LECTURE NOTES IN COMPUTATIONAL SCIENCE AND ENGINEERING

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Domain Decomposition Methods in Science and Engineering XXVIII

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Preface to the Proceedings of DD28

The proceedings of the 28th International Conference on Domain Decomposition Methods (DD28) constitute a record of advances in the analysis, algorithmic development, large-scale implementation, and application of domain decomposition methods in science and engineering. As applications addressed by participants have expanded beyond those modeled by partial differential equations posed on a domain, the conference subtitle was generalized to "Divide, Conquer and Combine Methods in Large-scale Simulation and Analytics." The conference was hosted by the King Abdullah University of Science and Technology in Thuwal, Saudi Arabia, from January 28 to February 1, 2024, and was organized jointly by the Scientific Committee of ddm.org, and a local organizing committee working with the KAUST Office of Research Funding & Services. The archival website is `https://dd28.kaust.edu.sa`. The 28th conference drew citizens from 38 nationalities from Europe, Asia and Pacific, the Middle East, and North and South America for a total of 193 participants.

1.1 Background of the Conference Series

The International Conference on Domain Decomposition Methods has been held in seventeen countries throughout Asia, Europe, the Middle East, and North America, beginning in Paris in 1987. Held annually for the first fourteen meetings, it has been spaced since DD15 at roughly 1.5-year intervals. The twenty-eighth conference was the first held in Saudi Arabia and the second in the Middle East, after DD18 in Jerusalem in 2008.

Over the four decades of the conference, demands of resolution and fidelity have driven the performance of simulations of first-principles mathematical models in science and engineering from MegaFlop/s to ExaFlop/s and their datasets from MegaBytes to ExaBytes – 12 orders of magnitude for each, or roughly a factor of one thousand per decade. Over the most recent decade, the application of machine learning to science and engineering systems, which came to the fore at DD28, has created a similar exascale demand for performance and associated storage. Lower levels of the software stack created for simulation have proved immediately useful for machine learning at scale. However, higher levels of the simulation software have not yet fulfilled their potential to lift the dominant algorithms for machine learning and inference today above relatively "brute force" implementations, resulting in massive costs for facilities and energy that slow progress and restrict access to the research frontier for many. At the same time, hardware optimized for machine learning applications possesses yet unrealized potential for traditional simulation. Increasingly, the same science or engineering application is more effectively addressable by simulation and learning working in tandem than by either alone, resulting in a confluence of two formerly distinct research communities.

With the number of floating point-capable cores in the highest-end scientific computing systems now exceeding ten million (up from mere hundreds when this conference series began), advances in the convergence of highly concurrent algorithms for distributed memory scaling have never been more essential. Today's most powerful computer systems cost up to a billion dollars to acquire and many tens of millions of dollars in annual operating costs for power and cooling. In such an environment, where saving a factor of two in execution time represents the annual cost of dozens of mathematicians, application- and architecture-adaptivity in algorithms are important pursuits. Many algorithms long ago achieved optimal log-linear parallel complexity asymptotically in discrete problem size. Initial successes with H-grad were extended to problems posed in H-div and H-curl spaces. Further ingenuity can undoubtedly improve their properties with respect to nonsymmetry, anisotropy, inhomogeneity, indefiniteness, and cross-coupling in multiphysics systems simultaneously possessing a mixture of behaviors.

Traditional "forward problems" have been extended to realms of computational science of greater impact: optimization, inversion, data assimilation, and uncertainty quantification. In these contexts, the "forward problem" of the early domain decomposition meetings is an inner loop, and thus must be highly performant and rapidly convergent.

