Preface

The proceedings of the 29th International Conference on Domain Decomposition Methods (DD29) provide a comprehensive record of recent advances in the analysis, algorithmic development, large-scale implementation, and applications of domain decomposition methods in science and engineering.

The conference was hosted by the Politecnico di Milano in Milan, Italy, from June 23 to June 27, 2025, and was jointly organized by the Scientific Committee of ddm.org and the local organizing committee. Further information about the event can be found on the archival website at https://www.dd29.polimi.it.

DD29 brought together 308 participants from institutions across 29 countries, spanning Europe, Asia-Pacific, the Middle East, and North and South America, reflecting the broad international interest and active development in this field.

Background of the Conference Series

The International Conference on Domain Decomposition Methods has been hosted in seventeen countries across Asia, Europe, the Middle East, and North America since its inception in Paris in 1987. The first fourteen meetings were held annually; since DD15, the conference has taken place at approximately 18-month intervals. The twenty-ninth conference marked the second time the series was held in Italy, 33 years after DD6, which took place in Como in 1992.

Over nearly four decades, increasing demands for resolution and fidelity have driven the performance of simulations based on first-principles mathematical models in science and engineering from megaflop to exaflop scales, and data volumes from megabytes to exabytes—representing growth of twelve orders of magnitude in each case, or roughly three orders of magnitude per decade. In the past decade, the rapid emergence of machine learning in scientific and engineering applications has introduced comparable demands in computational performance and data storage. Lower levels of the simulation software stack have proven immediately beneficial for large-scale machine learning, whereas higher-level methodologies have yet to fully realize their potential to improve upon the largely brute-force paradigms that still

dominate learning and inference. As a consequence, the associated computational and energy costs remain substantial, limiting accessibility to cutting-edge resources. At the same time, hardware architectures optimized for machine learning offer significant, as yet underexploited, opportunities for traditional simulation. Increasingly, complex scientific and engineering challenges are most effectively addressed through the combined use of simulation and learning, leading to a convergence of previously distinct research communities.

With the number of floating-point-capable cores in leading scientific computing systems now exceeding ten million—compared to only hundreds at the inception of this conference series—the need for scalable, highly concurrent algorithms has become more pressing than ever. Modern high-end computing systems require investments on the order of hundreds of millions to a billion dollars, with substantial annual operating costs. In this context, even modest gains in algorithmic efficiency can translate into significant economic impact. While many methods have achieved optimal or near-optimal asymptotic parallel complexity, continued advances are needed to address challenges such as nonsymmetry, anisotropy, inhomogeneity, indefiniteness, and multiphysics coupling across heterogeneous models.

Beyond traditional forward problems, the scope of computational science has expanded to include optimization, inverse problems, data assimilation, and uncertainty quantification. In these settings, the forward problem—central to the early domain decomposition meetings—serves as an inner computational kernel and must therefore be solved with high efficiency and rapid convergence.

The DD conference series has always been rooted in mathematical analysis, while being strongly motivated by the efficient use of distributed-memory architectures for complex applications in science and engineering. As such, it has consistently attracted contributions from mathematicians, computer scientists, engineers, and domain scientists. Although domain decomposition methods are now central to large-scale parallel simulation, they also play a key role in more modest computational settings—for example, in coupling heterogeneous models or exploiting specialized solvers across subproblems, as in fluid–structure interaction and other multiphysics applications. As parallelism becomes ubiquitous and multiphysics modeling continues to expand, domain decomposition remains a vibrant and inherently interdisciplinary field.

While research on domain decomposition methods is presented at many venues, the International Conference on Domain Decomposition Methods remains the only regularly held international forum specifically dedicated to fostering interaction between theoreticians and practitioners engaged in the development, analysis, implementation, and application of these methods, as well as to integrating new ideas from related fields.

