectively. $\mathbf{W}_\Gamma$ and Q_Γ are subdomain boundary velocity and pressure spaces, respectively. All functions in $\mathbf{W}_\Gamma$ and Q_Γ are continuous across the subdomain boundaries Γ; their degrees of freedom are shared by neighboring subdomains.

To formulate our algorithm, we introduce a partially sub-assembled subdomain boundary velocity space $\widetilde{\mathbf{W}}_\Gamma$,

$$\widetilde{\mathbf{W}}_\Gamma = \mathbf{W}_\Pi \bigoplus \mathbf{W}_\Delta = \mathbf{W}_\Pi \bigoplus \left(\bigoplus_{i=1}^{N} \mathbf{W}_\Delta^{(i)} \right).$$

Here $\mathbf{W}_\Pi$ is the continuous primal velocity space which forms the coarse level problem of the proposed algorithm. In this paper, we consider two choices of $\mathbf{W}_\Pi$. The first choice is with that $\mathbf{W}_\Pi$ is spanned by all the subdomain corner velocity nodal basis functions. In the second choice, $\mathbf{W}_\Pi$ is spanned by both subdomain corner velocity nodal basis functions and edge-average finite element basis functions. We note that the appropriate choice of $\mathbf{W}_\Pi$ depends on the preconditioner used in the algorithm. The first choice is sufficient for using the lumped preconditioner, but for the Dirichlet preconditioner the second one has to be used.

The space $\mathbf{W}_\Delta$ is the direct sum of subdomain dual interface velocity spaces $\mathbf{W}_\Delta^{(i)}$. The functions $\mathbf{w}_\Delta$ in $\mathbf{W}_\Delta$ are in general not continuous across Γ. In order to enforce their continuity, we construct a matrix B_Δ from $\{0, 1, -1\}$ such that for any $\mathbf{w}_\Delta$ in $\mathbf{W}_\Delta$, each row of $B_\Delta \mathbf{w}_\Delta = 0$ implies that the two independent degrees of freedom from the neighboring subdomains be the same. The range of B_Δ applied on $\mathbf{W}_\Delta$ is denoted by Λ, the vector space of the Lagrange multipliers. A positive scaling factor $\delta^\dagger(x)$ for each node x on the subdomain boundary Γ is defined as $\delta^\dagger(x) = 1/\mathcal{N}_x$, where $\mathcal{N}_x$ represents the number of subdomains sharing x. Multiplying the entries on each row of B_Δ by the corresponding scaling factor $\delta^\dagger(x)$ gives us $B_{\Delta,D}$.

The original linear system (2) is equivalent to: find $(\mathbf{u}_I,\ p_I,\ \mathbf{u}_\Delta,\ \mathbf{u}_\Pi,\ p_\Gamma,\ \lambda) \in \mathbf{W}_I \bigoplus Q_I \bigoplus \mathbf{W}_\Delta \bigoplus \mathbf{W}_\Pi \bigoplus Q_\Gamma \bigoplus \Lambda$, such that

$$\begin{bmatrix} A_{II} & B_{II}^T & A_{I\Delta} & A_{I\Pi} & B_{\Gamma I}^T & 0 \\ B_{II} & 0 & B_{I\Delta} & B_{I\Pi} & 0 & 0 \\ A_{\Delta I} & B_{I\Delta}^T & A_{\Delta\Delta} & A_{\Delta\Pi} & B_{\Gamma\Delta}^T & B_\Delta^T \\ A_{\Pi I} & B_{I\Pi}^T & A_{\Pi\Delta} & A_{\Pi\Pi} & B_{\Gamma\Pi}^T & 0 \\ B_{\Gamma I} & 0 & B_{\Gamma\Delta} & B_{\Gamma\Pi} & 0 & 0 \\ 0 & 0 & B_\Delta & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u}_I \\ p_I \\ \mathbf{u}_\Delta \\ \mathbf{u}_\Pi \\ p_\Gamma \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{f}_I \\ 0 \\ \mathbf{f}_\Delta \\ \mathbf{f}_\Pi \\ 0 \\ 0 \end{bmatrix}, \tag{5}$$

where the sub-blocks in the coefficient matrix represent the restrictions of A and B of (2) to appropriate subspaces. The leading three-by-three block can be ordered to become block diagonal with each diagonal block representing one independent subdomain problem.