The primary technical content of the DD conference series has always been mathematical, while the principal motivation is to make efficient use of distributed memory computers for complex applications arising in science and engineering. Thus, contributions from mathematicians, computer scientists, engineers, and scientists have always been sought. While domain decomposition methods are nowadays very important for the efficient simulation of large scale applications on massively parallel computers, there are also many interesting applications of domain decomposition that are not massively parallel. For example, efficiently connecting just two subproblems to effectively exploit a different solver on each remains a core area of research, and the same holds for coupling different phenomena, like fluid structure interaction. As multiprocessing is commonplace, multiphysics modeling is in ascendancy, so developments in domain decomposition remain as relevant and are as interdisciplinary as ever. While research in domain decomposition methods is presented at numerous venues, the International Conference on Domain Decomposition

Methods, whose history is below, is the only regularly occurring international forum dedicated to interdisciplinary technical interactions between theoreticians and practitioners working in the creation, analysis, software implementation, and application of domain decomposition methods and the absorption of new insights from other fields that enhance its power and component techniques.

1. Paris, France, January 7–9, 1987
2. Los Angeles, USA, January 14–16, 1988
3. Houston, USA, March 20–22, 1989
4. Moscow, USSR, May 21–25, 1990
5. Norfolk, USA, May 6–8, 1991
6. Como, Italy, June 15–19, 1992
7. University Park, Pennsylvania, USA, October 27–30, 1993
8. Beijing, China, May 16–19, 1995
9. Ullensvang, Norway, June 3–8, 1996
10. Boulder, USA, August 10–14, 1997
11. Greenwich, UK, July 20–24, 1998
12. Chiba, Japan, October 25–29, 1999
13. Lyon, France, October 9–12, 2000
14. Cocoyoc, Mexico, January 6–11, 2002
15. Berlin, Germany, July 21–15, 2003
16. New York, USA, January 12–15, 2005
17. St. Wolfgang-Strobl, July 3–7, Austria 2006
18. Jerusalem, Israel, January 12–17, 2008
19. Zhangjiajie, China, August 17–22, 2009
20. San Diego, California, February 7–11, 2011
21. Rennes, France, June 25–29, 2012
22. Lugano, Switzerland, September 16–20, 2013
23. Jeju Island, Korea, July 5–10, 2015
24. Svalbard, Norway, February 6–10, 2017
25. St. John's, Canada, July 23–27, 2018
26. Hong Kong, China, December 7–12, 2020 (virtual)
27. Prague, Czech Republic, July 25–29, 2022

The proceedings of the conference series are online at the ddm.org website and are published as follows:

1. R. Bank, M. Holst, O. Widlund and J. Xu, eds., *Proc. Twentieth Int. Conf. on Domain Decomposition Methods* (San Diego, 2011), Springer, Heidelberg, 2012.
2. M. Bercovier, M. J. Gander, R. Kornhuber and O. Widlund, eds., *Proc. Eighteenth Int. Conf. on Domain Decomposition Methods* (Jerusalem, 2008), Springer, Heidelberg, 2009.
3. P. Bjørstad, S. C. Brenner, L. Halpern, H. H. Kim, R. Kornhuber, T. Rahman and O. B Widlund, eds., *Proc. Twenty-fourth Int. Conf. on Domain Decomposition Methods* (Svalbard, 2017), Springer, Heidelberg, 2018.