1. Paris, France, January 7–9, 1987
2. Los Angeles, USA, January 14–16, 1988
3. Houston, USA, March 20–22, 1989
4. Moscow, USSR, May 21–25, 1990
5. Norfolk, USA, May 6–8, 1991

6. Como, Italy, June 15–19, 1992
7. University Park, Pennsylvania, USA, October 27–30, 1993
8. Beijing, China, May 16–19, 1995
9. Ullensvang, Norway, June 3–8, 1996
10. Boulder, USA, August 10–14, 1997
11. Greenwich, UK, July 20–24, 1998
12. Chiba, Japan, October 25–29, 1999
13. Lyon, France, October 9–12, 2000
14. Cocoyoc, Mexico, January 6–11, 2002
15. Berlin, Germany, July 21–15, 2003
16. New York, USA, January 12–15, 2005
17. St. Wolfgang-Strobl, July 3–7, Austria 2006
18. Jerusalem, Israel, January 12–17, 2008
19. Zhangjiajie, China, August 17–22, 2009
20. San Diego, California, February 7–11, 2011
21. Rennes, France, June 25–29, 2012
22. Lugano, Switzerland, September 16–20, 2013
23. Jeju Island, Korea, July 5–10, 2015
24. Svalbard, Norway, February 6–10, 2017
25. St. John's, Canada, July 23–27, 2018
26. Hong Kong, China, December 7–12, 2020 (virtual)
27. Prague, Czech Republic, July 25–29, 2022
28. KAUST, Saudi Arabia, January 28-February 1, 2024
29. Milan, Italy, June 23-27, 2025

The proceedings of the conference series are online at the ddm.org website.

About the Twenty-ninth Conference

The 29th conference was organized over six days – five-day conference (June 23-27, 2025) plus a well-attended short course on the preceding day – and featured 305 presentations, of five types:

- 12 invited plenary talks, selected by the International Scientific Committee;
- 263 talks invited by minisymposia organizers, arranged around a common topic, and grouped into 31 minisymposia;
- 9 contributed talks, grouped into 4 sessions;
- 17 posters, grouped around a reception and preceded by a poster blitz session;
- 4 lectures.

The conference employed ten parallel sessions.

The Scientific Committee members overseeing the 29th conference were:

1. Petter Bjørstad, University of Bergen, Norway
2. Susanne Brenner, Louisiana State University, USA
3. Xiao-Chuan Cai, University of Macau, China

4. Victorita Dolean, TU-Eindhoven, Netherlands
5. Martin Gander, University of Geneva, Switzerland
6. Laurence Halpern, University of Paris 13, France (Chair)
7. David Keyes, KAUST, Saudi Arabia
8. Hyea Hyun Kim, Kyung Hee University, Korea
9. Axel Klawonn, University of Cologne, Germany
10. Ralf Kornhuber, Freie Universität Berlin, Germany
11. Ulrich Langer, University of Linz, Austria
12. Luca Pavarino, University of Pavia, Italy
13. Olof Widlund, Courant Institute, New York University, USA
14. Jinchao Xu, KAUST, Saudi Arabia

The local organizing committee included was composed by 11 members, including one honorary chair and three co-chairs.

1. Prof. Paola F. Antonietti, MOX Lab - Dipartimento di Matematica, Politecnico di Milano, Milan, Italy (**co-chair**)
2. Prof. Silvia Bertoluzza, Istituto di Matematica Applicata e Tecnologie Informatiche del CNR, Pavia, Italy
3. Prof. Gabriele Ciaramella MOX Lab - Dipartimento di Matematica, Politecnico di Milano, Milan, Italy (**co-chair**)
4. Prof. Luca Pavarino, Dipartimento di Matematica, Università degli Studi di Pavia, Pavia, Italy (**co-chair**)
5. Prof. Simone Scacchi, Dipartimento di Matematica, Università di Milano, Milano, Italy
6. Prof. Lourenco Beirao da Veiga, Dipartimento di Matematica e Applicazioni, Università di Milano Bicocca, Milano, Italy
7. Prof. Claudio Canuto, Dipartimento di Scienze Matematiche, Politecnico di Torino, Torino, Italy
8. Prof. Paola Gervasio, DICATAM, Università degli Studi di Brescia, Brescia, Italy
9. Prof. Alfio M. Quarteroni, MOX Lab - Dipartimento di Matematica, Politecnico di Milano, Milano, Italy (**honorary chair**)
10. Prof. Christian Vergara, LABS, Dipartimento di Chimica, Materiali e Ingegneria Chimica, Politecnico di Milano, Milano, Italy