Corresponding to the one-dimensional null space of (2), a basis of the one-dimensional null space of (5) has the form

$$\left(0,\, 1_{p_I},\, 0,\, 0,\, 1_{p_\Gamma},\, -B_{\Delta,D}[B_{I\Delta}^T \;\; B_{\Gamma\Delta}^T] \begin{bmatrix} 1_{p_I} \\ 1_{p_\Gamma} \end{bmatrix} \right), \tag{6}$$

where $1_{p_I} \in Q_I$ and $1_{p_\Gamma} \in Q_\Gamma$ represent vectors with each entry equal to 1.

3 A reduced symmetric positive semi-definite system

The system (5) can be reduced to a Schur complement problem for the variables $(p_\Gamma,\, \lambda)$. The leading four-by-four block of the coefficient matrix in (5) is invertible and the variables $(\mathbf{u}_I,\, p_I,\, \mathbf{u}_\Delta,\, \mathbf{u}_\Pi)$ can be eliminated and we obtain

$$G \begin{bmatrix} p_\Gamma \\ \lambda \end{bmatrix} \;=\; g, \tag{7}$$

where

$$G = \begin{bmatrix} B_{\Gamma I} & 0 & B_{\Gamma\Delta} & B_{\Gamma\Pi} \\ 0 & 0 & B_\Delta & 0 \end{bmatrix} \begin{bmatrix} A_{II} & B_{II}^T & A_{I\Delta} & A_{I\Pi} \\ B_{II} & 0 & B_{I\Delta} & B_{I\Pi} \\ A_{\Delta I} & B_{I\Delta}^T & A_{\Delta\Delta} & A_{\Delta\Pi} \\ A_{\Pi I} & B_{I\Pi}^T & A_{\Pi\Delta} & A_{\Pi\Pi} \end{bmatrix}^{-1} \begin{bmatrix} B_{\Gamma I}^T & 0 \\ 0 & 0 \\ B_{\Gamma\Delta}^T & B_\Delta^T \\ B_{\Gamma\Pi}^T & 0 \end{bmatrix}, \tag{8}$$

and

$$g = \begin{bmatrix} B_{\Gamma I} & 0 & B_{\Gamma\Delta} & B_{\Gamma\Pi} \\ 0 & 0 & B_\Delta & 0 \end{bmatrix} \begin{bmatrix} A_{II} & B_{II}^T & A_{I\Delta} & A_{I\Pi} \\ B_{II} & 0 & B_{I\Delta} & B_{I\Pi} \\ A_{\Delta I} & B_{I\Delta}^T & A_{\Delta\Delta} & A_{\Delta\Pi} \\ A_{\Pi I} & B_{I\Pi}^T & A_{\Pi\Delta} & A_{\Pi\Pi} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{f}_I \\ 0 \\ \mathbf{f}_\Delta \\ \mathbf{f}_\Pi \end{bmatrix}. \tag{9}$$

We denote

$$\widetilde{A} = \begin{bmatrix} A_{II} & B_{II}^T & A_{I\Delta} & A_{I\Pi} \\ B_{II} & 0 & B_{I\Delta} & B_{I\Pi} \\ A_{\Delta I} & B_{I\Delta}^T & A_{\Delta\Delta} & A_{\Delta\Pi} \\ A_{\Pi I} & B_{I\Pi}^T & A_{\Pi\Delta} & A_{\Pi\Pi} \end{bmatrix} \quad \text{and} \quad B_C = \begin{bmatrix} B_{\Gamma I} & 0 & B_{\Gamma\Delta} & B_{\Gamma\Pi} \\ 0 & 0 & B_\Delta & 0 \end{bmatrix}. \tag{10}$$

It is easy to see that $-G$ is the Schur complement of the coefficient matrix of (5) with respect to the last two row blocks. By the Sylvester law of inertia, G is symmetric positive semi-definite. The null space of G can be derived from the null space of the original coefficient matrix of (5), and its basis has the form, cf. (6),

$$\left(1_{p_\Gamma},\, -B_{\Delta,D}[B_{I\Delta}^T \;\; B_{\Gamma\Delta}^T] \begin{bmatrix} 1_{p_I} \\ 1_{p_\Gamma} \end{bmatrix} \right).$$

Let $X = Q_\Gamma \bigoplus \Lambda$. The range of G, denoted by R_G, is the subspace of X, which is orthogonal to the null space of G and has the form

$$R_G = \left\{ \begin{bmatrix} g_{p_\Gamma} \\ g_\lambda \end{bmatrix} \in X \;\Big|\; g_{p_\Gamma}^T 1_{p_\Gamma} - g_\lambda^T \left(B_{\Delta,D} [B_{I\Delta}^T \;\; B_{\Gamma\Delta}^T] \begin{bmatrix} 1_{p_I} \\ 1_{p_\Gamma} \end{bmatrix} \right) = 0 \right\}. \tag{11}$$

The restriction of G to its range R_G is positive definite.