4. P. Bjørstad, M. Espedal and D. E. Keyes, eds., *Proc. Ninth Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Ullensvang, 1997), Wiley, New York, 1999.
5. S. C. Brenner, E. T. S. Chung, A. Klawonn, F. Kwok, J. Xu, J. Zou, eds., *Proc. Twenty-fifth Int. Conf. on Domain Decomposition Methods* (Hong Kong, 2020), Springer, Heidelberg, 2023.
6. T. F. Chan, R. Glowinski, J. Périaux and O. B. Widlund, eds., *Proc. Second Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Los Angeles, 1988), SIAM, Philadelphia, 1989.
7. T. F. Chan, R. Glowinski, J. Périaux, O. B. Widlund, eds., *Proc. Third Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Houston, 1989), SIAM, Philadelphia, 1990.
8. T. Chan, T. Kako, H. Kawarada and O. Pironneau, eds., *Proc. Twelfth Int. Conf. on Domain Decomposition Methods in Science and Engineering* (Chiba, 1999), DDM.org, Bergen, 2001.
9. N. Débit, M. Garbey, R. Hoppe, D. Keyes, Yu. A. Kuznetsov and J. Périaux, eds., *Proc. Thirteenth Int. Conf. on Domain Decomposition Methods in Science and Engineering* (Lyon, 2000), CINME, Barcelona, 2002.
10. T. Dickhopf, M. J. Gander, L. Halpern, R. Krause and L.F. Pavarino, eds., *Proc. Twenty-second Int. Conf. on Domain Decomposition Methods* (Lugano, 2013), Springer, Heidelberg, 2016.
11. S. Dostál, Kozubek, A. Klawonn, U. Langer, L. F. Pavarino, J. Šístek, and O. B. Widlund, eds., *Proc. Twenty-seventh Int. Conf. on Domain Decomposition Methods* (Prague, 2022), Springer, Heidelberg, 2024.
12. J. Erhel, M. J. Gander, L. Halpern, G. Pichot, T. Sassi and O. Widlund, eds., *Proc. Twenty-first Int. Conf. on Domain Decomposition Methods* (Rennes, 2012), Springer, Heidelberg, 2013.
13. R. Glowinski, G. H. Golub, G. A. Meurant and J. Périaux, eds., *Proc. First Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Paris, 1987), SIAM, Philadelphia, 1988.
14. R. Glowinski, Yu. A. Kuznetsov, G. A. Meurant, J. Périaux and O. B. Widlund, eds., *Proc. Fourth Int. Symp. on Domain Decomposition Methods for Partial Differential Equations* (Moscow, 1990), SIAM, Philadel- phia, 1991.
15. R. Glowinski, J. Périaux, Z.-C. Shi and O. B. Widlund, eds., *Proc. Eighth International Conference of Domain Decomposition Methods* (Beijing, 1995), Wiley, Strasbourg, 1997.
16. R. Haynes, S. MacLachlan, X.-C. Cai, L. Halpern, H. H. Kim, A. Klawonn and O. B Widlund, eds., *Proc. Twenty-fifth Int. Conf. on Domain Decomposition Methods* (Svalbard, 2018), Springer, Heidelberg, 2020.
17. I. Herrera, D. Keyes, O. Widlund and R. Yates, eds., *Proc. Fourteenth Int. Conf. on Domain Decomposition Methods in Science and Engineering* (Cocoyoc, 2003), National Autonomous University of Mexico (UNAM), Mexico City, 2003.

18. Y. Huang, R. Kornhuber, O. Widlund and J. Xu, eds., *Proc. Nineteenth Int. Conf. on Domain Decomposition Methods* (Zhangjiajie, 2009), Springer, Heidelberg, 2010.
19. D. E. Keyes, T. F. Chan, G. A. Meurant, J. S. Scroggs and R. G. Voigt, eds., *Proc. Fifth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations* (Norfolk, 1991), SIAM, Philadelphia, 1992.
20. D. E. Keyes and J. Xu, eds., *Proc. Seventh Int. Conf. on Domain Decomposition Methods for Partial Differential Equations* (University Park, 1993), AMS, Providence, 1995.
21. R. Kornhuber, R. Hoppe, J. Périaux, O. Pironneau, O. Widlund and J. Xu, eds., . *Fifteenth Int. Conf. on Domain Decomposition Methods* (Berlin, 2003), Springer, Heidelberg, 2004.
22. C.-H. Lai, P. Bjørstad, M. Cross and O. Widlund, eds., *Proc. Eleventh Int. Conf. on Domain Decomposition Methods* (Greenwich, 1999), DDM.org, Bergen, 2000.
23. U. Langer, M. Discacciati, D. Keyes, O. Widlund and W. Zulehner, eds., *Proc. Seventeenth Int. Conf. on Domain Decomposition Methods* (Strobl, 2006), Springer, Heidelberg, 2007.
24. C.-O. Lee, X.-C. Cai, D. E. Keyes, H. H. Kim, A Klawonn, E.-J. Park and O. B. Widlund, eds., *Proc. Twenth-third Int. Conf. on Domain Decomposition Methods* (Jeju, 2015), Springer, Heidelberg, 2017.
25. J. Mandel, C. Farhat, and X.-C. Cai, eds, *Proc. Tenth Int. Conf. on Domain Decomposition Methods in Science and Engineering* (Boulder, 1998), AMS, Providence, 1999.
26. A. Quarteroni, J. Périaux, Yu. A. Kuznetsov and O. B. Widlund, eds., *Proc. Sixth Int. Conf. on Domain Decomposition Methods in Science and Engineering* (Como, 1992), AMS, Providence, 1994.
27. O. Widlund and D. E. Keyes, eds., *Proc. Sixteenth Int. Conf. on Domain Decomposition Methods* (New York, 2005), Springer, Heidelberg, 2006.