DD29 had eight sponsoring academic partners, namely IMATI Pavia, Università di Milano Bicocca, Università degli Studi di Brescia, Università degli Studi di Milano, Universtità di Pavia, Politecnico di Milano (Dipartimento di Matematica - MOX Lab, Dipartimento di Chimica LaBS), Politecnico di Torino, and six sponsors, namely Department of Excellence DMAT (Politecnico di Milano), SemMat (Seminario Matematico di Brescia), Indam GNCS, M3E (Matematical Methods and Models for Engineering), ERC (European Research Council), Centro Nazionale di Ricerca in High Performance Computing, Big Data, and Quantum Computing funded by European Union-NextGenerationEU and by MUR, Dipartimento di Eccellenza 2023-27.

Social events included a welcoming reception, guided tours at the Pinacoteca di Brera and Basilica di Sant'Ambrogio, and an al fresco seaside banquet within the Garden of the Politecnico di Milano Rectorate.

Research Activity in Domain Decomposition at DD29

We survey contemporary research in domain decomposition through the contributions presented at DD29 and documented in these proceedings.

The plenary presentations are selected by the scientific committee. At DD29, there were 12 plenary talks:

1. Santiago Badia (Monash University):
   Recent advances in unfitted finite element methods
2. Erin Carson (Charles University):
   Mixed Precision Matrix Computations
3. Stéphanie Chaillat (CNRS – Laboratoire POems):
   Fast Boundary Element Methods Beyond Homogeneous Media: Towards Realistic Wave Propagation Modeling
4. Björn Engquist (The University of Texas Austin):
   Domain Decomposition for Molecular Dynamics
5. Patrick Farrell (University of Oxford):
   Fast high-order solvers on simplices for the de Rham complex
6. Marlis Hochbruck (Karlsruhe Institute of Technology -KIT):
   Higher-order locally implicit methods for linear Maxwell's equation
7. Pierre Jolivet (Sorbonne Université, CNRS, LIP6):
   Robust overlapping Schwarz methods and their applications
8. Alena Kopanicakova (University of Toulouse):
   Training of Deep Neural Networks Using Multilevel and Domain-Decomposition Methods
9. Jan Mandel - Olof B. Widlund Prize (University of Colorado):
   Domain Decomposition and Beyond
10. Ilario Mazzieri (Politecnico di Milano):
    Efficient space-time methods for solving wave propagation challenges
11. Nicole Spillane (CNRS – Ecole polytechnique):
    Preconditioning, weighting and deflation applied to non-symmetric linear systems
12. Xiaowen Xu (Institute of Applied Physics and Computational Mathematics – Beijing):
    Divide and Conquer: Explorations in Developing Intelligent AMG Solver for Sequences of Large-Scale Sparse Linear Systems

There were 31 minisymposia at DD29, with organizers as follows:

1. MS01 – Domain decomposition methods for wave-type problems
   Organizers: T. Buchholz, G. Ciaramella, R. Maier

2. MS02 – Recent advancements in preconditioning techniques for high-performance computing
   Organizers: C. Janna, A. Franceschini
3. MS03 – Advanced discretization methods and solvers for coupled problems
   Organizers: D. Grappein, T. Vanzan
4. MS04 – Iterative and direct solvers for optimization and inverse problems
   Organizers: M. Bonazzoli, L. Lu, T. Vanzan
5. MS05 – Domain decomposition methods for multiphysics and heterogeneous problems
   Organizers: M. Botti, I. Mazzieri
6. MS06 – Domain decomposition based methods for cardiovascular simulation
   Organizers: M. Bucelli, F. Renzi, C. Vergara, A. Quarteroni
7. MS07 – Efficient parallel solvers for non-Hermitian and indefinite problems
   Organizers: V. Dolean, V. Dwarka, P. Marchand, N. Spillane
8. MS08 – Advanced space-time discretization methods: theory, solvers and applications
   Organizers: I. Mazzieri, A. Moiola, G. Sangalli
9. MS09 – Fast solution techniques for polytopal methods and related applications: A Nemesis mini-symposium
   Organizers: P. Antonietti, L. Beirao da Veiga, D. A. Di Pietro, J. Droniou
10. MS10 – Scalable domain decomposition solvers for cellular reaction diffusion models in computational biology
    Organizers: E. Centofanti, N. M. M. Huynh, S. Scacchi
11. MS11 – Robust parallel solvers for linear, nonlinear, and multiphysics problems
    Organizers: S. Gong, W. Wang, C. Zhang
12. MS12 – Numerical methods for time-harmonic wave propagation problems
    Organizers: P.-H. Cocquet, M. J. Gander
13. MS13 – Machine learning-enhanced solvers: Advances in domain decomposition and operator learning for PDEs
    Organizers: V. Dolean, M. Verani
14. MS14 – Continuous and discrete approaches to DDM: solvers and preconditioners with applications
    Organizers: P. J. Lucero Lorca, C. McCoid, M. Outrata
15. MS15 – Efficient numerical methods with machine learning for PDEs with singularities
    Organizers: A. Heinlein, X. Xu, J. Zou
16. MS16 – Robust and scalable algorithms for problems beyond benchmarking in computational science and engineering
    Organizers: R. Chen, L. Luo, L. Pavarino
17. MS17 – Efficient solvers for Maxwell equations
    Organizers: V. Dolean, A. Heinlein, S. Kinnewig, T. Wick
18. MS18 – Discrete duality finite volume methods and applications
    Organizers: F. Hubert, S. Krell, K. Mikula
19. MS19 – Domain decomposition methods and model order reduction techniques
    Organizers: S. Deparis, M. Discacciati, P. Gervasio, M. Giacomini

20. MS20 – Localized model order reduction, multiscale and domain decomposition Methods
    Organizers: M.J. Gander, M. Ohlberger, S. Rave
21. MS21 – Multilevel domain decomposition methods and coarse spaces
    Organizers: M.J. Gander, A. Heinlein
22. MS22 – Robust and efficient domain decomposition methods for multiscale and multi-physics problems
    Organizers: E. Chung, H.H. Kim
23. MS23 – New advances in parallel-in-time methods
    Organizers: B. Delourme, L. Halpern, F. Kwok, J. Salomon
24. MS24 – Preconditioning strategies for saddle point problems
    Organizers: P. Brubeck Martinez, U. Zerbinati
25. MS25 – Solution techniques for nonstandard approximations: Theory and Applications
    Organizers: B. Ayuso de Dios, S. C. Brenner
26. MS26 – Transmission conditions in domain decomposition for steady and evolution problems with applications
    Organizers: M.J. Gander, Y. Xu
27. MS27 – Domain decomposition methods and nonlinear preconditioning for nonlinear PDEs
    Organizers: X.-C. Cai, A. Klawonn, M. Lanser
28. MS28 – Learning-based algorithms and applications
    Organizers: X.-C. Cai, A. Klawonn, J. Weber
29. MS29 – Next-geneneration numerical methods for computational life sciences
    Organizers: F. Bonizzoni, M. Corti, I. Fumagalli, S. Pagani
30. MS30 – Methods of Reflection in the context of domain decomposition with applications
    Organizers: M.J. Gander, L. Halpern
31. MSHI – High-performance computing and industrial applications
    Organizers: P. Antonietti, M. Botti, G. Ciaramella.

The plenary lectures and minisymposia at DD29 covered a broad spectrum of contemporary research topics in domain decomposition, reflecting both methodological advances and emerging applications. A large portion of the contributions focused on numerical techniques, including discretization methods (finite elements, boundary elements, polytopal and structure-preserving schemes), iterative and direct solvers, preconditioning strategies for symmetric and non-symmetric systems, multilevel and coarse space techniques, transmission conditions, and parallel-in-time and space-time methods. Increasing attention was devoted to machine learning and data-driven approaches, such as multilevel training strategies, operator learning, and the integration of domain decomposition with neural networks. At the same time, many sessions emphasized challenging applications, including wave propagation, electromagnetics (Maxwell equations), molecular dynamics, cardiovascular and biological systems, multiphysics and multiscale problems, as well as optimization, inverse problems, and uncertainty quantification. The contributions collected in these proceedings reflect this balance between methodological developments and

application-driven research. A further emerging theme, highlighted both in plenary talks and minisymposia discussions, is the interplay between simulation and machine learning, together with the need to design scalable algorithms capable of exploiting modern high-performance computing architectures.

Acknowledgments

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