The main operation in the implementation of multiplying G by a vector is the product of $\widetilde{A}^{-1}$ with a vector, cf. (8) and (9). We denote

$$A_{rr} = \begin{bmatrix} A_{II} & B_{II}^T & A_{I\Delta} \\ B_{II} & 0 & B_{I\Delta} \\ A_{\Delta I} & B_{I\Delta}^T & A_{\Delta\Delta} \end{bmatrix}, \;\; A_{\Pi r} = A_{r\Pi}^T = \begin{bmatrix} A_{\Pi I} & B_{I\Pi}^T & A_{\Pi\Delta} \end{bmatrix}, \;\; f_r = \begin{bmatrix} \mathbf{f}_I \\ 0 \\ \mathbf{f}_\Delta \end{bmatrix},$$

and define the Schur complement

$$S_\Pi = A_{\Pi\Pi} - A_{\Pi r} A_{rr}^{-1} A_{r\Pi}.$$

By the Sylvester law of inertia, S_Π is symmetric positive definite and defines the coarse level problem in the algorithm. The product

$$\begin{bmatrix} A_{II} & B_{II}^T & A_{I\Delta} & A_{I\Pi} \\ B_{II} & 0 & B_{I\Delta} & B_{I\Pi} \\ A_{\Delta I} & B_{I\Delta}^T & A_{\Delta\Delta} & A_{\Delta\Pi} \\ A_{\Pi I} & B_{I\Pi}^T & A_{\Pi\Delta} & A_{\Pi\Pi} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{f}_I \\ 0 \\ \mathbf{f}_\Delta \\ \mathbf{f}_\Pi \end{bmatrix}$$

can then be represented by

$$\begin{bmatrix} A_{rr}^{-1} f_r \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} -A_{rr}^{-1} A_{r\Pi} \\ I_\Pi \end{bmatrix} S_\Pi^{-1} \left(\mathbf{f}_\Pi - A_{\Pi r} A_{rr}^{-1} f_r \right),$$

which requires solving the coarse level problem once and independent subdomain Stokes problems with Neumann type boundary conditions twice.

4 Preconditioners and condition number bounds

Both the lumped and the Dirichlet preconditioners are proposed here for solving (7). We define

$$\widetilde{V} = \mathbf{W}_I \bigoplus Q_I \bigoplus \mathbf{W}_\Delta \bigoplus \mathbf{W}_\Pi,$$

and its subspace

$$\widetilde{V}_0 = \left\{ w = (\mathbf{w}_I,\; p_I,\; \mathbf{w}_\Delta,\; \mathbf{w}_\Pi) \in \widetilde{V} : B_{II}\mathbf{w}_I + B_{I\Delta}\mathbf{w}_\Delta + B_{I\Pi}\mathbf{w}_\Pi = 0 \right\}.$$

We note that $\langle \cdot,\cdot \rangle_{\widetilde{A}}$ defines an inner product on $\widetilde{V}_0$. We denote the restriction operator from $\widetilde{V}$ onto $\mathbf{W}_\Delta$ by $\widetilde{R}_\Delta$ such that for any $v = (\mathbf{w}_I,\ p_I,\ \mathbf{w}_\Delta,\ \mathbf{w}_\Pi) \in \widetilde{V}$, $\widetilde{R}_\Delta v = \mathbf{w}_\Delta$.

The lumped preconditioner is given by

$$M_L^{-1} = \begin{bmatrix} \frac{1}{h^2} I_{p_\Gamma} & \\ & M_{L,\lambda}^{-1} \end{bmatrix},$$

where I_{p_Γ} is the identity matrix of the same length as p_Γ and $M_{L,\lambda}^{-1} = B_{\Delta,D}\widetilde{R}_\Delta \widetilde{A} \widetilde{R}_\Delta^T B_{\Delta,D}^T$.

Let $M_{D,\lambda}^{-1} = B_{\Delta,D} H_\Delta B_{\Delta,D}^T$. Then the Dirichlet preconditioner is defined as

$$M_D^{-1} = \begin{bmatrix} \frac{1}{h^2} I_{p_\Gamma} & \\ & M_{D,\lambda}^{-1} \end{bmatrix},$$

where H_Δ is the direct sum of the discrete subdomain harmonic extension operators.

The following lemma is used for obtaining the upper bound estimate in Theorem 1, and it is valid for both preconditioners, denoted here by M^{-1}.