Many books and monographs have been published on domain decomposition in addition to the proceedings of the ddm.org conference series. With no pretense of comprehensiveness, various references beyond the proceedings follow:

1. E. Bueler, *PETSc for Partial Differential Equations*, SIAM, Philadelphia, 2020.
2. T. F. Chan and T. P. Mathew, *Domain Decomposition Algorithms*, Acta Numerica, 1994, pp. 61–143.
3. V. Dolean, P. Jolivet and F. Nataf, *An Introduction to Domain Decomposition Methods: Algorithms, Theory, and Parallel Implementation*, SIAM, Philadelphia, 2015.
4. C. Farhat and F.-X. Roux, *Implicit Parallel Processing in Structural Mechanics*, Computational Mechanics Advances 2, 1994, pp. 1–124.
5. W. Hackbusch, *Iterative Methods for Large Sparse Linear Systems*, Springer, Heidelberg, 1993.

6. D. E. Keyes, Y. Saad and D. G. Truhlar, eds., *Domain-based Parallelism and Problem Decomposition Methods in Science and Engineering*, SIAM, Philadelphia, 1995.
7. P. Le Tallec, *Domain Decomposition Methods in Computational Mechanics*, Computational Mechanics Advances 2, 1994, pp. 121–220.
8. T. Mathew, *Domain Decomposition Methods for the Numerical Solution of Partial Differential Equations*, Volume 61 of Lecture Notes in Computational Science & Engineering, Springer, Heidelberg, 2008.
9. L. Pavarino and A. Toselli, *Recent Developments in Domain Decomposition Methods*, Volume 23 of Lecture Notes in Computational Science & Engineering, Springer, Heidelberg, 2002.
10. C. Pechstein, *Finite and Boundary Element Tearing and Interconnecting Solvers for Multiscale Problems*, Volume 90 of Lecture Notes in Computational Science & Engineering, Springer, Heidelberg, 2013.
11. A. Quarteroni and A. Valli, *Domain Decomposition Methods for Partial Differential Equations*, Oxford, 1999.
12. P. J. Roache, *Elliptic Marching Methods and Domain Decomposition*, CRC Press, UK, 1995.
13. Y. Saad, *Iterative Methods for Sparse Linear Systems*, PWS, Boston, 1996.
14. B. F. Smith, P. E. Bjørstad and W. D. Gropp, *Domain Decomposition: Parallel Multilevel Algorithms for Elliptic Partial Differential Equations*, Cambridge Univ. Press, Cambridge, 1996.
15. A. Toselli and O. Widlund, *Domain Decomposition Methods: Algorithms and Theory*, Springer, New York, 2004.
16. O. Steinbach, *Stability Estimates for Hybrid Coupled Domain Decomposition Methods*, Lecture Notes in Mathematics, Springer, New York, 2002.
17. B. I. Wolhmuth, *Discretization Methods and Iterative Solvers Based on Domain Decomposition*, Volume 17 of Lecture Notes in Computational Science & Engineering, Springer, Heidelberg, 2001.
18. J. Xu, *Iterative Methods by Space Decomposition and Subspace Correction*, SIAM Review 34, 1991, pp. 581-613.

1.2 About the Twenty-eighth Conference

The 28th conference was organized over five days – three-and-a-half days of conference plus a well-attended short course on the preceding day – and featured 132 presentations, of four types:

- 11 invited plenary talks, selected by the International Scientific Committee from about three times this number of nominees;
- 96 talks invited by minisymposia organizers, arranged around a common topic, and grouped into 12 mini-symposia;
- 16 contributed talks, grouped into 4 sessions;
- 9 posters, grouped around a reception.

The conference employed four parallel sessions.

The Scientific Committee members overseeing the 28th conference were:

1. Petter Bjørstad, University of Bergen, Norway
2. Susanne Brenner, Louisiana State University, USA
3. Xiao-Chuan Cai, University of Macau, China
4. Victorita Dolean, TU-Eindhoven, Netherlands
5. Martin Gander, University of Geneva, Switzerland
6. Laurence Halpern, University of Paris 13, France (Chair)
7. David Keyes, KAUST, Saudi Arabia
8. Hyea Hyun Kim, Kyung Hee University, Korea
9. Axel Klawonn, University of Cologne, Germany
10. Ralf Kornhuber, Freie Universität Berlin, Germany
11. Ulrich Langer, University of Linz, Austria
12. Luca Pavarino, University of Pavia, Italy
13. Olof Widlund, Courant Institute, New York University, USA
14. Jinchao Xu, KAUST, Saudi Arabia

The local organizing committee included four members who have chaired previous conferences in the ddm.org series:

1. Prof. Tony Chan, KAUST President, Honorary Chair (DD2, Los Angeles)
2. Prof. David Keyes, KAUST, Co-chair (DD16, New York)
3. Prof. Jinchao Xu, KAUST, Co-chair (DD7, State College)
4. Prof. Daniele Boffi, KAUST
5. Prof. Rolf Krause, KAUST (DD22, Lugano)
6. Prof. Gabriel Wittum, KAUST
7. Dr. Stefano Zampini, KAUST
8. Dr. Abdulaziz Baiz, Saudi Aramco
9. Dr. Aniello Esposito, HPE-Cray

DD28 had two primary sponsors: KAUST's Office of Research Funding & Services and Saudi Aramco's Exploration and Petroleum Engineering Center's Advanced Research Center (EXPEC-ARC).

Social events included a welcoming reception following the short course, a musical recital by local organizers, an Arabic *majlis* in the seaview atrium of the KAUST library, and an *al fresco* seaside banquet opposite the KAUST Beacon.

1.3 Research Activity in Domain Decomposition at DD28

We inventory contemporary research activities in domain decomposition by examining the content of the DD28 conference and the present proceedings.

The plenary presentations are selected by the scientific committee. At DD28, there were 11 plenary talks, and 7 presenters submitted a paper to the proceedings.

Lanser and Klawonn devise a nonlinear Schwarz preconditioner that makes use of two levels of discretization, as is traditional in linear domain decomposition, and apply it to the Navier-Stokes equations in primitive variables for a driven cavity, where it compares very favorably to a Newton-Krylov-Schwarz approach using a two-level linear preconditioner for the global Jacobian.

Liu *et al.* devise a nonlinear preconditioner with a focus on identification of troublesome degrees of freedom for nonlinear elimination inside of a global Newton iteration, and apply it to the Navier-Stokes equations in streamfunction-vorticity variables for a backward-facing step, extending the robustness of Newton to higher Reynolds numbers.

Park and Xu propose a new convergence theory for parallel subspace correction methods for smooth convex optimization, which allows weaker assumptions and provides better estimates on convergence rates; they apply it to the s-Laplacian.

Ciaramella *et al.* draw from Schwarz Waveform Relaxation to advance their "unmapped tent pitching" algorithm from DD27, which favors redundant computation to eliminate the mapping process of the mapped tent pitching algorithm, for the domain decomposition solution of 3D-in-space-plus-time hyperbolic problems.

Three of the seven invited talks explore non-standard bases induced by domain decomposition and motivated by convenient local computation:

Discacciati *et al.* build up a basis to a high dimensional problem by progressive enrichment in the style of an overlapping domain decomposition method, using a proper generalized decomposition (as opposed to proper orthogonal decomposition) and apply it to parameterized elliptic boundary value problems.