Lemma 1. *For any $v \in \widetilde{V}_0$, $\left\langle M^{-1} B_C v, B_C v \right\rangle \leq \Phi(H,h) \left\langle \widetilde{A} v, v \right\rangle$. Here, for the lumped preconditioner, $\Phi(H,h) = C(H/h)(1+\log(H/h))$ with only corner variables in the coarse space; $\Phi(H/h) = C(H/h)$ with both corner and edge-average variables. For the Dirichlet preconditioner, $\Phi(H,h) = C(1+\log(H/h))^2$ with both corner and edge-average coarse variables.*

The second lemma is used for the lower bound estimate. For the lumped preconditioner, the corner primal constraints are sufficient for the coarse space to prove this lemma. However, for the Dirichlet preconditioner, both corner and edge-average constraints have to be included in the coarse space.

Lemma 2. *For any given $y = (g_{p_\Gamma}, g_\lambda) \in R_G$, there exits $v \in \widetilde{V}_0$, such that $B_C v = y$, and $\left\langle \widetilde{A} v, v \right\rangle \leq \frac{C}{\beta^2} \left\langle M^{-1} y, y \right\rangle$.*

Theorem 1. *For all $x = (p_\Gamma, \lambda) \in R_{M^{-1}G}$,*

$$c\beta^2 \langle Mx, x \rangle \leq \langle Gx, x \rangle \leq \Phi(H,h) \langle Mx, x \rangle,$$

where $\Phi(H,h)$ is as defined in Lemma 1 and β is the inf-sup constant of (3).

5 Numerical experiments

We solve the incompressible Stokes problem in the square domain $\Omega = [0,1] \times [0,1]$. Zero Dirichlet boundary conditions are used. The right-hand side function $\mathbf{f}$ is chosen such that the exact solution is

Table 1 Performance with the lumped preconditioner M_L^{-1}.

		Vertex			Vertex and edge		
H/h	#sub	λ_{min}	λ_{max}	iter	λ_{min}	λ_{max}	iter
8	4×4	0.31	32.28	31	0.31	4.30	19
	8×8	0.31	37.25	46	0.31	4.50	20
	16×16	0.31	38.40	51	0.31	4.53	21
	24×24	0.31	38.62	51	0.31	4.55	21
	32×32	0.31	38.68	51	0.31	4.55	21
#sub	H/h	λ_{min}	λ_{max}	iter	λ_{min}	λ_{max}	iter
8×8	4	0.30	15.92	34	0.30	3.21	18
	8	0.31	37.25	46	0.30	4.50	20
	12	0.31	60.62	56	0.31	6.65	24
	16	0.31	85.32	62	0.31	8.87	27
	24	0.31	137.49	73	0.31	13.40	32

$$\mathbf{u} = \begin{bmatrix} \sin^3(\pi x)\sin^2(\pi y)\cos(\pi y) \\ -\sin^2(\pi x)\sin^3(\pi y)\cos(\pi x) \end{bmatrix} \quad \text{and} \quad p = x^2 - y^2.$$

The Q_2-Q_1 Taylor-Hood mixed finite element is used for the finite element solution. The preconditioned system is solved by a CG iteration; the iteration is stopped when the L^2-norm of the residual is reduced by a factor of 10^{-6}.

Table 1 shows the minimum and maximum eigenvalues of the iteration matrix $M_L^{-1}G$, and the iteration counts. Two different coarse level spaces are tested in the experiments: the coarse space spanned by only the subdomain corner velocities, and the coarse space spanned by both the subdomain corner and the subdomain edge-average velocities. The additional edge-average velocity components in the coarse level problem improve the convergence rate even though they are not necessary for the analysis.

Table 2 shows the performance of our algorithm for solving the same problem with the Dirichlet preconditioner. For this case, the additional edge-average velocity components included in the coarse level space are necessary, which is consistent with our theory.

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Table 2 Performance with the Dirichlet preconditioner M_D^{-1}.

		Vertex			Vertex and edge		
H/h	#sub	λ_{min}	λ_{max}	iter	λ_{min}	λ_{max}	iter
8	4×4	0.30	4.40	18	0.30	3.04	17
	8×8	0.29	5.03	24	0.30	3.50	18
	16×16	0.26	5.28	25	0.30	3.92	19
	24×24	0.24	5.33	25	0.30	4.10	19
	32×32	0.23	5.36	25	0.30	4.18	19
#sub	H/h	λ_{min}	λ_{max}	iter	λ_{min}	λ_{max}	iter
8×8	4	0.27	4.15	21	0.30	3.15	17
	8	0.29	5.03	24	0.30	3.50	18
	12	0.29	5.60	25	0.30	3.92	18
	16	0.30	6.04	25	0.30	4.24	18
	24	0.30	6.70	26	0.30	4.71	19

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