Sangalli *et al.* tackle isogeometric analysis with a Krylov solver preconditioned by possibly inexact, but fast subdomain preconditioners that exploit tensor product structure over subdomains whose bases need not form a direct sum of the global basis but may contain linearly dependent elements.

Scacchi *et al.* build a BCCD preconditioner for a virtual element discretization of saddle point problems discretized over very general polygonal or polyhedral shapes.

There were twelve minisymposia at DD28, with organizers as follows:

1. Nonlinear Preconditioning Techniques and Applications (Xiao-Chuan Cai, Axel Klawonn, Martin Lanser, and Li Luo)
2. HPC for Training Large Neural Network Models (Aleksandr Mikhalev and Ivan Oseledets)
3. Novel Solution Techniques: Theory and Applications (Blanca Ayuso De Dios and Susanne Brenner)
4. Domain Decomposition and Machine Learning Algorithms (Xiao-Chuan Cai, Alexander Heinlein, Axel Klawonn, and Janine Weber)
5. Solvers for Innovative PDE Discretizations and Applications (Luca Pavarino)
6. Schwarz Methods for Steady and Evolution Problems (Lahcen Laayouni and Martin Gander)
7. Time Parallel Time Integration (Thibaut Lunet and Martin Gander)
8. Domain Decomposition Methods for High Frequency Wave Propagation (Victorita Dolean and Martin Gander)

9. Coarse Spaces and Transmission Conditions (Serge Criekingen and Martin Gander)
10. Polygonal Finite Elements, Discontinuous Galerkin and Related Methods (Eun-Jae Park and Dongwook Shin)
11. Transmission Conditions in Domain Decomposition and Optimal Control (Liu-Di Lu and Martin Gander)
12. Solvers for Large-Scale Parallel Computations (Gabriel Wittum and Rolf Krause)

The majority of minisymposia focused on technique (discretization, decomposition, iterative solution, preconditioning (linear and nonlinear), coarse spaces, transmission and boundary conditions, time parallelization, optimization, machine learning, PINNs, etc.) while others majored in applications (fluid flow, linear and nonlinear elasticity, porous media flow, phase fields, fluid-structure interaction, Helmholtz and Maxwell equations, etc.).

The minisymposium papers submitted the proceedings reflect this mix between application focused contributions and domain decomposition technique-focused contributions.

A minor theme in the proceedings, but a major point of discussion at the conference, is porting domain decomposition techniques to computer architectures built from GPUs.

1.4 Domain Decomposition Methods in the Future

The present proceedings volume is a testimony that domain decomposition is a vibrant, active field of research, even as it matures. The Arabic *majlis* (community "town hall" meeting) held in the late afternoon of the third day of the conference raised many themes for discussion including:

- In what directions are trends in HPC architecture driving algorithms?
- How should HPC applications and algorithms be driving architecture?
- Are there fundamental limits to scaling, in the sense of diminishing or negative returns, for: explicit PDEs, implicit PDE, graphs, neural networks, etc.?
- What are the relative advantages of different types of decomposition: by domain, by parameters, by operators, by physics, etc.?
- Are there good divide-and-conquer preconditioning strategies for Hessians of second-order training algorithms in machine learning?
- Can we "machine-learn" new coarse spaces for domain decomposition preconditioners?
- Can we find better coarse spaces for time-harmonic waves?
- What is the state of the art for preconditioning nonsymmetric nonnormal matrices?
- What is the limit to scaling time parallelization?
- Can we generate new types of coarse spaces by stochastic means?

- Given the decadal drumbeat of discovery of optimal algorithms – FFT, multigrid, FMM, sparse grids, MLMC, hierarchical matrices, randomized algorithms – where should we be looking for the next optimal algorithm?

There are three driving forces that thrust this field forward: the mathematical tools of divide-conquer-and-combine, applications that demand computational approaches, and the ever increasing capability of manycore distributed-memory computers, which constitutes a prime means of placing needed computational power into the hands of scientists and engineers. New interdisciplinary researchers have been drawn into the community culture of `ddm.org` and we look forward eagerly to their further contributions.

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